Neutrino Physics: Lecture 5

Neutrino oscillations in constant matter density

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Outline

1. Effective Hamiltonian in $2 - \nu$ system
2. Matter potential for neutrinos
3. $\nu_e - \nu_\mu$ evolution in matter: MSW resonance
4. $\nu_\mu \leftrightarrow \nu_{\text{sterile}}$ in atmospheric neutrinos?

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Effective Hamiltonian in vacuum

$$\nu_1 \to \nu_1 : \qquad H_{11} = \quad p + m_1^2/(2E)$$
$$\nu_2 \to \nu_2 : \qquad H_{22} = \quad p + m_2^2/(2E)$$

ν_1–ν_2 basis (mass basis):

$$H = \begin{pmatrix} p + m_1^2/(2E) & 0 \\ 0 & p + m_2^2/(2E) \end{pmatrix}$$
$$= p + \frac{m_1^2 + m_2^2}{2E} + \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix}$$

$$\Delta \equiv \frac{m_2^2 - m_1^2}{4E}$$

$$H_{eff} = \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix}$$

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Evolution in vacuum: two flavours

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}-\Delta & 0\\ 0 & \Delta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

$$\begin{pmatrix}\nu_1(t)\\ \nu_2(t)\end{pmatrix} = \begin{pmatrix}e^{i\Delta t} & 0\\ 0 & e^{-i\Delta t}\end{pmatrix}\begin{pmatrix}\nu_1(0)\\ \nu_2(0)\end{pmatrix}$$

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$$\nu_1(t) = e^{i\Delta t}\,\nu_1(0)$$

$$\nu_2(t) = e^{-i\Delta t}\,\nu_2(0)$$

Evolution in flavour basis

Change of basis:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

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Evolution:

$$\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = U^\dagger \begin{pmatrix} e^{i\Delta t} & 0 \\ 0 & e^{-i\Delta t} \end{pmatrix} U \begin{pmatrix} \nu_\alpha(0) \\ \nu_\beta(0) \end{pmatrix}$$

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Outline

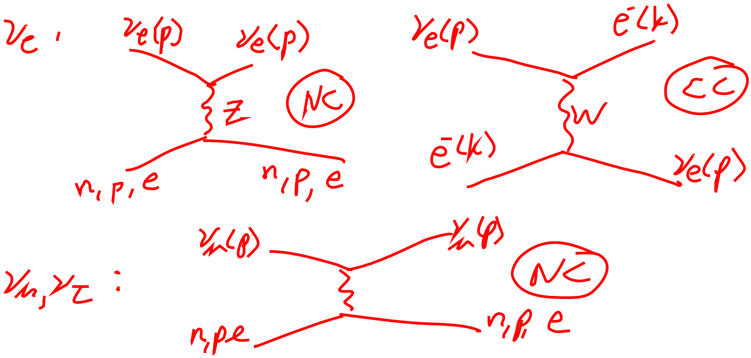
Forward scattering interactions with matter

$$\nu_e \to \nu_e : \quad H_{ee} = V_C + V_N$$
$$\nu_\mu \to \nu_\mu : \quad H_{\mu\mu} = V_N$$
$$\nu_\tau \to \nu_\tau : \quad H_{\tau\tau} = V_N$$

$$V_C = \sqrt{2} G_F n_e \quad , \quad V_N = -G_F n_n / \sqrt{2}$$

Detailed explanation: J. Linder, hep-ph/0504264

Forward scattering interactions with matter



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Effective potential in matter

Three flavours:

$$V = \begin{pmatrix} V_C + V_N & 0 & 0 \\ 0 & V_N & 0 \\ 0 & 0 & V_N \end{pmatrix}$$

Two flavours (ν_e-$\nu_{\mu/\tau}$):

$$V_f = \begin{pmatrix} V_C & 0 \\ 0 & 0 \end{pmatrix}$$

Two flavours (ν_μ-ν_τ):

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Evolution in flavour basis: constant density

$$H_f = U \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix} U^\dagger + \begin{pmatrix} V_C & 0 \\ 0 & 0 \end{pmatrix}$$

$$H_f = \begin{pmatrix} -\Delta \cos 2\theta + V_C & \Delta \sin 2\theta \\ \Delta \sin 2\theta & \Delta \cos 2\theta \end{pmatrix}$$

Evolution:

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = e^{-i\, H_f t} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix}$$

Not simple....

