# Inverse Supernova Neutrino Problem

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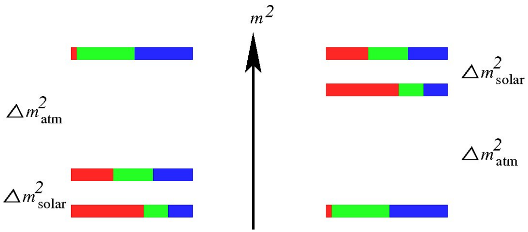
# Mixing parameters relevant for SN neutrinos

Mixing of $\nu_e$, $\nu_\mu$, $\nu_\tau$ $\Rightarrow$ $\nu_1, \nu_2, \nu_3$ (mass eigenstates)




- Mass ordering: Normal or Inverted ?
- Value of $\theta_{13}$: how small ?

# Outline

1. Neutrino emission and primary spectra
2. Flavor conversions inside the star
   - Collective effects and spectral swaps
   - MSW resonances inside the star
   - Predicting swapped and unswapped regimes
3. Observable signals at the Earth
   - Earth matter effects
   - Shock wave effects
   - Neutronization burst signal
4. Inverse SN neutrino problem

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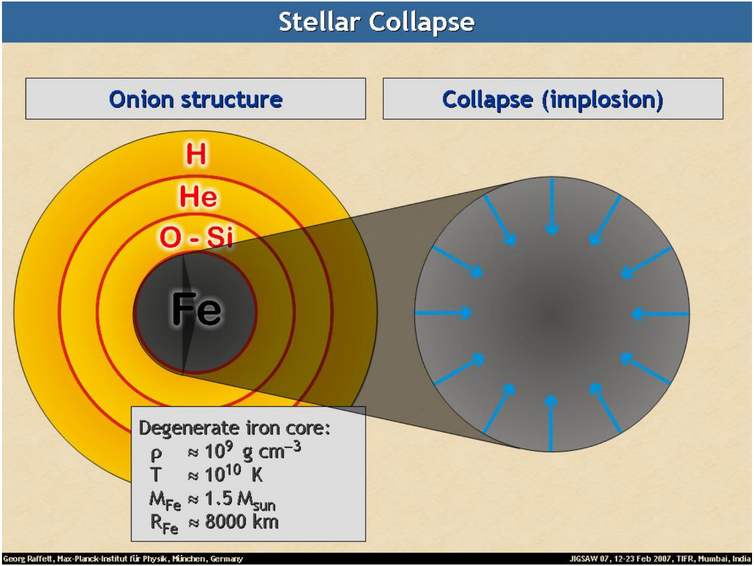
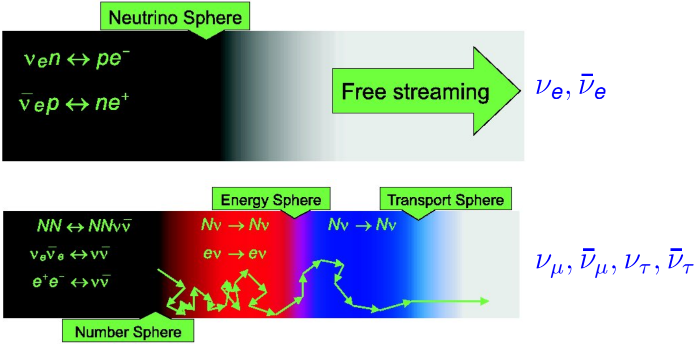
# The would-be supernova before the collapse



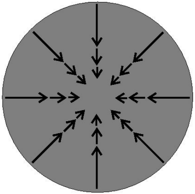
# Trapped neutrinos before the collapse

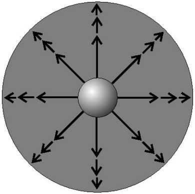
- Neutrinos trapped inside "neutrinospheres" around $\rho \sim 10^{10}$g/cc
- Free-streaming when $\rho \lesssim 10^{10}$g/cc



# Core collapse, shock wave, and explosion

Gravitational core collapse $\Rightarrow$ Shock Wave





Neutronization burst: $\nu_e$ emitted for $\sim$ 10 ms

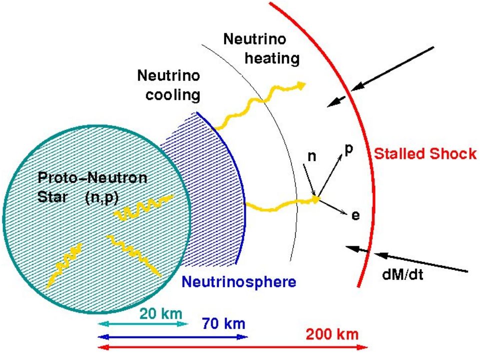
**Cooling through neutrino emission: $\sim 10^{58}$ neutrinos**

