Neutrino Physics 2010: Assignment 4

(Given 10/05/2010, To be submitted 25/05/2010)

1. The Lagrangian for a boson A with mass m_A is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu$$

   (a) Find the equation of motion satisfied by A_α.

   (b) Show that the first term is invariant under the relevant gauge transformation, but the second term is not.

2. Let the electroweak symmetry be broken as

$$\langle \phi_2 \rangle = \langle \phi_3 \rangle = \langle \phi_4 \rangle = 0 \ , \quad \langle \phi_1 \rangle = w/\sqrt{2}$$

   (a) Determine the masses of W, Z bosons and the photon A. Calculate the Weinberg angle θ_W.

   (b) Show that electric charge is not conserved. Which is the quantity that is conserved instead (write in terms of T_3 and Y) ?

3. In the Dirac representation

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \ , \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

   show that $C \equiv i\gamma^2\gamma^0$ satisfies all the requirements of a CP-conjugate operator, i.e.

   (a) Unitarity

   (b) Matching condition $C\gamma^{\mu T} = -\gamma^\mu C$

   (c) Antisymmetry $C = -C^T$

4. Given the current 3σ bounds on neutrino mixing parameters, show the allowed values of $|m_{\beta\beta}|$ (the effective Majorana mass measured in neutrinoless double beta decay) as a function of m_0 (the mass of the lightest neutrino). You may find it convenient to draw a scatter plot. On the same plot, with different symbols, show the allowed regions for

   (a) Normal mass ordering

   (b) Inverted mass ordering

5. In Type-II seesaw mechanism, one adds three Higgs particles $\Delta_0, \Delta_-, \Delta_{--}$ to the standard model, which form a triplet under $SU(2)_L$. Draw all the vertices involving $\Delta_0, \Delta_-, \Delta_{--}$ and other standard model particles: (the strengths of vertices not needed, as long as they are nonzero)

   (a) Before electroweak symmetry breaking
   (b) After electroweak symmetry breaking

6. Consider the seesaw mechanism with three left handed neutrinos and n right handed neutrinos. Let the neutrino Dirac matrix be $[M_D]_{3\times n}$ and the Majorana matrix for the right-handed neutrinos be $[M_R]_{n\times n}$. The effective neutrino mass matrix is then
$$\mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} .$$
   We want to block-diagonalize $\mathcal{M}$ by a unitary matrix $\mathcal{U}$ such that
$$\mathcal{U}^T \mathcal{M} \mathcal{U} = M^B = \begin{pmatrix} [M_1]_{3\times 3} & [0]_{3\times n} \\ [0]_{n\times 3} & [M_2]_{n\times n} \end{pmatrix} .$$

   (a) Show that the matrix
$$\mathcal{U} \equiv \begin{pmatrix} \left[1 - \frac{WW^\dagger}{2}\right]_{3\times 3} & [W]_{3\times n} \\ \left[-W^\dagger\right]_{n\times 3} & \left[1 - \frac{W^\dagger W}{2}\right]_{n\times n} \end{pmatrix}$$
   is unitary, as long as we neglect terms of $\mathcal{O}(W^3)$. This is OK since from the one-flavour seesaw, we expect $W \sim m_D/m_R \ll 1$

   (b) Calculate M^B, keeping terms up to $\mathcal{O}(m_D W) \sim \mathcal{O}(M_R W W)$ and neglecting the higher order ones.

   (c) Determine the "mixing angle matrix" W, the "light neutrino mass matrix" M_1 and the "heavy neutrino mass matrix" M_2 in terms of the matrices m_D and M_R. Note that matrices do not commute, so the order of multiplication needs to be taken care of.

7. If neutrino Majorana mass matrix M_M in the flavour basis is μ–τ symmetric, show that it can be diagonalized by U_{PMNS} with $\theta_{23} = 45°$ and $\theta_{13} = 0$. Calculate θ_{12}.

8. If the neutrino mass matrix is of the form
$$M = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix} ,$$
   and $\Delta m^2_{21} = 0$, write the neutrino masses in terms of $a \equiv \sqrt{\Delta m^2_{31}}$, for

   (a) normal mass ordering,
   (b) inverted mass ordering.