Beyond the gauge principle

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Tata Institute of Fundamental Research

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Outline

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Fritz London (1928)

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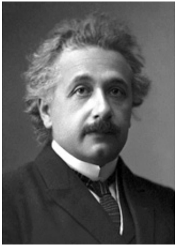

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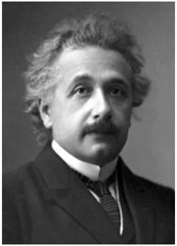

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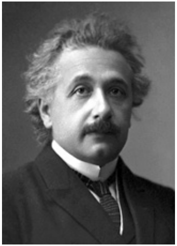

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- These equations were based on the principle of general relativity which basically asserts that the laws of nature take the same form in any choice of space-time coordinates.
- This principle can in fact be re-cast as a type of gauge invariance.

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There holds, as we now know, a principle of gauge invariance in nature; but it does not connect the electromagnetic potentials ϕ_μ, as I had assumed, with Einstein's gravitational potentials $g_{\mu\nu}$, but ties them to the four components of the wave field ... which ... represent the electron.

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- However, Weyl's nomenclature "gauge invariance" survived.

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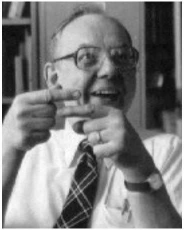

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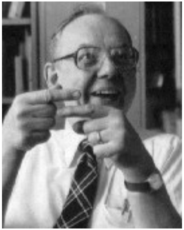

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- Thus, the gauge principle governs all the basic interactions observed in nature. This is experimentally verified beyond a shadow of doubt.

Outline

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- Notice that these are invariant under the transformations:
$$\vec{A} \to \vec{A} - \vec{\nabla}\lambda, \quad \phi \to \phi + \dot{\lambda}$$
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where the parameter $\lambda(\vec{x}, t)$ is an arbitrary function of space and time.
- This is the simplified form of gauge transformations in a situation where only electromagnetism (and no matter) is present.

- Now suppose a particle of mass m and electric charge e propagates in an electromagnetic field. Experimentally we know it obeys the Lorentz force law:

$$m\ddot{\vec{x}} = e\,\dot{\vec{x}} \times \vec{B} + e\vec{E}$$

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- In quantum mechanics the above equation should take the form of Heisenberg's equation of motion:

$$\frac{dO}{dt} = \frac{i}{\hbar}[H, O] + \frac{\partial O}{\partial t}$$

where $O = m\dot{\vec{x}}$ and H is some Hamiltonian operator.

- A simple calculation tells us that the Hamiltonian must be:

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where $\vec{p} = -i\hbar\vec{\nabla}$.

- The observation of Fock and London amounts to saying that this equation is invariant under:

$$\psi \to e^{-i\frac{e}{\hbar}\lambda(\vec{x},t)}\psi, \quad \vec{A} \to \vec{A} - \vec{\nabla}\lambda, \quad \phi \to \phi + \dot{\lambda}$$

for an arbitrary function λ. This now is the full gauge transformation and we see that it includes a phase multiplication on the wave-function.

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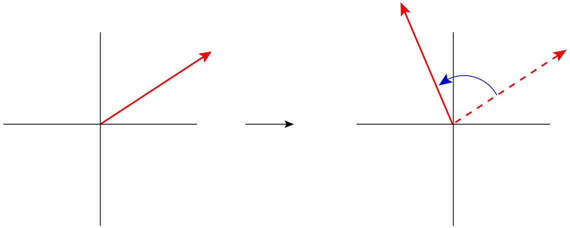

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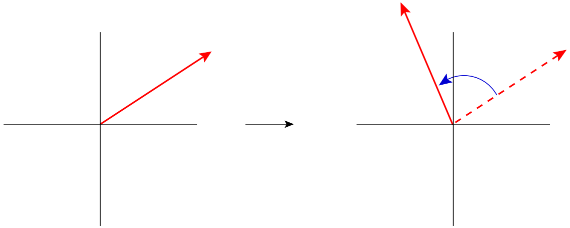

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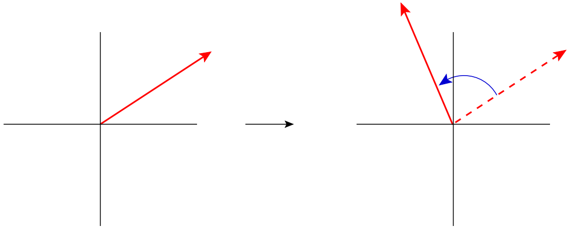

- A gauge transformation performs this rotation independently at each point of space and time. Therefore it is also called a local symmetry transformation.
- One should keep in mind that the rotation is not in real space but in "internal space" in which the wave function is valued.

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- In this case, the derivatives acting on ψ would bring down extra factors. The transformation of $(\vec{A}, \phi)$ just cancels these factors.
- Without the electromagnetic field $(\vec{A}, \phi)$ we could not possibly have gauge invariance! Conversely, imposing gauge invariance on matter fields requires the electromagnetic field to exist.