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$ Duration: About 10 sec

Emission of 99% of the SN collapse energy in neutrinos

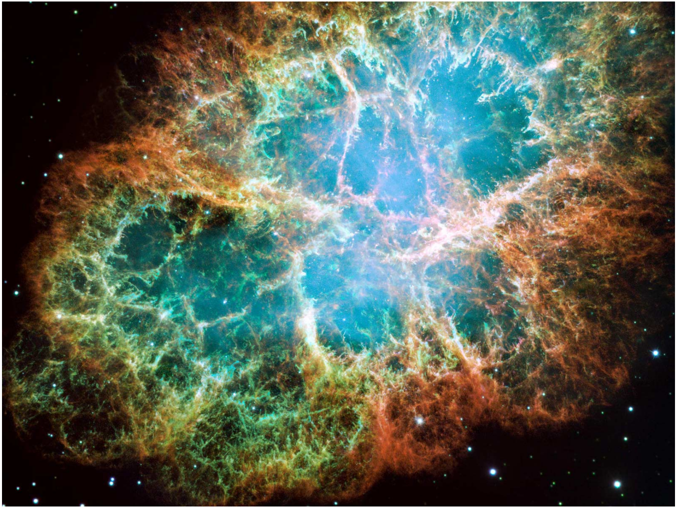
¿¿¿ Explosion ???

# Role of neutrinos in explosion



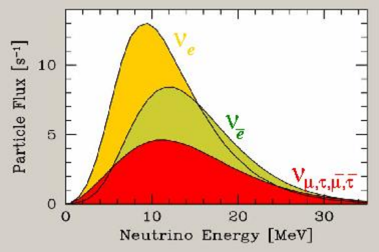

- Neutrino heating needed for pushing the shock wave
- Large scale convection also needed for explosion

# The star after explosion



(Crab nebula, supernova seen in 1054)

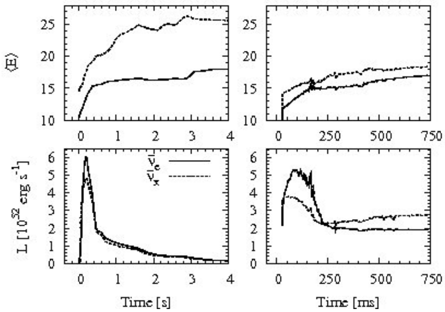
# Primary fluxes and spectra




- Almost blackbody spectra, slightly “pinched”
- Energy hierarchy:
  $\langle E_0(\nu_e)\rangle < \langle E_0(\bar{\nu}_e)\rangle < \langle E_0(\nu_x)\rangle = \langle E_0(\nu_y)\rangle$
- $\langle E_0(\nu_e)\rangle \approx$ 10–12 MeV
  $\langle E_0(\bar{\nu}_e)\rangle \approx$ 13–16 MeV
  $\langle E_0(\nu_x)\rangle \approx$ 15–25 MeV
- $L_{\nu_e} \approx L_{\bar{\nu}_e}$

$$\nu_x \equiv \cos\theta_{23}\nu_\mu - \sin\theta_{23}\nu_\tau, \quad \nu_y \equiv \sin\theta_{23}\nu_\mu + \cos\theta_{23}\nu_\tau$$

# Time-dependence, Flavor-dependence, Model-dependence




solid line: $\bar{\nu}_e$

dotted line: $\bar{\nu}_x$

Differing model predictions in cooling phase:

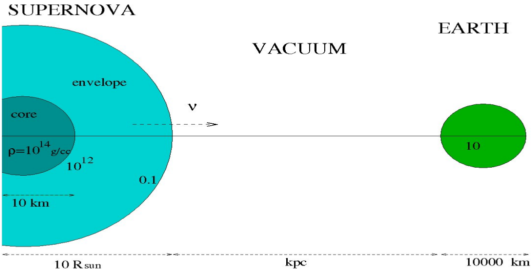
| Model | $\langle E_0(\nu_e)\rangle$ | $\langle E_0(\bar{\nu}_e)\rangle$ | $\langle E_0(\nu_x)\rangle$ | $\frac{\Phi_0(\nu_e)}{\Phi_0(\nu_x)}$ | $\frac{\Phi_0(\bar{\nu}_e)}{\Phi_0(\nu_x)}$ |
|---|---|---|---|---|---|
| Garching (G) | 12 | 15 | 18 | 0.85 | 0.75 |
| Livermore (L) | 12 | 15 | 24 | 2.0 | 1.6 |


