Neutrino Physics: Lecture 10

Three flavour mixing with complex mixing matrix

Amol Dighe

Department of Theoretical Physics
Tata Institute of Fundamental Research

Mar 23, 2010

Outline

1. Parameterization of complex U_{PMNS}
2. Neutrinos $\leftrightarrow$ antineutrinos and CP violation
3. Matter-induced CP violation

Pontecorvo
Maki
Nakagawa
Sakata

Outline

Counting number of complex phases

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- Complex 3×3 matrix: 18 parameters
- Unitarity: $U^\dagger_{PMNS} U = I$: 9 conditions $\Rightarrow$ 9 parameters left
- Only 3 angles of rotation $(\theta_{12}, \theta_{23}, \theta_{13})$
- 6 parameters must be complex phases

Counting number of complex phases

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- Complex 3×3 matrix: 18 parameters
- Unitarity: $U^\dagger_{PMNS} U = I$: 9 conditions $\Rightarrow$ 9 parameters left

- Only 3 angles of rotation $(\theta_{12}, \theta_{23}, \theta_{13})$
- 6 parameters must be complex phases

A general parameterization

$$U_{PMNS} = \Phi(\chi_1, \chi_2, \chi_3) R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) \Phi(\phi_1, \phi_2, 0)$$

$$\Phi(\chi_1, \chi_2, \chi_3) = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix}, \Phi(\alpha_1, \alpha_2, 0) = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\widetilde{U}_{PMNS} \equiv R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_i \equiv \cos\theta_i$, $s_i \equiv \sin\theta_i$

A general parameterization

$$U_{PMNS} = \Phi(\chi_1,\chi_2,\chi_3) R_{23}(\theta_{23}) U_{13}(\theta_{13},\delta) R_{12}(\theta_{12}) \Phi(\phi_1,\phi_2,0)$$

$$\Phi(\chi_1,\chi_2,\chi_3) = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix}, \quad \Phi(\phi_1,\phi_2,0) = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\widetilde{U}_{PMNS} \equiv R_{23}(\theta_{23}) U_{13}(\theta_{13},\delta) R_{12}(\theta_{12}) =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_i \equiv \cos\theta_i$, $s_i \equiv \sin\theta_i$

A general parameterization

$$U_{PMNS} = \Phi(\chi_1, \chi_2, \chi_3) R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) \Phi(\phi_1, \phi_2, 0)$$

$$\Phi(\chi_1, \chi_2, \chi_3) = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix}, \quad \Phi(\alpha_1, \alpha_2, 0) = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\widetilde{U}_{PMNS} \equiv R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_i \equiv \cos\theta_i, \; s_i \equiv \sin\theta_i$

The six phases

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix} \cdot \widetilde{U}_{PMNS} \cdot \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

All ν_i can be multiplied by a common phase $\Rightarrow$ No need of ϕ_3

Flavour phases (also "unphysical phases") χ_i

IF phases of flavour eigenstates ν_α can be chosen,

χ_1, χ_2, χ_3 can be rotated away to zero.

Majorana phases ϕ_i

IF phases of mass eigenstates ν_i can be chosen,

ϕ_1, ϕ_2 can be rotated away to zero.

Dirac phase δ

Cannot be rotated away, observable in oscillation experiments

The six phases

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix} \cdot \widetilde{U}_{PMNS} \cdot \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

All ν_i can be multiplied by a common phase $\Rightarrow$ No need of ϕ_3

Flavour phases (also "unphysical phases") χ_i

IF phases of flavour eigenstates ν_α can be chosen, independently

χ_1, χ_2, χ_3 can be rotated away to zero.

Majorana phases ϕ_i

IF phases of mass eigenstates ν_i can be chosen,

ϕ_1, ϕ_2 can be rotated away to zero.

Dirac phase δ

Cannot be rotated away, observable in oscillation experiments

The six phases

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix} \cdot \widetilde{U}_{PMNS} \cdot \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

All ν_i can be multiplied by a common phase $\Rightarrow$ No need of ϕ_3

Flavour phases (also "unphysical phases") χ_i

IF phases of flavour eigenstates ν_α can be chosen,

χ_1, χ_2, χ_3 can be rotated away to zero.

