Neutrino Physics 2010: Assignment 3

(Given 14/04/2010, To be submitted 03/05/2010)

1. Consider a short baseline experiment, where only one Δm^2 is relevant (single mass-squared dominance approximation). Take the standard parameterization of U_{PMNS}, assume no CP violation.

   (a) Calculate the survival probability $P_{\mu\mu} \equiv P_{\nu_\mu \to \nu_\mu}$. Comparing it with the standard two-neutrino oscillation form, determine the "effective disappearance mixing angle" $\theta_{\mu\mu}^{\rm eff}$.

   (b) Calculate the conversion probability $P_{\mu e} \equiv P_{\nu_\mu \to \nu_e}$. Comparing it with the standard two-neutrino oscillation form, determine the "effective appearance mixing angle" $\theta_{\mu e}^{\rm eff}$.

   (c) Show the equi-probability contours of $P_{\nu_\mu \to \nu_e}$ in the plane of $\log(\Delta m^2/{\rm eV}^2)$ vs. $\sin^2 2\theta_{eff}$, with $P_{\mu e} = 0, 0.2, 0.5$.
   Use $L = 10$ MeV, $E = 1$ km. Indicate the effects of decoherence at high Δm^2.

2. We want to calculate how Δ and θ change in the presence of matter with constant potential V_c, using the perturbation theory technique. This is a warm-up problem with two flavours.

   (a) Calculate the effective Hamiltonian in the flavour basis,
   $H_f = U H_{vac} U^\dagger + V$, where

   $$H_{vac} = \begin{pmatrix} 0 & 0 \\ 0 & 2\Delta \end{pmatrix} , \quad V = \begin{pmatrix} V_c & 0 \\ 0 & 0 \end{pmatrix}$$

   and U is the neutrino mixing matrix.

   (b) Separate H_f as $H_0 + H_1$, where $H_0 = H_f(V_c = 0)$.

   (c) Find eigenvalues $\epsilon_j^{(0)}$ and corresponding eigenvectors $v_j^{(0)}$ of H_0.

   (d) Using perturbation theory techniques, calculate the first order corrections to the eigenvalues and eigenvectors of H_0, i.e. calculate $\epsilon_j^{(1)}$ and $v_j^{(1)}$.

   (e) Find the net eigenvalues $\epsilon_j = \epsilon_j^{(0)} + \epsilon_j^{(1)}$, hence determine Δ_m.

   (f) Find the net eigenvectors $v_j = v_j^{(0)} + v_j^{(1)}$. Normalize v_j (only keep terms to first order in the small parameter V_c) and determine the new mixing matrix U_m. Hence find the mixing angle θ_m.

   (g) Compare the expressions for Δ_m and θ_m with the known exact expressions.

3. We want to calculate the conversion probability $P_{\nu_e \to \nu_\mu}$ in three flavours in matter with constant potential V_c.

   (a) Calculate the effective Hamiltonian in the flavour basis, $H_f = U H_{vac} U^\dagger + V$, where

$$H_{vac} = 2\Delta_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad V = 2\Delta_{31} \begin{pmatrix} \widehat{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

   Here $\widehat{A} \equiv V_c/(2\Delta_{31}), \alpha \equiv \Delta_{21}/\Delta_{31}$, and U is the neutrino mixing matrix.

   (b) Separate H_f as $H_0 + H_1$, where $H_0 = H_f(\theta_{13} = 0, \alpha = 0)$.

   (c) Find eigenvalues $\epsilon_j^{(0)}$ and corresponding eigenvectors $v_j^{(0)}$ of H_0.

   (d) Using perturbation theory techniques, calculate the first order corrections to the eigenvalues and eigenvectors of H_0, i.e. calculate $\epsilon_j^{(1)}$ and $v_j^{(1)}$.

   (e) Determine the net eigenvalues $\epsilon_j = \epsilon_j^{(0)} + \epsilon_j^{(1)}$ and the net eigenvectors $v_j = v_j^{(0)} + v_j^{(1)}$.

   (f) Normalize v_j (only keep terms to first order in the small parameters θ_{13} and α) and determine the new mixing matrix U_m.

   (g) Using the net conversion probability

$$P_{\alpha\beta} = \left| \sum_j (U_m)^*_{\beta j} (U_m)_{\alpha j} e^{-i\epsilon_j L} \right|^2 ,$$

   calculate $P_{e\mu}$.

4. Let there be a sterile neutrino, which is heavier than all the active neutrinos, with $\Delta m^2 \approx 1$ eV^2. The net 4×4 neutrino mixing matrix is given by

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

   Calculate

   (a) $P_{\mu\mu}$ relevant for atmospheric neutrinos. Simplify to the extent possible by neglecting / averaging out appropriate terms.

   (b) $P_{\mu e}$ relevant for the LSND experiment. Simplify to the extent possible by neglecting / averaging out appropriate terms.

   (c) P_{ee} relevant for the KamLAND experiment. Simplify to the extent possible by neglecting / averaging out appropriate terms.