G. G. Raffelt, M. T. Keil, R. Buras, H. T. Janka and M. Rampp, astro-ph/0303226

T. Totani, K. Sato, H. E. Dalhed and J. R. Wilson, Astrophys. J. 496, 216 (1998)

# Outline

# Propagation through matter of varying density




**Inside the SN:** *flavor conversion*

Collective effects and MSW matter effects

**Between the SN and Earth:** *no flavor conversion*

Mass eigenstates travel independently

**Inside the Earth:** *flavor oscillations*

MSW matter effects (*if detector is shadowed by the Earth*)

# Outline

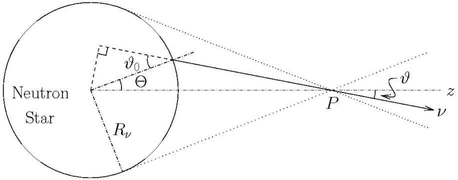
# Nonlinear effects due to $\nu - \nu$ coherent interactions

- Large neutrino density $\Rightarrow$ substantial $\nu$–$\nu$ potential

$$H = H_{vac} + H_{MSW} + H_{\nu\nu}$$

$$\begin{aligned} H_{vac}(\vec{p}) &= M^2/(2p) \\ H_{MSW} &= \sqrt{2} G_F n_{e^-} diag(1,0,0) \\ H_{\nu\nu}(\vec{p}) &= \sqrt{2} G_F \int \frac{d^3q}{(2\pi)^3} (1 - \cos\theta_{pq}) (\rho(\vec{q}) - \bar{\rho}(\vec{q})) \end{aligned}$$

- $d\rho/dt = i[H(\rho), \rho] \quad \Rightarrow \quad$ Nonlinear effects !



# "Collective" effects: qualitatively new phenomena

## Synchronized oscillations:

$\nu$ and $\bar{\nu}$ of all energies oscillate with the same frequency

S. Pastor, G. Raffelt and D. Semikoz, PRD65, 053011 (2002)

## Bipolar/pendular oscillations:

Coherent $\nu_e\bar{\nu}_e \leftrightarrow \nu_x\bar{\nu}_x$ oscillations even for extremely small $\theta_{13}$

S. Hannestad, G. Raffelt, G. Sigl, Y. Wong, PRD74, 105010 (2006)

## Spectral split/swap:

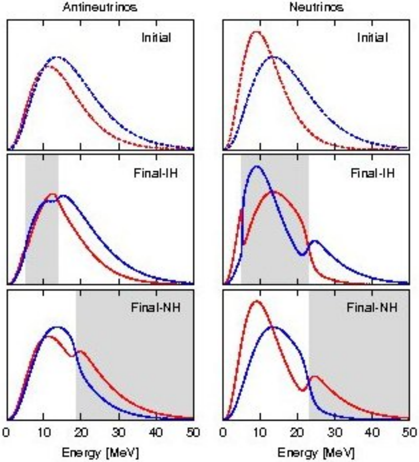
$\nu_e$ and $\nu_y$ ($\bar{\nu}_e$ and $\bar{\nu}_y$) spectra interchange completely, only within certain energy ranges.

G.Raffelt, A.Smirnov, PRD76, 081301 (2007), PRD76, 125008 (2007)

B. Dasgupta, AD, G.Raffelt, A.Smirnov, PRL103,051105 (2009)

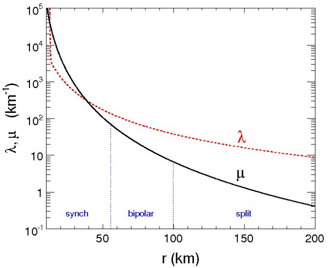
# Spectral split/swap

cooling-phase Garching fluxes




B. Dasgupta, AD, G.Raffelt, A.Smirnov, arXiv:0904.3542 [hep-ph], PRL

# Sequential dominance of phenomena (Fe-core SN)




- $\mu \equiv \sqrt{2} G_F (N_\nu + N_{\bar{\nu}})$
- $\lambda \equiv \sqrt{2} G_F N_e$

- $r \lesssim 200$ km: collective effects dominate
- $r \gtrsim 200$ km: standard MSW matter effects dominate

G.L.Fogli, E. Lisi, A. Marrone, A. Mirizzi, JCAP 0712, 010 (2007)

# Three-flavor collective effects

## Three-flavor results by combining two-flavor ones

- Factorization in two two-flavor evolutions possible

B.Dasgupta and AD, PRD77, 113002 (2008)

## New three-flavor effects

- In early accretion phase, large $\mu$-$\tau$ matter potential causes interference between MSW and collective effects, *sensitive to deviation of* $\theta_{23}$ *from maximality*

A.Esteban-Pretel, S.Pastor, R.Tomas, G.Raffelt, G.Sigl, PRD77, 065024 (2008)

- Spectral splits develop at two energies, in a stepwise process, during neutronization burst of a O-Ne-Mg SN.