Majorana phases ϕ_i

IF phases of mass eigenstates ν_i can be chosen,

ϕ_1, ϕ_2 can be rotated away to zero.

Dirac phase δ

Cannot be rotated away, observable in oscillation experiments

The six phases

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix} \cdot \widetilde{U}_{PMNS} \cdot \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

All ν_i can be multiplied by a common phase $\Rightarrow$ No need of ϕ_3

Flavour phases (also "unphysical phases") χ_i

IF phases of flavour eigenstates ν_α can be chosen,
χ_1, χ_2, χ_3 can be rotated away to zero.

Majorana phases ϕ_i

IF phases of mass eigenstates ν_i can be chosen,
ϕ_1, ϕ_2 can be rotated away to zero.

Dirac phase δ

Cannot be rotated away, observable in oscillation experiments

Oscillation probability and phases

$$P_{\alpha\beta} = \delta_{\alpha\beta} - \sum_{i<j} 4\mathrm{Re}(\Box_{\alpha\beta ij}) \sin^2(\Delta_{ji}) - 2\sum_{i<j} \mathrm{Im}(\Box_{\alpha\beta ij}) \sin(2\Delta_{ji})$$

$$\Box_{\alpha\beta ij} \equiv U_{\alpha_i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}$$

- $\Box_{\alpha\beta ij}$ not affected by $\chi_1, \chi_2, \chi_3, \phi_1, \phi_2$ $\Rightarrow$ Rephase invariant
- Only the Dirac phase δ survives
- Valid even for neutrinos propagating in matter
- For oscillation experiments, $\widetilde{U}_{PMNS}$ suffices

Oscillation probability and phases

$\nu_\alpha \to \nu_\beta$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - \sum_{i<j} 4\mathrm{Re}(\square_{\alpha\beta ij}) \sin^2(\Delta_{ji}) - 2\sum_{i<j} \mathrm{Im}(\square_{\alpha\beta ij}) \sin(2\Delta_{ji})$$

$$\square_{\alpha\beta ij} \equiv U_{\alpha_i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}$$

- $\square_{\alpha\beta ij}$ not affected by $\chi_1, \chi_2, \chi_3, \phi_1, \phi_2 \Rightarrow$ Rephase invariant
- Only the Dirac phase δ survives
- Valid even for neutrinos propagating in matter
- For oscillation experiments, $\widetilde{U}_{PMNS}$ suffices

$R_{23}(\theta_{23})\, U_{13}(\theta_{13}, \delta)\, R_{12}(\theta_{12})$

Outline

1. Parameterization of complex U_{PMNS}
2. Neutrinos $\leftrightarrow$ antineutrinos and CP violation
3. Matter-induced CP violation

U_{PMNS} for antineutrinos

CPT conservation:

$$\langle \nu_\alpha | \nu_i \rangle = \langle \bar{\nu}_i | \bar{\nu}_\alpha \rangle = \langle \bar{\nu}_\alpha | \bar{\nu}_i \rangle^*$$

$U_{\alpha i}$ → ... → $\bar{U}_{\alpha i}^*$

$$U_{\alpha i} = \overline{U}^*_{\alpha i}$$

$U_{PMNS} \leftrightarrow \overline{U}_{PMNS}$

- Flavour phases $\chi_i \leftrightarrow -\chi_i$
- Majorana phases $\phi_i \leftrightarrow -\phi_i$
- Dirac phase $\delta \leftrightarrow -\delta$
- Mixing angles $\theta_{ij} \leftrightarrow \theta_{ij}$
- Plaquette $\square_{\alpha\beta ij} \leftrightarrow \square^*_{\alpha\beta ij}$