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Matter basis: where H is diagonal

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = U_m \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

In matter basis:

$$H_m = U_m^\dagger H_f U_m = \text{Constant} \times I + \begin{pmatrix} -\Delta_m & 0 \\ 0 & \Delta_m \end{pmatrix}$$

Flavour evolution:

$$\begin{pmatrix} \nu_{1m}(t) \\ \nu_{2m}(t) \end{pmatrix} = \begin{pmatrix} e^{i\Delta_m t} & 0 \\ 0 & e^{-i\Delta_m t} \end{pmatrix} \begin{pmatrix} \nu_{1m}(0) \\ \nu_{2m}(0) \end{pmatrix}$$

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Evolution in flavour basis again

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = U_m^\dagger \begin{pmatrix} e^{i\Delta_m t} & 0 \\ 0 & e^{-i\Delta_m t} \end{pmatrix} U_m \begin{pmatrix} \nu_e(0) \\ \nu_\mu(0) \end{pmatrix}$$

Exactly like vacuum oscillations, with “m” suffix added !

$$P_{ee} = 1 - \sin^2(2\theta_m)\sin^2(\Delta_m L)$$

Need to determine Δ_m, θ_m

Evolution in flavour basis again

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Diagonalising H_f

$$H_f = \begin{pmatrix} -\Delta\cos 2\theta + V_C & \Delta\sin 2\theta \\ \Delta\sin 2\theta & \Delta\cos 2\theta \end{pmatrix}$$

Solve

$$H_m = U_m^\dagger H_f U_m = Constant + \begin{pmatrix} -\Delta_m & 0 \\ 0 & \Delta_m \end{pmatrix}$$

Mixing angle:

$$\tan 2\theta_m = \frac{\Delta\sin 2\theta}{2\Delta\cos 2\theta - V_C}$$

Mass squared difference in matter:

$$\Delta_m = \sqrt{(\Delta\cos 2\theta - V_C)^2 + (\Delta\sin 2\theta)^2}$$

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$$\Delta_m = \sqrt{(\Delta\cos 2\theta - \frac{V_C}{2})^2 + (\Delta\sin 2\theta)^2}$$

New amplitude, new wavelength

Vacuum → Resonance → High densities

$\theta_m = 45^\circ$

Δ_m minimum

↳ $\Delta \sin 2\theta$

$\sin^2 2\theta_m \simeq 1$

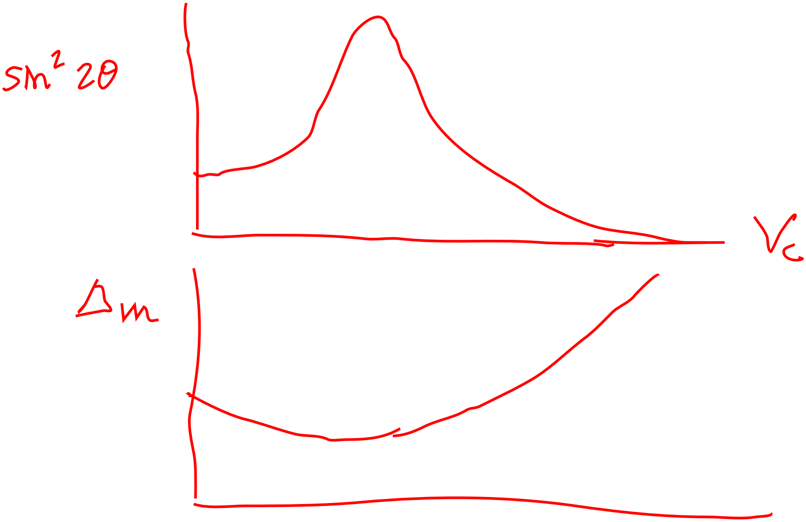
maximum

$\theta_m = 90^\circ$

$\sin^2 2\theta_m \approx 0$

No oscillations

MSW resonance: minimum Δ_m, maximum $\sin^2 2\theta$



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Atmospheric oscillations: $\nu_\mu \rightarrow ?$

- A candidate: $\nu_\mu \leftrightarrow \nu_{sterile}$
- Matter potential ($\nu_\mu - \nu_s$ basis):

$$V_f = \begin{pmatrix} V_N & 0 \\ 0 & 0 \end{pmatrix}$$

- Oscillation probability different from vacuum oscillations

$$P_{\mu\mu} = 1 - \sin^2(2\theta_m)\sin^2(\Delta_m L)$$

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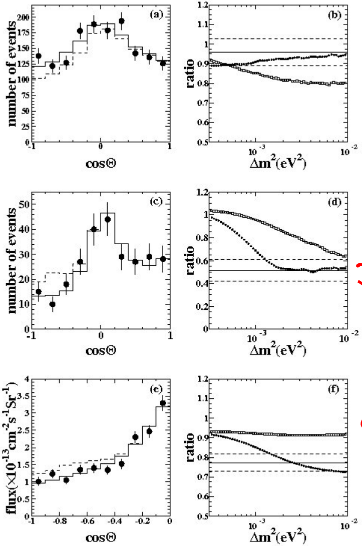
Ruling out $\nu_\mu \leftrightarrow \nu_s$ in atmospheric neutrinos



multi ring

PC

} observed band

upward going μ

U/D ratio

Super-Kamiokande