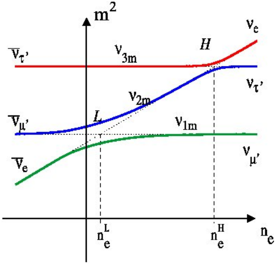
H.Duan, G.M.Fuller and Y.Z.Qian, PRD77, 085016 (2008)

B.Dasgupta, AD, A.Mirizzi and G. G. Raffelt, PRD77, 113007 (2008)

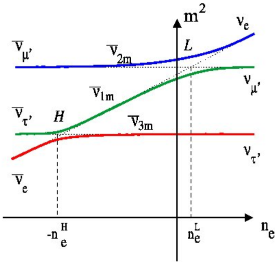
# Outline

# MSW Resonances inside a SN

## Normal mass ordering




## Inverted mass ordering




AD, A.Smirnov, PRD62, 033007 (2000)

**$H$ resonance: $(\Delta m^2_{\rm atm}, \theta_{13})$, $\rho \sim 10^3$–$10^4$ g/cc**

- In $\nu(\bar{\nu})$ for normal (inverted) hierarchy
- Adiabatic (non-adiabatic) for $\sin^2\theta_{13} \gtrsim 10^{-3} (\lesssim 10^{-5})$

**$L$ resonance: $(\Delta m^2_{\odot}, \theta_{\odot})$, $\rho \sim 10$–$100$ g/cc**

- Always adiabatic, always in $\nu$

# Fluxes arriving at the Earth

Mixture of initial fluxes:

$$F_{\nu_e} = p\, F^0_{\nu_e} + (1-p)\, F^0_{\nu_x} \,,$$
$$F_{\bar{\nu}_e} = \bar{p}\, F^0_{\bar{\nu}_e} + (1-\bar{p})\, F^0_{\nu_x} \,,$$

## $p$ and $\bar{p}$ in the swapped and unswapped energy regimes

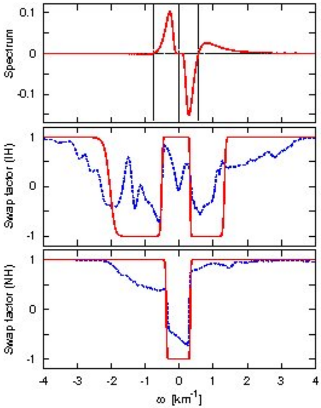
| | | $\Theta_{13}$ | $p$ unswapped | $p$ swapped | $\bar{p}$ unswapped | $\bar{p}$ swapped |
|---|---|---|---|---|---|---|
| A | NH | Large | 0 | $\sin^2\theta_{12}$ | $\cos^2\theta_{12}$ | 0 |
| B | IH | Large | $\sin^2\theta_{12}$ | 0 | 0 | $\cos^2\theta_{12}$ |
| C | NH | small | $\sin^2\theta_{12}$ | 0 | $\cos^2\theta_{12}$ | 0 |
| D | IH | small | $\sin^2\theta_{12}$ | 0 | $\cos^2\theta_{12}$ | 0 |

- "Small": $\sin^2\theta_{13} \lesssim 10^{-5}$, "Large": $\sin^2\theta_{13} \gtrsim 10^{-3}$.

AD, TAUP09 Talk

# Outline

# Mass ordering, spectral crossings, swaps, and splits



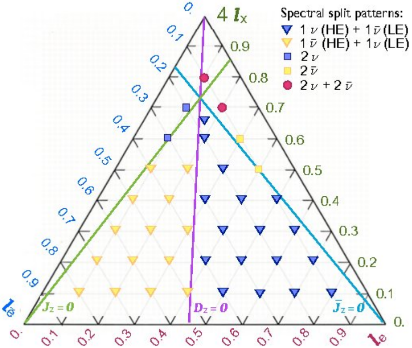

- Neutrinos:
  $\omega \equiv 1/E$
- Antineutrinos:
  $\omega \equiv -1/E$
- Spectrum
  $g(|\omega|) = F_{\nu_e}(\omega) - F_{\nu_x}(\omega)$
  $g(-|\omega|) = F_{\bar{\nu}_x}(\omega) - F_{\bar{\nu}_e}(\omega)$
- Swap
  $S(\omega) = \frac{g(\omega)_{final}}{g(\omega)_{initial}}$

**Swap $S(\omega) = -1 \Rightarrow$**

- Inverted Hierarchy: positive crossing
- Normal Hierarchy: negative crossing
- Nearby swaps may overlap to reduce number of splits

