Neutrino Physics 2010: Assignment 2

(Given 18/03/2010, To be submitted 05/04/2010)

1. Let two neutrino flavours, ν_α and ν_β, mix to form two mass eigenstates ν_1 and ν_2, such that $\nu_\alpha = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$, $\nu_\beta = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$. Let $\nu(0) = \nu_\alpha$. Find the probability of detecting a ν_β after the neutrino has travelled a distance L in vacuum, when the heavier neutrino ν_2 decays with a lifetime τ to particles that remain undetected.

   Plot the conversion probability as a function of L, and find its value at L much greater than the coherence length, i.e. after the wavepackets have separated. (Use any appropriate values for kΔm^2 and θ that will bring out the main features in the plot.)

2. If the solution to atmospheric neutrinos was via oscillations to sterile neutrinos ($\nu_\mu \leftrightarrow \nu_s$),

   (a) Plot $P_{\mu\mu}$ as a function of the zenith angle Θ for three values of energy: $E = 0.2, 2, 20$ GeV (on the same plot). Show the numerical values on both the axes explicitly.

   (b) Plot the up-down asymmetry $(U - D)/(U + D)$ as a function of energy. The "up" events are defined as those with $\cos\Theta < -0.2$ and the "down" events are those with $\cos\Theta > 0.2$. You may do the integrals numerically. Assume that the atmospheric neutrino flux is isotropic.

   Use the paremeter values $\Delta m^2 = 2.4 \times 10^{-3}$ eV^2 and $\theta = 45°$. Take the density of the Earth to be constant at 5 g/cc. Neglect the density of the atmosphere.

   Compare the results with those of the ν_μ-ν_τ oscillation problem in the earlier assignment.

3. Consider two-neutrino mixing between ν_e and ν_μ, with $\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$, $\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$. Let the neutrinos travel through matter, so that they experience a matter potential V_c. Plot the values of $\sin^2 2\theta_m$ and Δ_m as a function of V_c for $E = 10$ MeV. Take $m_1 = 0.10000$ eV, $m_2^2 = m_1^2 + \Delta m^2$, and $\theta = 20°$, with

   (a) $\Delta m^2 = 10^{-4}$ eV^2

   (b) $\Delta m^2 = -10^{-4}$ eV^2

   Show the numerical values on both the axes explicitly. What would happen to the $\sin^2 2\theta_m$ plots if the mixing angle in vacuum were $\theta = 1°$?

4. Plot the survival probability of solar neutrinos as a function of energy (0.5 MeV $< E <$ 15 MeV) for the following scenarios:

   - VAC: $\Delta m^2 = 10^{-9}$ eV^2, $\theta = 45°$
   - SMA: $\Delta m^2 = 5 \times 10^{-6}$ eV^2, $\theta = 2°$
   - LOW: $\Delta m^2 = 10^{-7}$ eV^2, $\theta = 40°$
   - LMA: $\Delta m^2 = 8 \times 10^{-5}$ eV^2, $\theta = 32°$

   For SMA, LMA and LOW, use $P_f = Exp(-\pi\gamma/2)$, with $\gamma \equiv \frac{\Delta m^2}{2E} \frac{sin^2 2\theta}{\cos 2\theta} \frac{1}{|V_C'/V_C|_{\text{resonance}}}$.
   The density profile of the Sun is

   $$\rho(r) = 245 \times 10^{-10.54(r/r_\odot)} \quad \text{cm}^{-3}$$

   with $r_\odot = 700,000$ km. For VAC, use the non-adiabatic limit for P_f.

5. For neutrinos with energy 10 MeV, calculate the day-night asymmetry if the solution was

   - SMA
   - LMA.

   Use the parameters from the problem above. Take the Earth to have an uniform density of 5 g/cc.