U_{PMNS} for antineutrinos

CPT conservation:

$$\langle \nu_\alpha | \nu_i \rangle = \langle \bar{\nu}_i | \bar{\nu}_\alpha \rangle = \langle \bar{\nu}_\alpha | \bar{\nu}_i \rangle^*$$

$$U_{\alpha i} = \overline{U}^*_{\alpha i}$$

$U_{PMNS} \leftrightarrow \overline{U}_{PMNS}$

- Flavour phases $\chi_i \leftrightarrow -\chi_i$
- Majorana phases $\phi_i \leftrightarrow -\phi_i$
- Dirac phase $\delta \leftrightarrow -\delta$
- Mixing angles $\theta_{ij} \leftrightarrow \theta_{ij}$
- Plaquette $\square_{\alpha\beta ij} \leftrightarrow \square^*_{\alpha\beta ij}$

Survival probability for neutrinos vs. antineutrinos

$\nu_\alpha \to \nu_\beta$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - \sum_{i<j} 4\mathrm{Re}(\Box_{\alpha\beta ij}) \sin^2(\Delta_{ji}) - 2\sum_{i<j} \mathrm{Im}(\Box_{\alpha\beta ij}) \sin(2\Delta_{ji})$$

$\overline{\nu_\alpha} \to \overline{\nu_\beta}$

$$\overline{P}_{\alpha\beta} = \delta_{\alpha\beta} - \sum_{i<j} 4\mathrm{Re}(\Box_{\alpha\beta ij}) \sin^2(\Delta_{ji}) + 2\sum_{i<j} \mathrm{Im}(\Box_{\alpha\beta ij}) \sin(2\Delta_{ji})$$

CP violating quantity

$$P_{\alpha\beta} - \overline{P}_{\alpha\beta} = -4\sum_{i<j} \mathrm{Im}(\Box_{\alpha\beta ij}) \sin(2\Delta_{ji})$$

Jarlskog invariant and CP violation

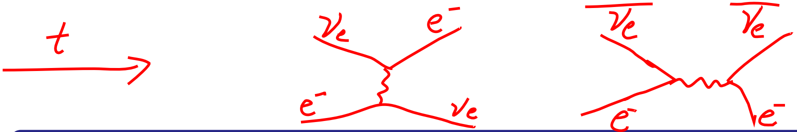
$$J \equiv \mathrm{Im}(\square_{\alpha\beta ij}) = \pm s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 s_\delta$$

- $|J|$ independent of α, β, i, j
- CP violation in vacuum proportional to J
- CP violation vanishes if $\theta_{13} = 0$ or $\delta = 0$

CP quantity $\propto$ $\theta_{13} \sin\delta$

Outline

Matter potential for ν and $\bar{\nu}$



Charged-current potential

- For neutrinos: $V_C = \sqrt{2} G_F N_e$
- For antineutrinos: $V_C = -\sqrt{2} G_F N_e$

Neutral current potential

- For neutrinos: $V_N = -G_F N_n / \sqrt{2}$
- For antineutrinos: $V_N = G_F N_n / \sqrt{2}$

Matter potential for ν and $\bar{\nu}$

Charged-current potential

- For neutrinos: $V_C = \sqrt{2} G_F N_e$
- For antineutrinos: $V_C = -\sqrt{2} G_F N_e$

Neutral current potential

- For neutrinos: $V_N = -G_F N_n / \sqrt{2}$
- For antineutrinos: $V_N = G_F N_n / \sqrt{2}$

relative -ve sign

Mixing angles in matter for ν and $\bar{\nu}$

ν_e - ν_μ mixing

$$\tan 2\theta_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta - G_F N_e/\sqrt{2}} \, , \tan 2\overline{\theta}_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta + G_F N_e/\sqrt{2}}$$

- $\theta_m > \theta_0 \Leftrightarrow \overline{\theta}_m < \theta_0$
- Resonance in ν corresponds to suppressed mixing in $\bar{\nu}$
- $\nu \leftrightarrow \bar{\nu}$ equivalent to $\Delta \leftrightarrow -\Delta$
- Matter effects sensitive to $\Delta > 0$ or $\Delta < 0$ (normal / inverted mass ordering)
- $P_{\alpha\beta} \neq \overline{P}_{\alpha\beta}$ in matter even with real U_{PMNS} ("fake CP violation")