B. Dasgupta, AD, G.Raffelt, A.Smirnov, arXiv:0904.3542 [hep-ph], PRL

# In general, the final answer is complicated




(Inverted hierarchy)

G.Fogli, E.Lisi, A.Marrone, I.Tamborra, JCAP 0910:002 (2009)

*Is there a pattern ?*

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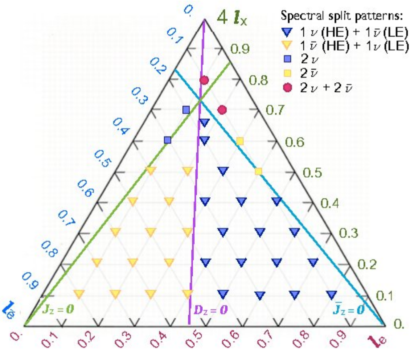
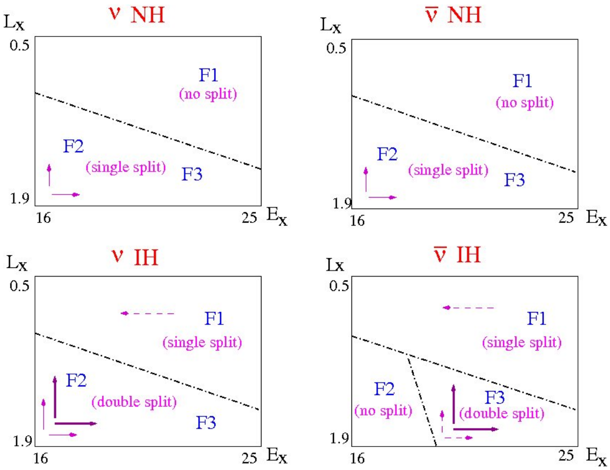



(Inverted hierarchy)

G.Fogli, E.Lisi, A.Marrone, I.Tamborra, JCAP 0910:002 (2009)

*Is there a pattern ?*

# "Phase diagrams" of spectral splits




$\langle E_0(\nu_e)\rangle = 12$ MeV, $\langle E_0(\bar{\nu}_e)\rangle = 15$ MeV, $L_{\nu_e} = L_{\bar{\nu}_e} = 1.0$

S.Choubey, B.Dasgupta, AD, A. Mirizzi, *Work in Progress*

# Survival probabilities with flux-mixing combinations

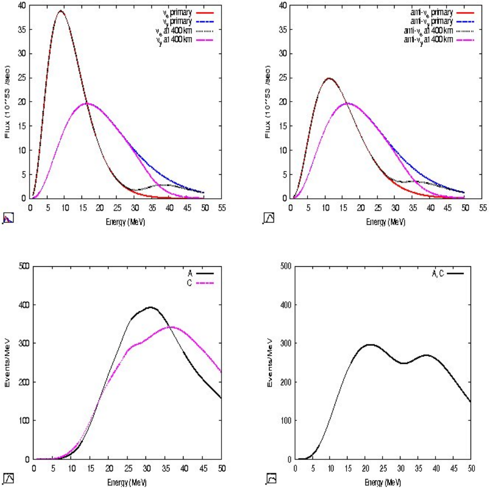
## $p$ at low, intermediate, high energies

| | $F_1$ | | | $F_2$ | | | $F_3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 | 0 | $s^2$ | 0 | 0 | $s^2$ |
| B | $s^2$ | 0 | 0 | $s^2$ | 0 | $s^2$ | $s^2$ | 0 | $s^2$ |
| C | $s^2$ | $s^2$ | $s^2$ | $s^2$ | $s^2$ | 0 | $s^2$ | $s^2$ | 0 |
| D | $s^2$ | 0 | 0 | $s^2$ | 0 | $s^2$ | $s^2$ | 0 | $s^2$ |

## $\bar{p}$ at low, intermediate, high energies

| | $F_1$ | | | $F_2$ | | | $F_3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| A | $c^2$ | $c^2$ | $c^2$ | $c^2$ | $c^2$ | 0 | $c^2$ | $c^2$ | 0 |
| B | 0 | $c^2$ | $c^2$ | 0 | 0 | 0 | 0 | $c^2$ | 0 |
| C | $c^2$ | $c^2$ | $c^2$ | $c^2$ | $c^2$ | 0 | $c^2$ | $c^2$ | 0 |
| D | $c^2$ | 0 | 0 | $c^2$ | $c^2$ | $c^2$ | $c^2$ | 0 | $c^2$ |

