Neutrino Physics: Lecture 16

Neutrino mass matrix

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Outline

1. Structure of neutrino mass matrix
2. Mass matrices consistent with current data

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Majorana mass matrix: symmetric

- Single flavour:

$$\mathcal{L}_M = -\frac{1}{2} m_M \, \overline{\nu^C} \nu = \frac{1}{2} m_M \, \nu^T C^\dagger \nu$$

Operations of $\dagger$ and T in spinor space (indices p, q, r, s)

$$\mathcal{L}_M = m_M \, [\nu]_p \, [C^\dagger]_{pq} \, [\nu]_q$$

- Three flavours (indices i, j, k)

$$\mathcal{L}_M = [M_M]_{ij} \, [\nu]_{ip} \, [C^\dagger]_{pq} \, [\nu]_{jq}$$

- Majorana mass matrix M_M symmetric (in flavour indices i, j)
(Need not be real / Hermitian)
- Suppress spinor indices: $\mathcal{L} = [\nu]_i \, [M_M]_{ij} \, [\nu]_j = \nu^T \, [M_M] \, \nu$
- Dirac mass matrix M_D: no such condition

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Mass matrix from Seesaw Type I

Three ν_L flavours, n right-handed neutrinos:

$$\mathcal{L}_M = -\frac{1}{2} \begin{pmatrix} [\overline{\nu_L}]_{1\times 3} & \left[\overline{\nu_R^c}\right]_{1\times n} \end{pmatrix} \begin{pmatrix} [0]_{3\times 3} & [M_D]_{3\times n} \\ \left[M_D^T\right]_{n\times 3} & [M_R]_{n\times n} \end{pmatrix} \begin{pmatrix} \left[\nu_L^c\right]_{3\times 1} \\ [\nu_R]_{n\times 1} \end{pmatrix}$$

$$M_{eff} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

Block-diagonalize:

$$\mathcal{U}^T M_{eff} \mathcal{U} = M^B = \begin{pmatrix} [M_1]_{3\times 3} & 0 \\ 0 & [M_2]_{n\times n} \end{pmatrix}$$

$$M_1 = -M_D^T \, M_R^{-1} \, M_D$$

Mostly (normal) hierarchical neutrino masses

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$(3+n)\times(3+n)$

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Mostly (normal) hierarchical neutrino masses

$$\theta \sim \frac{m_D}{m_R}$$

Mass matrix from Seesaw Type I + II

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Quasi-degenerate masses possible

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Quasi-degenerate masses possible

Diagonalizing Majorana mass matrix

M_M: Symmetric matrix ($M_M = M_1$ for seesaw mechanisms)

Diagonalization:

$$U_{PMNS}^T \, M_M \, U_{PMNS} = M^D = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Reconstructing mass matrix M_M from masses and U_{PMNS}:

$$M_M = U_{PMNS}^* \, M^D \, U_{PMNS}^\dagger$$

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Reconstructing mass matrix M_M from masses and U_{PMNS}:

$$M_M = U^*_{PMNS}\, M^D\, U^\dagger_{PMNS}$$

Diagonalization of M_M and H_{eff}

Neutrinos with momentum p:

$$H_{eff} = \left(p^2 + M_M^\dagger M_M\right)^{1/2} = p + \frac{M_M^\dagger M_M}{2p}$$

M_M symmetric $\Rightarrow$ $U_{PMNS}^T M_M U_{PMNS} = M^D$

$M_M^\dagger M_M$ Hermitian $\Rightarrow$

$$U_{PMNS}^\dagger (M_M^\dagger M_M) U_{PMNS} = U_{PMNS}^\dagger (M_M^\dagger) U_{PMNS}^* U_{PMNS}^T (M_M) U_{PMNS} = (M^D)^\dagger (M^D)$$

U_{PMNS} diagonalizes M_M through $U^T M U$ $\Rightarrow$
U_{PMNS} diagonalizes H_{eff} through $U^\dagger H_{eff} U$

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Diagonalization of M_M and H_{eff}

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$$H_{eff} = \left(p^2 + M_M^\dagger M_M\right)^{1/2} = p + \frac{M_M^\dagger M_M}{2p}$$

$$\begin{aligned}
M_M \text{ symmetric} \quad &\Rightarrow \quad U_{PMNS}^T M_M U_{PMNS} = M^D \\
M_M^\dagger M_M \text{ Hermitian} \quad &\Rightarrow \quad U_{PMNS}^\dagger (M_M^\dagger M_M) U_{PMNS} \\
&\qquad = U_{PMNS}^\dagger (M_M^\dagger) U_{PMNS}^* U_{PMNS}^T (M_M) U_{PMNS} \\
&\qquad = (M^D)^\dagger \, (M^D)
\end{aligned}$$

