Neutrino Physics: Lecture 7

Neutrinos through varying matter density

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Outline

1. Propagating through a MSW resonance
2. Adiabaticity and flip probability
3. The fate of ν_e produced in the core of the Sun
4. Day-night asymmetry

Outline

2-ν level crossing: MSW resonance

- Effective Hamiltonian:

$$H = \begin{pmatrix} -\Delta\cos 2\theta + V_C & \Delta\sin 2\theta \\ \Delta\sin 2\theta & \Delta\cos 2\theta \end{pmatrix}$$

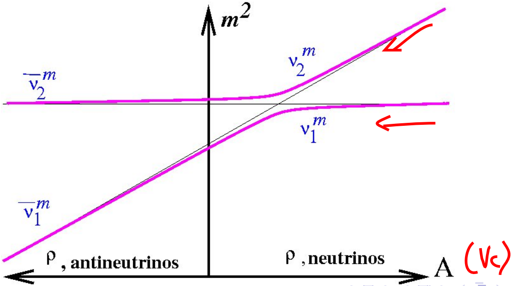
$$(V_C = \sqrt{2} G_F N_e)$$

- Eigenvalues:

$$h_\pm = \frac{V_C}{2} \pm \frac{1}{2}\sqrt{(\Delta\cos 2\theta - \frac{V_C}{2})^2 + (\Delta\sin 2\theta)^2}$$

- Effective mixing angle:

$$\tan 2\theta_m = \frac{\Delta\sin 2\theta}{\Delta\cos 2\theta - (V_C/2)}$$



Passing through a MSW resonance

Instantaneous mass eigenstates in matter:

$$\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = U^\dagger(\theta_m) \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \quad .$$

Time evolution:

$$\begin{aligned} i\frac{d}{dt}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} &= i\frac{dU^\dagger(\theta_m)}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} + iU^\dagger(\theta_m)\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \\ &= i\frac{dU^\dagger(\theta_m)}{dt}U(\theta_m)\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} + U^\dagger(\theta_m)H_f U(\theta_m)\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} \end{aligned}$$

$$i\frac{d}{dt}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} -\Delta_m & -id\theta_m/dt \\ id\theta_m/dt & \Delta_m \end{pmatrix}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

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Adiabaticity

Off-diagonal elements small $(2d\theta_m/dt \ll 2|\Delta_m|) \Rightarrow$
Mass eigenstates travel independently

$$\nu_{jm}(t) = \nu_{jm}(0)\exp\left(-i\int \frac{m_{jm}^2}{2E}dt\right)$$

Adiabaticity parameter

$$\gamma \equiv \frac{|\Delta_m|}{|d\theta_m/dt|} = \left|\frac{4\Delta_m^3}{\Delta \sin 2\theta}\left(\frac{dV_c}{dt}\right)^{-1}\right|$$

- Adiabaticity $\Rightarrow \gamma \gg 1$ everywhere

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Adiabaticity breaking

γ_{res}: a measure of adiabaticity breaking

Minimum γ near MSW resonance:

$$\gamma_{res} = \frac{\Delta m^2}{2E} \frac{\sin^2 2\theta}{\cos 2\theta} \left| \frac{1}{V_C} \frac{dV_C}{dt} \right|_{res}^{-1}$$

Away from the resonance

Automatic adiabaticity:

$$\Delta_m \to \infty \Rightarrow \gamma \gg 1$$

Flip probability $P_{\nu_{2m} \to \nu_{1m}}$

General expression

$$P_f \approx \left[\frac{exp\left(-\frac{\pi}{2}\gamma_{res} F\right) - exp\left(-\frac{\pi}{2}\gamma_{res}\frac{F}{\sin^2\theta}\right)}{1 - exp\left(-\frac{\pi}{2}\gamma_{res}\frac{F}{\sin^2\theta}\right)} \right]$$

Kuo, Pantaleone, PRD 39, 1930 (1989)

Two useful limits ($F \approx 1$ for solar density profile)

- $\gamma_{res} > 1$:

$$P_f \equiv \exp\left(-\frac{\pi\gamma_{res}}{2}\right)$$

- $\gamma_{res} \ll 1$:

$$P_f \approx \cos^2\theta$$

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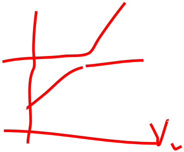
The fate of $\nu(0) = \nu_{2m}(0) = \nu_e$

Just after the MSW resonance region:



$$\nu(t_R) = \sqrt{P_f}\, e^{i\chi_1}\, \nu_{1m} + \sqrt{1 - P_f}\, e^{i\chi_2}\, \nu_{2m}$$

Propagation till the detector:

$$\begin{aligned}\nu(t) &= \sqrt{P_f}\, e^{i\chi_1}\, \nu_{1m} \exp\left(-i \int \frac{m_{1m}^2}{2E} dt\right) \\ &\quad + \sqrt{1 - P_f}\, e^{i\chi_2}\, \nu_{2m} \exp\left(-i \int \frac{m_{2m}^2}{2E} dt\right)\end{aligned}$$