# $\nu_e$ and $\bar{\nu}_e$ spectra at detectors




F3

$$\langle E_0(\nu_e)\rangle = 12 \text{ MeV}$$
$$\langle E_0(\bar{\nu}_e)\rangle = 15 \text{ MeV}$$
$$\langle E_0(\nu_y)\rangle = 22 \text{ MeV}$$
$$L_{\bar{\nu}_e} = L_{\nu_e}$$
$$L_{\nu_y} = L_{\bar{\nu}_y} = 1.7 L_{\nu_e}$$

# Outline

# A recent nearby supernova: SN1987A

(Hubble image)

- Confirmed the SN cooling mechanism through neutrinos
- Number of events too small to say anything concrete about neutrino mixing
- Some constraints on SN parameters obtained

# Signal expected from a galactic SN (10 kpc)

## Water Cherenkov detector:

- $\bar{\nu}_e p \rightarrow n e^+$: $\approx 7000 - 12000^*$
- $\nu e^- \rightarrow \nu e^-$: $\approx 200 - 300^*$
- $\nu_e + ^{16}O \rightarrow X + e^-$: $\approx$ 150–800$^*$

$^*$ Events expected at Super-Kamiokande with a galactic SN at 10 kpc

## Carbon-based scintillation detector:

- $\bar{\nu}_e p \rightarrow n e^+$ ($\sim$ 300 per kt)
- $\nu + {}^{12}C \rightarrow \nu + X + \gamma$ (15.11 MeV)

## Liquid Argon detector:

- $\nu_e + {}^{40}Ar \rightarrow {}^{40}K^* + e^-$ ($\sim$ 300 per kt)

# Outline

# Earth matter effects

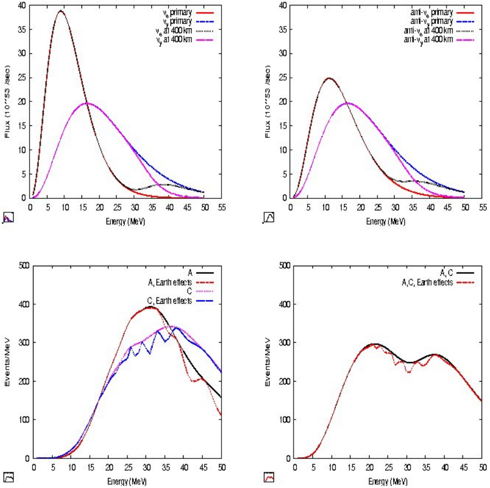
- If $F_{\nu_1}$ and $F_{\nu_2}$ reach the earth,

$$F^D_{\nu_e}(L) - F^D_{\nu_e}(0) \quad = \quad (F_{\nu_2} - F_{\nu_1}) \times \sin 2\theta^{\oplus}_{12} \sin(2\theta^{\oplus}_{12} - 2\theta_{12}) \sin^2\left(\frac{\Delta m^2_{\oplus} L}{4E}\right)$$

(Sign changes for antineutrinos)

- Nonzero Earth matter effects require
  - Neutrinos: $p \neq 0$
  - Antineutrinos: $\bar{p} \neq 0$
- Can distinguish scenarios depending on Earth effects in different energy regimes
- A more efficient way of detecting split positions

# Spectra at detectors with Earth effects

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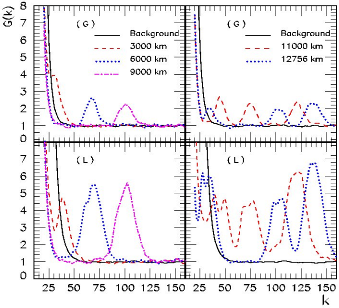
$L_{\bar{\nu}_e} = L_{\nu_e}$

$L_{\nu_y} = L_{\bar{\nu}_y} = 1.7 L_{\nu_e}$

# Earth effects: oscillations at a single detector

Fourier power spectrum: $G_N(k) = \frac{1}{N}\left|\sum_{events} e^{iky}\right|^2$
$(y \equiv 25\ MeV/E)$

- Peak positions model independent, at known frequencies




AD, M. Kachelrieß, G. Raffelt,

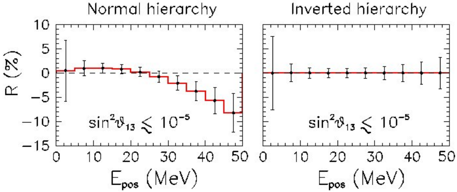
R. Tomàs, JCAP 0401:004 (2004)

# Comparison between two detectors

- Ratio of luminosities at IceCube and a megaton water Cherenkov, as a function of time

AD, M. Keil, G. Raffelt, JCAP 0306:005 (2003)

- Comparing spectra at two megaton water Cherenkovs




$$R \equiv \frac{N_{shadowed} - N_{unshadowed}}{N_{unshadowed}}$$

F1 . F2 and F3: NH $\leftrightarrow$ IH

B.Dasgupta, AD, A. Mirizzi, PRL101, 171801 (2008)

