Neutrino Physics 2010: Midterm Examination

10 April 2010, 10:00 am

(Max points: 60)

1. Calculate the difference in speeds of ν_1 and ν_2, both having energy $E = 10$ MeV ($\gg m_1, m_2$), given $\Delta m^2_{21} = 8 \times 10^{-5}$ eV^2.

   (5 points)

2. Write down the mass eigenstate ν_3 as a linear combination of flavour eigenstates ν_e, ν_μ, ν_τ, in terms of the mixing angles θ_{ij}, CP violating Dirac phase δ, Majorana phases ϕ_i and flavour phases χ_i.

   (5 points)

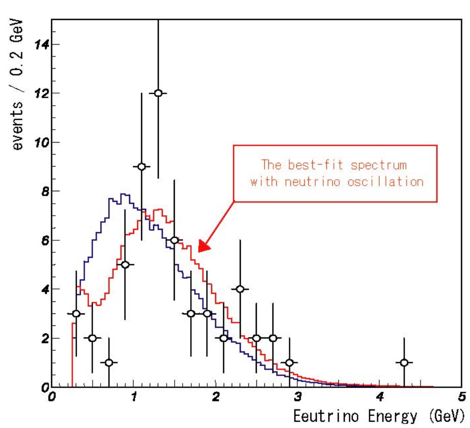
3. The expected flux of ν_e with $E > 5$ MeV at Super-Kamiokande is $\Phi \approx 10^7/\text{cm}^2/\text{sec}$. Taking the average cross section of neutrino-electron elastic scattering in this energy range to be $\sigma \approx 10^{-47}$ m^2, estimate the number of neutrino events expected per day (without oscillations) inside this 40 kiloton detector full of water.

   (10 points)

4. An astrophysical source emits neutrons, which decay on their way, giving rise to $\bar{\nu}_e$. These antineutrinos travel for intergalactic distances before reaching the Earth. Calculate the ratio of fluxes of $\bar{\nu}_e, \bar{\nu}_\mu$ and $\bar{\nu}_\tau$ arriving at the Earth. Assume $\theta_{13} = 0$.

   (10 points)

5. The figure shows data from the K2K experiment, where ν_μ produced at an accelerator are detected a distance L away. The dark line shows the expected spectrum in the absence of oscillations. Given that the data give the best fit at $\Delta m^2 \approx 2.5 \times 10^{-3}$ eV^2, estimate the distance between the accelerator and the detector.



   (10 points)

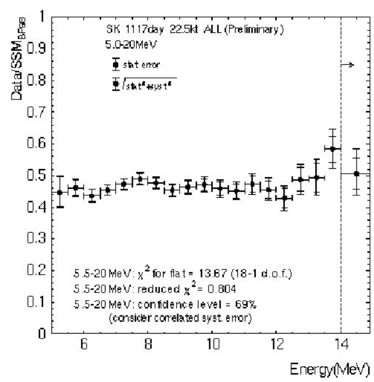
6. The MSW resonance layer is the region for which $\sin^2 2\theta_m > 0.5$. The density of the Sun is parameterized as $\rho = \rho_0 \exp(-r/r_0)$. Calculate the width of the MSW resonance layer inside the Sun as a multiple of r_0. Use $\sin 2\theta = 0.6$.

(10 points)

7. At Super-Kamiokande, solar neutrinos are detected through the neutrino-electron elastic scattering process.
(a) Draw all the Feynman diagrams that contribute to this reaction.
(b) The figure shows Data/SSM for solar neutrinos, measured at Super-Kamiokande. The data give a good fit to an energy-independent value

$$\text{Data/SSM} = 0.47 \pm 0.01.$$

Given that the solar neutrino solution is LMA, determine the value of the solar mixing angle $\theta_\odot$ (and the error on it).



(10 points)

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Useful expressions:

$$\widetilde{U}_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\begin{aligned} \sigma(\nu_e e^- \to \nu_e e^-) &= 9.5 \times 10^{-49}\ \mathrm{m}^2 (E/MeV) \\ \sigma(\nu_\mu e^- \to \nu_\mu e^-) &= 1.6 \times 10^{-49}\ \mathrm{m}^2 (E/MeV) \end{aligned}$$