Neutrino Physics: Lecture 13

Interactions and masses in the Standard Model

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+19

Outline

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1. The Standard Model Lagrangian: a reminder
2. Mass terms in the SM

The electroweak Lagrangian in compact form

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}\vec{\mathbf{W}}_{\mu\nu}\cdot\vec{\mathbf{W}}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& +\overline{L}\gamma^{\mu}\left(i\partial_{\mu} - \frac{g}{2}\vec{\mathbf{W}}_{\mu}\cdot\vec{\tau} - \frac{g'}{2}B_{\mu}Y\right)L \\
& +\overline{R}\gamma^{\mu}\left(i\partial_{\mu} - \frac{g'}{2}B_{\mu}Y\right)R \\
& +\left|\left(i\partial_{\mu} - \frac{g}{2}\vec{\mathbf{W}}_{\mu}\cdot\vec{\tau} - \frac{g'}{2}B_{\mu}Y\right)\Phi\right|^{2} - V(\Phi) \\
& -(\lambda_{d}\overline{L}\Phi R + \lambda_{u}\overline{L}\Phi^{C}R + H.c.)
\end{aligned}
$$

Only particles, no interactions

$$
\begin{aligned}
\mathcal{L} = &\\
&+\overline{L}\gamma^{\mu}\left(i\partial_{\mu}\right)L\\
&+\overline{R}\gamma^{\mu}\left(i\partial_{\mu}\right)R\\
&+\left|\left(i\partial_{\mu}\right)\Phi\right|^{2} - V(\Phi)
\end{aligned}
$$

Adding the weak hypercharge $U(1)_Y$

$U(1)_Y$ gauge symmetry

$$\begin{aligned}\psi(x) &\rightarrow e^{i\,\frac{g'}{2}\alpha(x)Y}\psi(x)\\ B_\mu &\rightarrow B_\mu - \frac{g'}{2}(\partial_\mu\alpha)\\ \partial_\mu &\rightarrow \partial_\mu + i\frac{g'}{2}B_\mu Y\end{aligned}$$

Y eigenvalues

$$\begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}: -1\,,\; e_R: -2\,,\; \begin{pmatrix}u_L\\ d_L\end{pmatrix}: \frac{1}{3}\,,\; u_R: \frac{4}{3}\,,\; d_R: -\frac{2}{3}\,,\; \Phi: 1$$

The new gauge boson B_μ

- No mass term
- Kinetic term $(-1/4)B_{\mu\nu}B^{\mu\nu}$ $\quad (B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu)$

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SM Lagrangian with $U(1)_Y$

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\end{aligned}
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Adding the weak isospin $SU(2)_L$

$SU(2)_L$ gauge symmetry

$$\begin{aligned} \psi(x) &\rightarrow e^{i\,\frac{g}{2}\vec{\alpha}(x)\cdot\vec{\tau}}\,\psi(x) \\ \vec{\mathbf{W}}_\mu &\rightarrow \vec{\mathbf{W}}_\mu - \frac{g}{2}(\partial_\mu\vec{\alpha}) \\ \partial_\mu &\rightarrow \partial_\mu + i\frac{g}{2}\vec{\mathbf{W}}_\mu\cdot\vec{\tau} \end{aligned}$$

$SU(2)_L$ eigenvalues (T_3)

$$+\frac{1}{2} : \nu_{eL}, u_L, \Phi^+ \,, \quad -\frac{1}{2} : e_L, d_L, \Phi^0 \,, \quad 0 : e_R, u_R, d_R$$

The new vector gauge boson $\vec{\mathbf{W}}_\mu$

- No mass term
- Kinetic term $(-1/4)\vec{\mathbf{W}}_{\mu\nu}\cdot\vec{\mathbf{W}}^{\mu\nu}$

$$(\vec{\mathbf{W}}_{\mu\nu} = \partial_\mu\vec{\mathbf{W}}_\nu - \partial_\nu\vec{\mathbf{W}}_\mu - g\vec{\mathbf{W}}_\mu\times\vec{\mathbf{W}}_\nu)$$

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SM Lagrangian with $SU(2)_L \times U(1)_Y$

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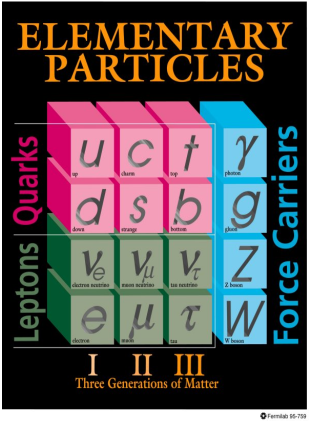
Outline

Only possible interactions: Yukawa



- $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} : (2, \frac{1}{3})$
- $u_R : (1, \frac{4}{3}),\ d_R : (1, -\frac{2}{3})$
- $l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} : (2, -1)$
- $e_R : (1, -2)$
- $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} : (2, 1)$
- $\Phi^C = i\tau^2 \Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi_- \end{pmatrix} : (2, -1)$

- Only possible interaction terms:
  $\overline{q_L}\Phi^c u_R, \quad \overline{q_L}\Phi d_R, \quad \overline{l_L}\Phi e_R \quad (+\text{ h.c.})$

expand...

- No masses yet...

