

$$\bar{\psi} = -\psi^T C^\dagger$$

$p, q, r, s \rightarrow$  spinor indices

$\alpha, \beta, \gamma, \delta \rightarrow$  Dirac (spacetime)

$a, b, c \rightarrow$   $SU(2)$  indices

$i, j, k \rightarrow$  flavour indices

$$\mathcal{L} = \underline{v^T} \underline{M_m} \underline{v}$$

$$\mathcal{L} = (v_e \ v_m \ v_\tau) \begin{pmatrix} M_m \end{pmatrix} \begin{pmatrix} v_e \\ v_m \\ v_\tau \end{pmatrix}$$

Seeraw type I

$$\mathcal{L} = \begin{pmatrix} \overline{v}_L & \overline{v}_R^c \end{pmatrix} \begin{pmatrix} \overbrace{0}^{3 \times 3} & \overbrace{m_D}^{3 \times n} \\ \underbrace{m_D^T}_{n \times 3} & \underbrace{m_R}_{n \times n} \end{pmatrix} \begin{pmatrix} \overline{v}_L^c | \beta \\ \overline{v}_R | n \end{pmatrix}$$

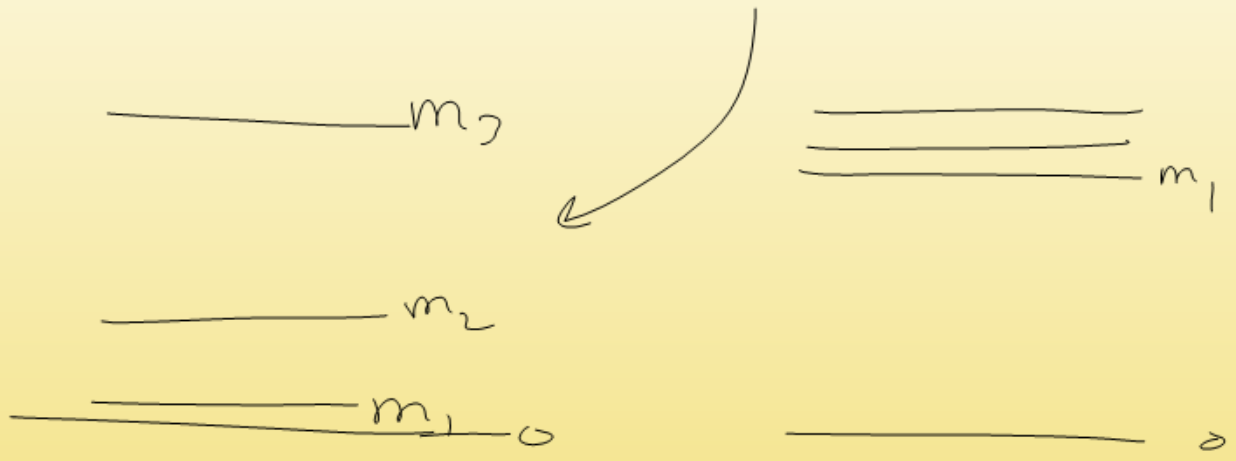
~~$$\begin{pmatrix} \overline{v}_L & \overline{v}_R \end{pmatrix}$$~~

$$\begin{pmatrix} \underbrace{\overline{v}_e \ \overline{v}_\mu \ \overline{v}_\tau}_3 \quad \underbrace{\overline{v}_{R1}^c \ \overline{v}_{R2}^c \ \dots}_n \end{pmatrix}$$

$$m_e = \frac{m_D^2}{M_{R1}}$$

$$m_{\mu} = \frac{m_D^2}{M_{R2}}$$

$$m_{\tau} = \frac{m_D^2}{M_{R3}}$$



Normal mass ordering

\_\_\_\_\_  $m_3$

\_\_\_\_\_  $m_2$

\_\_\_\_\_  $m_1$

Inverted  
mass ordering

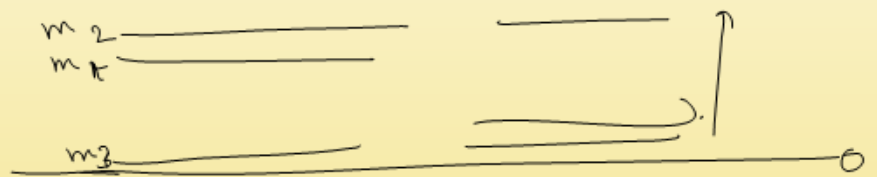
\_\_\_\_\_  $m_2$

\_\_\_\_\_  $m_1$

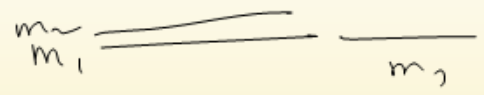
\_\_\_\_\_  $m_3$

# Hierarchy

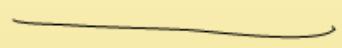
quasi-hierarchical  
inverted ordering



normal  
hierarchy



quasi-  
degenerate



Type I + II

$$m_e = \underline{\underline{m_{Le}}} + \frac{m_D^2}{M_{R2}}$$

$$m_\mu = \underline{\underline{m_{L\mu}}} + \frac{m_D^2}{M_{R2}}$$

↓

quasi-degenerate .

$$U^T M_M U = M^D = \begin{pmatrix} m_1 & & \\ & m_2 & 0 \\ 0 & & m_3 \end{pmatrix}$$

$$\boxed{M_M} = \underline{\underline{U^*}} \underline{\underline{M^D}} \underline{\underline{U^+}}$$



$$\begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & 1 \end{pmatrix} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} \begin{pmatrix} e^{-i\phi_1} & & \\ & e^{-i\phi_2} & \\ & & 1 \end{pmatrix}$$

↓

$$\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$