U_{PMNS} diagonalizes M_M through $U^T M U \Rightarrow$
U_{PMNS} diagonalizes H_{eff} through $U^\dagger H_{eff} U$

Reconstruction of neutrino mass matrix

$$
\begin{aligned}
M_M &= U^*_{PMNS} M^D U^\dagger_{PMNS} \\
&= \Phi(-\chi_1, -\chi_2, -\chi_3) R_{23}(\theta_{23}) U_{13}(\theta_{13}, -\delta) R_{12}(\theta_{12}) \\
&\quad \Phi(-\phi_1, -\phi_2, 0) \;\; M^D \;\; \Phi(-\phi_1, -\phi_2, 0) R^T_{12}(\theta_{12}) \\
&\quad U^T_{13}(\theta_{13}, -\delta) R^T_{23}(\theta_{23}) \Phi(-\chi_1, -\chi_2, -\chi_3)
\end{aligned}
$$

$$
\begin{aligned}
M^\dagger_M M_M &= U_{PMNS} \left(M^{D\dagger} M_D \right) U^\dagger_{PMNS} \\
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&\quad \Phi(\phi_1, \phi_2, 0) \;\; \left(M^{D\dagger} M^D \right) \;\; \Phi(-\phi_1, -\phi_2, 0) R^T_{12}(\theta_{12}) \\
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Independent of Majorana phases ϕ_1, ϕ_2

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$$U^\dagger (M^\dagger M) U = M_D^\dagger M^D$$

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Independent of Majorana phases ϕ_1, ϕ_2 ← H_{eff}

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Masses: ordering and hierarchy

Masses

- Eigenvalues: m_1, m_2, m_3
- $|m_2|^2 - |m_1|^2 = \Delta m^2_\odot$
- $|m_3|^2 - |m_2|^2 = \pm\Delta m^2_{atm}$
- Normal vs. Inverted mass ordering
- Normal hierarchy, Inverted hierarchy, Quasi-degeneracy

Mixing angles: some "aesthetic" values

Mixing angles

$$\theta_{12} \approx 32^\circ\ , \quad \theta_{23} \approx 45^\circ\ , \quad \theta_{13} \approx 0$$

- Bimaximal mixing:

$$\sin\theta_{12} = 1/\sqrt{2}\,, \sin\theta_{23} = 1/\sqrt{2}\,, \sin\theta_{13} = 0$$

- Trimaximal mixing:

$$\sin\theta_{12} = 1/\sqrt{3}\,, \sin\theta_{23} = 1/\sqrt{3}\,, \sin\theta_{13} = 1/\sqrt{3}$$

- Tri-bimaximal mixing:

$$\sin\theta_{12} = 1/\sqrt{3}\,, \sin\theta_{23} = 1/\sqrt{2}\,, \sin\theta_{13} = 0$$

ok

- Quark-lepton complementarity (one of the scenarios)

$$\theta_{12} \approx 45^\circ - \frac{\theta_{12q}}{\sqrt{2}}\,, \theta_{23} \approx 45^\circ - \theta_{23q}\,, \theta_{13} \approx \frac{\theta_{12q}}{\sqrt{2}}$$

ok

Tri-bimaximal mixing and mass matrix

$$U_{TBM} = \Phi(0,0,\pi)\, R_{23}(\pi/4) R_{13}(0) R_{12}(\arcsin\sqrt{1/3})\, \Phi(\phi_1,\phi_2,0)$$

$$= \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \Phi(\phi_1,\phi_2,0)$$

$$M_M = U^*_{PMNS} M^D U^\dagger_{PMNS}$$

$$= \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

where

$$A = \frac{1}{3}(2\, m_1\, e^{-2i\phi_1} + m_2 e^{-2i\phi_2}), \quad B = \frac{1}{3}(m_2 e^{-2i\phi_2} - m_1\, e^{-2i\phi_1}), \quad D = m_3 .$$

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Some possible symmetries of neutrino mass matrix

μ–τ exchange symmetry

$$M_M = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\mu} & m_{\mu\tau} & m_{\mu\mu} \end{pmatrix}$$

- $\theta_{23} = 45^\circ, \theta_{13} = 0$

$L_e - L_\mu - L_\tau$ symmetry

$$M_M = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}$$

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- Inverted mass ordering
- $\Delta m^2_\odot = 0$

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More models for neutrino mass matrix...

- Textures
- Extra $U(1)$ gauge symmetries
- Extra discrete symmetries