Mass terms for bosons and fermions

Equation of motion from lagrangian

$$\partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \Psi)} \right) = \frac{\partial \mathcal{L}}{\partial \Psi}$$

Cpmplex scalar field

- Lagrangian: $\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^2$
- Equation of motion: $\partial_\alpha \partial^\alpha \phi - m^2 \phi = 0$ Klein-Gordan

Gauge bosons

- Lagrangian: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$
- Equation of motion: $\partial_\alpha \partial^\alpha A_\beta - m^2 A_\beta = 0$ Klein-Gordan

Fermions

- Lagrangian: $\mathcal{L} = \overline{\psi} \gamma^\mu i \partial_\mu \psi - m \overline{\psi} \psi$
- Equation of motion: $i \partial_\alpha \gamma^\alpha \psi - m \psi = 0$ Dirac

Gauge boson and Fermion mass terms forbidden

Gauge bosons

- $\frac{1}{2}m_B^2 B_\mu B^\mu$ violates $U(1)_Y$
- $\frac{1}{2}m_W^2 \overrightarrow{\mathbf{W}}_\mu \cdot \overrightarrow{\mathbf{W}}^\mu$ violates $SU(2)_L$

Fermion masses

- $m_u \overline{u_L} u_R$, $m_d \overline{d_L} d_R$, $m_e \overline{e_L} e_R$
- Violate both, $SU(2)_L$ as well as $U(1)_Y$

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Electroweak symmetry breaking

$$V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mu^2 < 0 \Rightarrow V(\Phi)_{min} \text{ at } \mu^2 = -2\lambda\Phi^\dagger\Phi$$

Minima on the surface of a 3-sphere:

$$\Phi^\dagger\Phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$$

Break symmetry $\Rightarrow$ choose a point $\Rightarrow \phi_1 = \phi_2 = \phi_4 = 0, \phi_3 = v$

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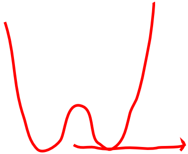

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$$\Phi^\dagger \Phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda}$$

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Symmetry broken: $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$

$$\phi_{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}} \qquad \Phi = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \frac{1}{\sqrt{2}}$$

$v \neq 0$ in vacuum $\Rightarrow$

- T_3 not conserved [$T_3(\Phi^0) = -1/2$]
- Y not conserved [$Y(\Phi^0) = 1$]
- However, conserved quantity in new vacuum:

$$T_3(\Phi^0) + \frac{Y(\Phi^0)}{2} = 0$$

- Electric charge $Q = T_3 + \frac{Y}{2}$ conserved by vacuum

Symmetry broken: $SU(2)_L \times U(1)_Y \to U(1)_Q$

$$\Phi = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \frac{1}{\sqrt{2}}$$

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Terms relevant for EWSB

$$\mathcal{L} = -\frac{1}{4}\overrightarrow{\mathbf{W}}_{\mu\nu}\cdot\overrightarrow{\mathbf{W}}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}\cdot B^{\mu\nu}$$

$$+\left|\left(-\frac{g}{2}\overrightarrow{\mathbf{W}}_{\mu}\cdot\overrightarrow{\tau} - \frac{g'}{2}B_{\mu}Y\right)\Phi\right|^2 - V(\Phi)_{min}$$

$$\phi = \begin{pmatrix} 0 \\ v \end{pmatrix}\frac{1}{\sqrt{2}}$$

Mass for W^+ and W^-

$$\left|\left(-\frac{g}{2}\overrightarrow{\mathbf{W}}_\mu \cdot \overrightarrow{\tau} - \frac{g'}{2}B_\mu Y\right)\Phi\right|^2$$

$$= \frac{1}{8}\left|\begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

$$= \frac{1}{2}\left(\frac{1}{2}gv\right)^2 |W_1|^2 + \frac{1}{2}\left(\frac{1}{2}gv\right)^2 |W_2|^2$$

$$+\frac{1}{8}v^2 \begin{pmatrix} W_3 & B \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_3 \\ B \end{pmatrix}$$

Diagonalize $\Rightarrow$

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$$+ \frac{1}{8} v^2 \begin{pmatrix} W_3 & B \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}$$

Diagonalize $\Rightarrow$

Mass for Z, no mass for A

- Eigenvalues: $0, (g^2 + g'^2)$

$$\Rightarrow \frac{1}{8}v^2 \begin{pmatrix} A & Z \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & (g^2 + g'^2) \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}$$

- Eigenvectors:

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}$$

- Mixing angle: $\tan 2\theta_W = 2gg'/(g^2 - g'^2) \Rightarrow \tan\theta_W = g'/g$

$$M_W = \frac{1}{2}gv\ ,\ M_Z = \frac{1}{2}v\sqrt{g^2 + g'^2}\ ,\ \rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} = 1$$

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Weinberg

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Fermion masses in SM: Dirac masses

Interaction terms:

$$-\lambda_u \overline{q}_L \Phi^c u_R\ , \quad -\lambda_d \overline{q}_L \Phi d_R\ , \quad -\lambda_e \overline{l}_L \Phi e_R$$

After electroweak symmetry breaking:

$$(\lambda_u v)\overline{u}_L u_R\ , \quad (\lambda_d v)\overline{d}_L d_R\ , \quad (\lambda_e v)\overline{e}_L e_R$$

Dirac masses

$$m_u = \lambda_u v\ , \quad m_d = \lambda_d v\ , \quad m_e = \lambda_e v$$

Fermion masses in SM: Dirac masses

Interaction terms:

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After electroweak symmetry breaking:

$$-\frac{(\lambda_u v)}{\sqrt{2}} \bar{u}_L u_R \ , \ -\frac{(\lambda_d v)}{\sqrt{2}} \bar{d}_L d_R \ , \ -\frac{(\lambda_e v)}{\sqrt{2}} \bar{e}_L e_R$$

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Dirac masses

$$m_u = \frac{\lambda_u v}{\sqrt{2}} \,, \quad m_d = \frac{\lambda_d v}{\sqrt{2}} \,, \quad m_e = \frac{\lambda_e v}{\sqrt{2}}$$

No mass for neutrinos yet !