Mixing angles in matter for ν and $\bar{\nu}$

$$\tan 2\theta_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta - G_F N_e/\sqrt{2}}, \tan 2\overline{\theta}_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta + G_F N_e/\sqrt{2}}$$

- $\theta_m > \theta_0 \Leftrightarrow \overline{\theta}_m < \theta_0$
- Resonance in ν corresponds to suppressed mixing in $\bar{\nu}$
- $\nu \leftrightarrow \bar{\nu}$ equivalent to $\Delta \leftrightarrow -\Delta$
- Matter effects sensitive to $\Delta > 0$ or $\Delta < 0$ (normal / inverted mass ordering)
- $P_{\alpha\beta} \neq \overline{P}_{\alpha\beta}$ in matter even with real U_{PMNS} ("fake CP violation")

Mixing angles in matter for ν and $\bar{\nu}$

$$\tan 2\theta_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta - G_F N_e/\sqrt{2}}\,, \tan 2\overline{\theta}_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta + G_F N_e/\sqrt{2}}$$

- $\theta_m > \theta_0 \Leftrightarrow \overline{\theta}_m < \theta_0$
- Resonance in ν corresponds to suppressed mixing in $\bar{\nu}$
- $\nu \leftrightarrow \bar{\nu}$ equivalent to $\Delta \leftrightarrow -\Delta$
- Matter effects sensitive to $\Delta > 0$ or $\Delta < 0$ (normal / inverted mass ordering)
- $P_{\alpha\beta} \neq \overline{P}_{\alpha\beta}$ in matter even with real U_{PMNS} ("fake CP violation")

Mixing angles in matter for ν and $\bar{\nu}$

$$\tan 2\theta_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta - G_F N_e/\sqrt{2}}\,, \tan 2\overline{\theta}_m = \frac{\Delta \sin 2\theta}{\Delta \cos 2\theta + G_F N_e/\sqrt{2}}$$

- $\theta_m > \theta_0 \Leftrightarrow \overline{\theta}_m < \theta_0$
- Resonance in ν corresponds to suppressed mixing in $\bar{\nu}$
- $\nu \leftrightarrow \bar{\nu}$ equivalent to $\Delta \leftrightarrow -\Delta$
- Matter effects sensitive to $\Delta > 0$ or $\Delta < 0$ (normal / inverted mass ordering)
- $P_{\alpha\beta} \neq \overline{P}_{\alpha\beta}$ in matter even with real U_{PMNS} ("fake CP violation")

Net three-flavour picture

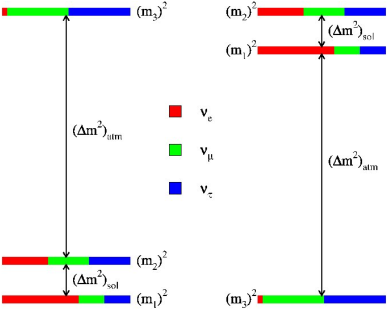
Mass squared differences

- $\Delta m^2_{21} = \Delta m^2_{\odot} \approx 8 \times 10^{-5}\ \mathrm{eV}^2$
- $\Delta m^2_{31} \approx \Delta m^2_{32} \approx \Delta m^2_{atm} \approx \pm 2.5 \times 10^{-3}\ \mathrm{eV}^2$

Mixing angles

- $\nu_\alpha = R_{23}(\theta_{23}) R_{13}(\theta_{13}) R_{12}(\theta_{12})\ \nu_i$ → $U_{13}(\theta_{13}, \delta)$
- $\theta_{13} \approx 0$
- $\theta_{23} \approx \theta_{atm} \approx 45°$
- $\theta_{12} \approx \theta_{\odot} \approx 32°$
- δ: unknown

Neutrino mass-flavour spectrum



Major unknown neutrino mixing parameters

- Mixing angle θ_{13}
- Mass ordering (normal vs. inverted)
- CP violating phase δ