Net survival probability $P_{ee} = |\langle \nu_e | \nu(t) \rangle|^2$:

$$\begin{aligned}P_{ee} &= P_f \cos^2\theta + (1 - P_f)\sin^2\theta \\ &\quad + 2\sqrt{P_f(1 - P_f)} \cos\theta \sin\theta \, \cos\left(\chi_1 - \chi_2 + \int 2\Delta_m dt\right)\end{aligned}$$

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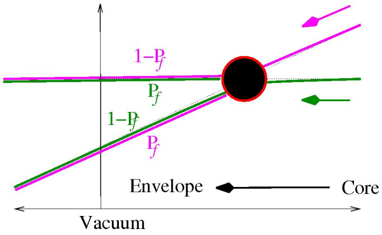
When coherence between mass eigenstates is lost

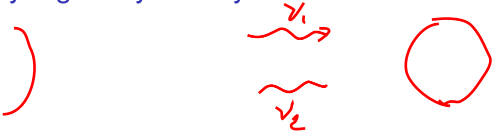


Net survival probability $P_{ee} = |\langle \nu_e | \nu(t) \rangle|^2$:

$$P_{ee} = P_f \cos^2\theta + (1 - P_f)\sin^2\theta$$

Close to the actual situation for solar neutrinos !

Outline



Incoherent ν_1 and ν_2 at the earth

When ν_1 and ν_2 arrive at the Earth decoherently:

$$\Phi_{\nu_1} = P_f \Phi_0 \ , \quad \Phi_{\nu_2} = (1 - P_f)\Phi_0$$

If neutrinos travel a distance L through the Earth:

$$\begin{aligned} |\nu_1(t)\rangle &= \cos(\theta_m - \theta)|\nu_{1m}\rangle e^{-i\frac{m_{1m}^2 L}{2E}} + \sin(\theta_m - \theta)|\nu_{2m}\rangle e^{-i\frac{m_{2m}^2 L}{2E}} \\ |\nu_2(t)\rangle &= -\sin(\theta_m - \theta)|\nu_{1m}\rangle e^{-i\frac{m_{1m}^2 L}{2E}} + \cos(\theta_m - \theta)|\nu_{2m}\rangle e^{-i\frac{m_{2m}^2 L}{2E}} \end{aligned}$$

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

ν_e from incoherent incoming ν_1 and ν_2

ν_e produced from incoming ν_1:

$$
\begin{aligned}
P_{1e} &= |\langle \nu_e | \nu_1(t) \rangle|^2 \\
&= \left| \cos\theta_m \cos(\theta_m - \theta) e^{-i\frac{m_{1m}^2 L}{2E}} + \sin\theta_m \sin(\theta_m - \theta) e^{-i\frac{m_{2m}^2 L}{2E}} \right|^2 \\
&= \cos^2\theta \cos^2(\theta_m - \theta) + \sin^2\theta \sin^2(\theta_m - \theta) \\
&\quad + \cos\theta \sin\theta \cos(\theta_m - \theta) \sin(\theta_m - \theta) \cos(2\Delta_m L)
\end{aligned}
$$

ν_e produced from incoming ν_2:

$$
\begin{aligned}
P_{2e} &= |\langle \nu_e | \nu_2(t) \rangle|^2 \\
&= \left| -\cos\theta_m \sin(\theta_m - \theta) e^{-i\frac{m_{1m}^2 L}{2E}} + \sin\theta_m \cos(\theta_m - \theta) e^{-i\frac{m_{2m}^2 L}{2E}} \right|^2 \\
&= \cos^2\theta \sin^2(\theta_m - \theta) + \sin^2\theta \cos^2(\theta_m - \theta) \\
&\quad - \cos\theta \sin\theta \cos(\theta_m - \theta) \sin(\theta_m - \theta) \cos(2\Delta_m L)
\end{aligned}
$$

Net survival probability at night

Net ν_e flux at night:

$$\Phi_e(\text{night}) = P_f \Phi_0 P_{1e} + (1 - P_f)\Phi_0 P_{2e}$$

Net ν_e flux during day:

$$\Phi_e(\text{day}) = P_f \Phi_0 \cos^2\theta + (1 - P_f)\Phi_0 \sin^2\theta$$

Day-night asymmetry

$$A_{DN} = P_f(P_{1e} - \cos^2\theta) + (1 - P_f)(P_{2e} - \sin^2\theta)$$

In general, nonzero day-night asymmetry !

denominator

Possible vanishing of day-night asymmetry

When $\theta_m - \theta \approx 0$:

$$P_{1e} \approx \cos^2\theta \;, \quad P_{2e} \approx \sin^2\theta$$

Happens when $\theta \approx 45^\circ$ (see from $\tan 2\theta_m$ expression)

Observables to look out for

- Magnitude of the total flux ✓
- Energy dependence of the flux ✓
- Day-night asymmetry ✓
- Seasonal variation ✓

$$A_{DN} = \frac{\Phi_{day} - \Phi_{night}}{\Phi_{day} + \Phi_{night}} = \frac{P_f(P_{1e} - \cos^2\theta) + (1-P_f)(P_{2e} - \sin^2\theta)}{P_f(P_{1e} + \cos^2\theta) + (1-P_f)(P_{2e} + \sin^2\theta)}$$