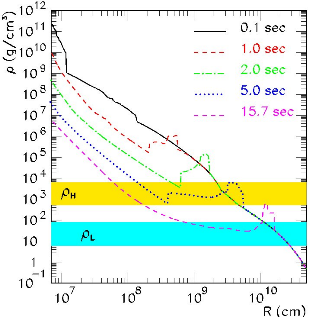
## Robust experimental signature

- Earth effects can distinguish hierarchies even for $\theta_{13} \lesssim 10^{-10}$

# Outline

# Shock wave and adiabaticity breaking

When shock wave passes through a resonance region (density $\rho_H$ or $\rho_L$):




- adiabatic resonances may become momentarily non-adiabatic
- Sharp changes in the final spectra even if the primary spectra change smoothly

R. C. Schirato, G. M. Fuller, astro-ph/0205390

G. L. Fogli, E. Lisi, D. Montanino and A. Mirizzi, PRD 68, 033005 (2003)

# Shock signals at a megaton water Cherenkov

- Time-dependent dip/peak features in $N_{\nu_e,\bar{\nu}_e}(E)$, $\langle E_{\nu_e,\bar{\nu}_e}\rangle$, $\langle E^2_{\nu_e,\bar{\nu}_e}\rangle$, etc.
- Times at which dips/peaks appear in $N_{\bar{\nu}_e}(E)$ are the times at which the shock waves enter the densities

$$\rho(E) = \frac{m_N \Delta m^2_{atm}}{2\sqrt{2} G_F Y_e E}$$

- $\Rightarrow$ Tracking of shock wave while it is still inside the mantle

R.Tomas, M.Kachelriess, G.Raffelt, AD, H.T.Janka and L.Scheck, JCAP **0409**, 015 (2004)

## Identifying mixing scenario

- Shock wave present in $\nu_e$ only for scenario A
- Shock wave present in $\bar{\nu}_e$ only for scenario B

# Shock signals at a megaton water Cherenkov

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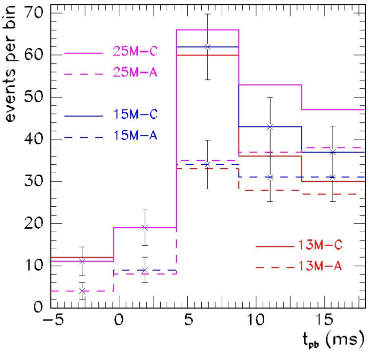
# Outline

# Vanishing $\nu_e$ burst




M. Kachelriess, R. Tomas, R. Buras,

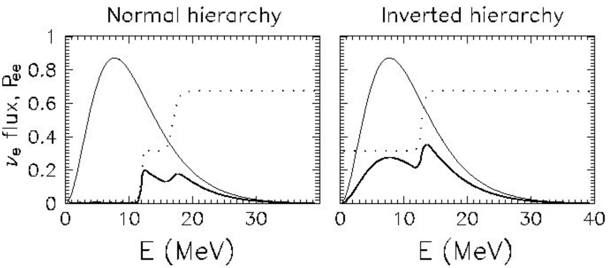
H. T. Janka, A. Marek and M. Rampp

PRD **71**, 063003 (2005)

- Time resolution of the detector crucial for separating $\nu_e$ burst from the accretion phase signal

Burst signal vanishes for mixing scenario A

# Stepwise spectral split in O-Ne-Mg supernovae




Neutronization burst

- MSW resonances deep inside collective regions

H. Duan, G. M. Fuller, J. Carlson, Y.Z.Qian, PRL100, 021101 (2008)

C. Lunardini, B. Mueller, H. T. Janka, arXiv:0712.3000

- “MSW-prepared” spectral splits: two for IH, one for NH

H.Duan, G.Fuller, Y.Z.Qian, PRD77, 085016 (2008)

- Positions of splits fixed by initial spectra

B.Dasgupta, AD, A. Mirizzi, G.G.Raffelt, PRD77, 1130007 (2008)

- $\nu_e$ suppression much more at low energy
- Identification of O-Ne-Mg supernova ??

