Neutrino Physics: Lecture 15

Neutrino mass models

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Outline

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Dirac and Majorana mass terms for neutrinos

- Dirac mass term:

$$\begin{aligned} \mathcal{L}_D &= -y_D \overline{l_L} \Phi^C \nu_R + h.c. \\ \rightarrow EWSB &\rightarrow -m_D \overline{\nu_L} \nu_R + h.c. \end{aligned}$$

- Majorana mass term for ν_R:

$$\mathcal{L}_R = -\frac{1}{2} m_R (\overline{\nu_R^C} \nu_R + h.c.)$$

- "After EWSB" Majorana mass term for ν_L:

$$\mathcal{L}_L = -\frac{1}{2} m_L (\overline{\nu_L} \nu_L^C + h.c.)$$

Needs physics beyond the SM

Dirac and Majorana mass terms for neutrinos

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Needs physics beyond the SM

Dirac mass + Majorana mass for ν_R

$$\mathcal{L} = -\frac{1}{2} m_D \overline{\nu_L} \nu_R - \frac{1}{2} m_D \overline{\nu_R^C} \nu_L^C - \frac{1}{2} m_R \overline{\nu_R^C} \nu_R + h.c.$$

$$= -\frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

Dirac
Dirac
Majorana

Diagonalization ($m_D \ll m_R$)

- Eigenvalues $m_1 \approx -\frac{m_D^2}{m_R}$, $m_2 \approx m_R$
- Mixing angle $\theta \approx m_D/m_R$
- New basis:

$$\begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$$

Dirac mass + Majorana mass for ν_R

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Majorana does not let Dirac alone !

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{2}\begin{pmatrix} \overline{\nu_{1L}} & \overline{\nu^C_{2L}} \end{pmatrix}\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \nu^C_{1L} \\ \nu_{2L} \end{pmatrix} + h.c. \\
&= -\frac{1}{2} m_1 \overline{\nu_{1L}} \nu^C_{1L} - \frac{1}{2} m_2 \overline{\nu^C_{2L}} \nu_{2L} \quad + h\,c
\end{aligned}
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- Two Majorana neutrinos: ν_{1L} and ν_{2L}
- Light neutrinos are also now Majorana !

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$$= -\frac{1}{2}m_1\overline{\nu_{1L}}\nu_{1L}^C - \frac{1}{2}m_2\overline{\nu_{2L}^C}\nu_{2L} \quad + h c$$

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The seesaw of masses

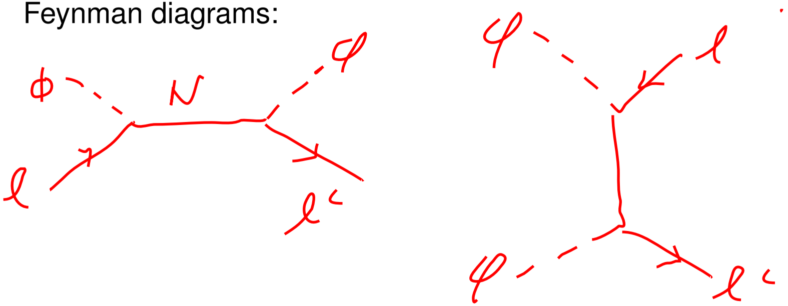
“Naturally” light masses for neutrinos

- Light neutinos get a mass $m_\nu \approx m_D^2/m_R$
- With $m_D \sim$ GeV and $m_R \sim 10^9$ GeV, $m_\nu \sim$ eV
- “Seesaw mechanism” $\Rightarrow$ light neutrino masses

$$m_\nu = - \frac{m_D^2}{m_R} \qquad\qquad m_\nu \bar{\nu}_L \nu_L$$

Type-I seesaw: neutrino mass from singlet N

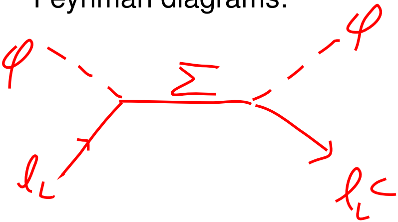
Feynman diagrams:

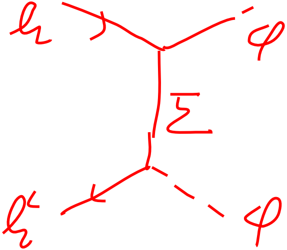


$$
\begin{aligned}
-\mathcal{L}_{\rm eff} &\sim \frac{y_D^2}{M_N}\,(\overline{\ell_L^C}\Phi)(\Phi^T\ell_L) \\
&= \frac{\kappa_{abcd}}{M}(\overline{\ell_L^C})_a(\ell_L)_b\Phi_c\Phi_d \sim \kappa_5\ell_L\ell_L\Phi\Phi
\end{aligned}
$$

Type-III seesaw: neutrino mass from $SU(2)_L$ triplet Σ

Feynman diagrams:





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Type-II seesaw: neutrino mass from triplet Higgs

- $m_L \overline{\nu_L} \nu_L^c$ forbidden by $SU(2)_L \times U(1)$:
  $\overline{\ell_L} \ell_L^c \sim (2,1) \times (2,1) = (1,2) + (3,2)$
- If a Higgs triplet $\Delta \equiv \begin{pmatrix} \Delta_0 \\ \Delta_- \\ \Delta_{--} \end{pmatrix} : (3,-2)$ exists, then

$$\mathcal{L}_m = -f \overline{\nu_L} (\tau \cdot \Delta) \nu_L^c - \mu \Phi^T (\tau \cdot \Delta) \Phi \qquad \text{possible}$$

- Feynman diagram:
- Effective mass $m_L = f \langle \Delta_0 \rangle / \sqrt{2}$

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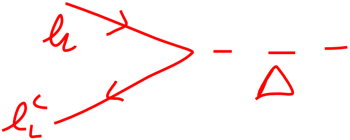

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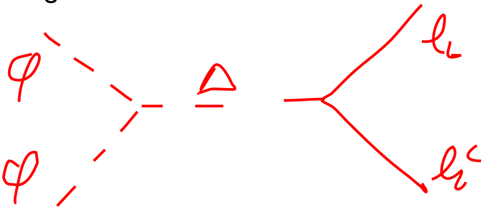
Details of Type-II seesaw

Feynman diagrams:



$$\mathcal{L}_{\rm eff} \sim \frac{f\mu}{M_\Delta^2}\,(\overline{\ell_L^C}\vec{\tau}\ell_L)\cdot(\Phi^T\vec{\tau}\Phi)$$

$$= \frac{\kappa_{abcd}}{M}(\overline{\ell_L^C})_a(\ell_L)_b\Phi_c\Phi_d \sim \kappa_5\ell_L\ell_L\Phi\Phi$$

Seesaw summary

Type I:

$$-\mathcal{L}_{\rm eff} \sim \frac{y_D^2}{M_N}(\overline{\ell_L^C}\Phi)(\Phi^T\ell_L)$$

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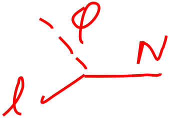

Type III:

$$-\mathcal{L}_{\rm eff} \sim \frac{y_\Sigma^2}{M_\Sigma}(\overline{\ell_L^C}\vec{\tau}\Phi)\cdot(\Phi^T\vec{\tau}\ell_L)$$

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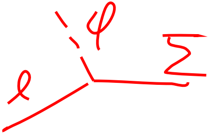

Type II:

$$\mathcal{L}_{\rm eff} \sim \frac{f\mu}{M_\Delta^2}(\overline{\ell_L^C}\vec{\tau}\ell_L)\cdot(\Phi^T\vec{\tau}\Phi)$$

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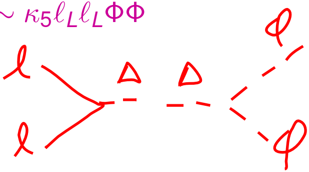


Constraints on seesaw models

hard to establish

Type-I seesaw

- If N is heavy, cannot be produced in the lab
- If N is light, $y_D \ll 1 \Rightarrow$ cannot be produced in the lab

Type-III seesaw

- Σ can be produced at LHC through gauge interactions

Type-II seesaw

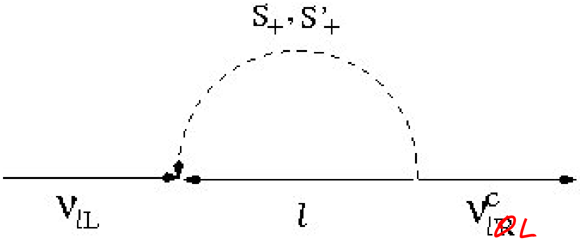
- $\langle\Delta_0\rangle$ also affects M_W and M_Z
- Measurements of $\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1+2(\langle\Delta_0\rangle/\langle\Phi_0\rangle)^2}{1+4(\langle\Delta_0\rangle/\langle\Phi_0\rangle)^2}$ restricts $\langle\Delta_0\rangle/\langle\Phi_0\rangle < 0.07$

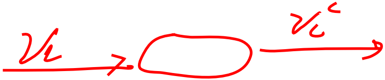
Outline

Zee model

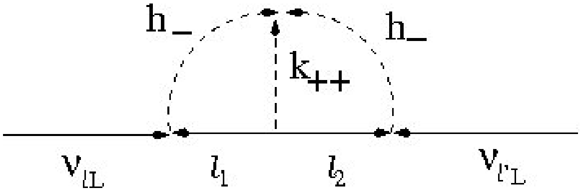
- $\overline{\ell_L}\ell_L^c \sim (2,1)\times(2,1) = (1,2)+(3,2)$
- Can form an invariant with h_- : $(1,-2)$
- A consistent model needs 2 Higgses, Φ and Φ'
- Neutrino mass through loop diagram: (S_+, S'_+ are some combination of Φ_+, Φ'_+ and h_+)

$\bar{\ell}_L \ell_L^c h_-$





Babu's model: $h_{-}(1, -2)$ and $k_{++}(1, 4)$



- Both h_{+} and k_{++} carry two units of $B - L$ charge
- k_{++} : $(1, 4)$ neutralises $\overline{l^c_R} l_R$: $(1, -4)$

$(1,-2)$ $(1,-2)$ $\overline{l^c_R} l_R k_{++}$

Outline

Spontaneous $B - L$ violation

- Majorana mass implies broken $B - L$ symmetry
- Breaking possible explicitly through, e.g., $M_R \overline{\nu_R^c} \nu_R$
- Alternatively, coupling $\overline{\nu_R^c} S \nu_R$, B-L = -1 ← → B-L = -1
  "Higgs" S with lepton number (or $B - L$ charge) of +-2
- S may get a vacuum expectation value by spontaneous symmetry breaking, which breaks $B - L$

Consequences:

- $\Rightarrow$ A massless Goldstone boson, the "Majoron" J
- Limits on J through the processes
  $\mu \to e + J$, $\gamma + e \to e + J$
- Stringent limits from cooling rates of stars
- One or more Majorons may also be emitted in $0\nu\beta\beta$

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Left-right symmetric model

- $G_{\text{weak}} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
- $Q = I_{3L} + I_{3R} + (B - L)/2$
- $SU(2)_R$ broken, $SU(2)_L$ unbroken $\Rightarrow \Delta I_{3R} = \Delta(B - L)/2$
- Type II Seesaw mechanism possible, with $H(2, 1)$ and $\Delta(3, -2)$
- Can give rise to $\mu^- \to e^- e^- e^+$, $\mu^- \to e^- \nu_e \bar{\nu}_\mu$, $\mu^+ e^- \to \mu^- e^+$
- Constraints on the coupling $\sim 10^{-3} G_F$

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Many more models...

- GUT-based models: SU(5), SO(10)
- Supersymmetric models (RPV, LR symmetric)
- ...