# Outline

# Identifying neutrino mixing scenario

## Multiple independent signals

| | Earth Matter Effects | | | | Shock effects | | $\nu_e$ burst vanishing | Harder spectrum |
|---|---|---|---|---|---|---|---|---|
| | $\nu_e$ | | $\bar{\nu}_e$ | | $\nu_e$ | $\bar{\nu}_e$ | | |
| | Peak | Tail | Peak | Tail | | | | |
| AF1 | X | X | $\surd$ | $\surd$ | $\surd$ | X | $\surd$ | $\nu_e$ |
| AF2 | X | $\surd$ | $\surd$ | X | $\surd$ | X | $\surd$ | $\bar{\nu}_e$ |
| AF3 | X | $\surd$ | $\surd$ | X | $\surd$ | X | $\surd$ | $\bar{\nu}_e$ |
| BF1 | X | X | $\surd$ | $\surd$ | X | $\surd$ | X | $\nu_e$ |
| BF2 | X | $\surd$ | X | X | X | $\surd$ | X | $\bar{\nu}_e$ |
| BF3 | X | $\surd$ | $\surd$ | X | X | $\surd$ | X | $\bar{\nu}_e$ |
| CF1 | $\surd$ | $\surd$ | $\surd$ | $\surd$ | X | X | X | ? |
| CF2 | $\surd$ | X | $\surd$ | X | X | X | X | Same |
| CF3 | $\surd$ | X | $\surd$ | X | X | X | X | Same |
| DF1 | X | X | X | X | X | X | X | Same |
| DF2 | X | $\surd$ | $\surd$ | $\surd$ | X | X | X | ? |
| DF3 | X | $\surd$ | X | $\surd$ | X | X | X | ? |

# Smoking gun signals and caveats

## Earth matter effects

- Hierarchy identification even for extremely small $\theta_{13}$ values
- If primary fluxes are similar, identifying Earth effects is hard
- [flux-mixing scenario] $\leftrightarrow$ [split positions] mapping still preliminary

## Shock wave effects

- Presence / absence independent of collective effects
- Stochastic density fluctuations: may partly erase the shock wave imprint
- Turbulent convections behind the shock wave: gradual depolarization effects

## Neutronization burst signal

- Robust, but needs Ar detector with good time resolution

# Some open questions

## Better analytical understanding of collective effects

- Development of "pendular oscillations"

B.Dasgupta, G.G.Raggelt, I.Tamborra, arXiv:1001.5396 [hep-ph]

- Prediction of positions and widths of spectral swaps
- Multi-angle decoherence effects
- Effects of turbulence

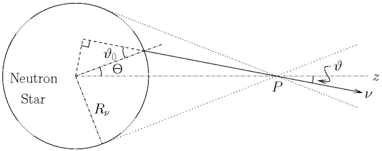
A. Friedland, arXiv:1001.0996 [hep-ph]

## Effect of $\nu$ oscillations on SN astrophysics

- Shock wave dynamics
- R-process nucleosynthesis

# Extra slides

# Multi-angle vs. single-angle approximation




H.Duan, G.M.Fuller, J.Carlson

Y.-Z. Qian, PRD74, 105014 (2006)

- Multi-angle effects only smear the spectra to some extent

G.L.Fogli, E. Lisi, A. Marrone, A. Mirizzi, JCAP 0712, 010 (2007)

- "Multi-angle decoherence" during collective oscillations suppressed by $\nu$–$\bar{\nu}$ asymmetry

A.Esteban-Pretel, S.Pastor, R.Tomas, G.Raffelt, G.Sigl, PRD76, 125018 (2007)

- If matter density is sufficiently high (may be possible during the accretion phase), multi-angle decoherence possible.

A.Esteban-Pretel, A.Mirizzi, S.Pastor, R.Tomas, G.G. Raffelt,

P.D.Serpico, G. Sigl, PRD78, 085012 (2008)

Single-angle approximation used unless specified

# Polarization-vector formalism

## 2-ν flavors : Formalism

- Expand all matrices in terms of Pauli matrices as

$$X = \frac{I}{2} + \frac{1}{2} \sum_{i=1,2,3} \mathrm{X}_i \, \sigma_i$$

- The following vectors result from the matrices

$$\rho_\mathrm{p} \Leftrightarrow \mathbf{P}_\omega$$

$$H_\mathrm{p}^0 \Leftrightarrow \omega \, \mathbf{B}$$

$$V \Leftrightarrow \sqrt{2} G_F N_e \, \mathbf{L} \equiv \lambda \, \mathbf{L}$$

$$H_\mathrm{p}^{\nu\nu} \Leftrightarrow \sqrt{2} G_F (n + \bar{n}) \int d\omega \, f(\omega) \, \mathbf{P}_\omega \, \mathrm{sgn}(\omega) \equiv \mu \, \mathbf{D}$$

- EOM resembles spin precession

$$\frac{d}{dr} \mathbf{P}_\omega = (h\omega \, \mathbf{B} + \lambda \, \mathbf{L} + \mu \, \mathbf{D}) \times \mathbf{P}_\omega \equiv \mathbf{H}_\omega \times \mathbf{P}_\omega$$