- All configurations related by gauge transformations are supposed to describe the same physical situation:

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- In non-relativistic physics gauge symmetry is just an elegant property, but as we will soon see, in relativistic physics it is crucial for consistency and has predictive power.

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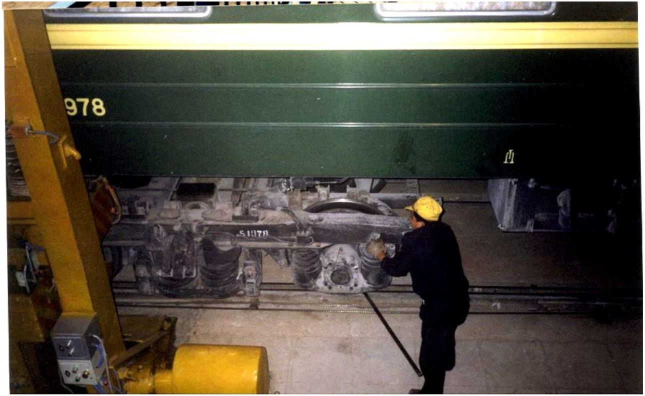

Gauge transformation being performed on a coach of the trans-Siberian railway

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Gauge symmetry in relativistic physics

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- The electromagnetic potentials $(\vec{A}, \phi)$ also combine into a quantum field A_μ that creates and destroys photons.
- The photons created by A_μ would have four polarisations, one for each $\mu = 0, 1, 2, 3$. This flatly contradicts experiment! It also contradicts unitarity because the state:

$$|\mu\rangle \sim A_\mu|0\rangle$$

must, by Lorentz invariance, satisfy:

$$\langle\mu|\nu\rangle \sim \eta_{\mu\nu} \quad \text{(Minkowski metric)}$$

and therefore some components have a negative norm.

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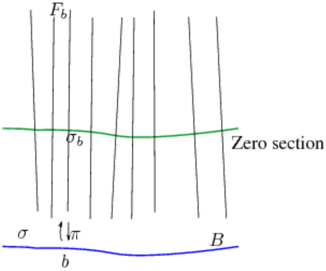
- As a result we are in agreement with experiment, as well as with positivity of probabilities.
- It is curious that gauge symmetry requires the photon to exist in the first place, thereby creating a potential problem with unitarity, and then solves that same problem!

- In mathematics, the gauge principle is related to connections on vector bundles.



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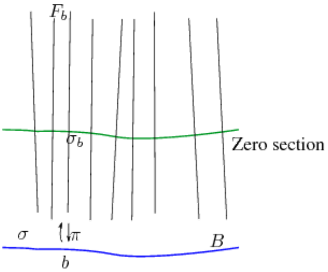

- The statement of gauge invariance becomes a statement about cohomology. So it would not be wrong to say the photon has two polarisations rather than four because it is a cohomology class!

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- This generalisation of Lorentz symmetry is called "local Lorentz symmetry".

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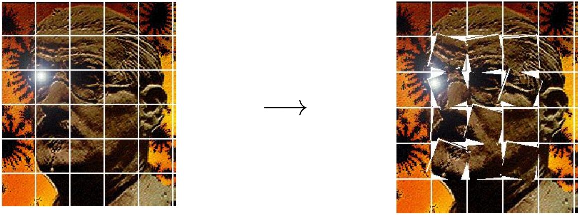

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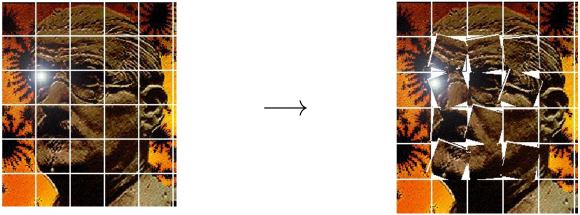

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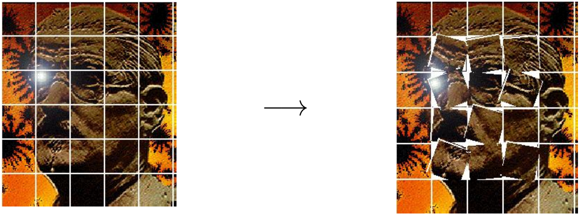

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- Therefore here too, a type of gauge symmetry correctly predicts a field and its interactions.

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Yang-Mills gauge symmetry

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- If $\boldsymbol{U}$ depends on $(\vec{x}, t)$ then this is a local gauge transformation. It requires a compensating field $\boldsymbol{A}_\mu(\vec{x}, t)$ which is a 2×2 Hermitian matrix.

- A new feature with respect to electrodynamics is that two transformations by matrices $\boldsymbol{U}^{(1)}$ and $\boldsymbol{U}^{(2)}$ do not necessarily commute:

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  where $\boldsymbol{T}^a$ form a representation of the Lie algebra.
- They obey the relation:

$$[\boldsymbol{T}^a, \boldsymbol{T}^b] = f^{ab}{}_c \boldsymbol{T}^c$$

  where $f^{ab}{}_c$ are the structure constants of the algebra.

- The key observation of Yang and Mills was that the gauge transformation must be non-linear and the associated field strength is given by:

$$\boldsymbol{F}_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu - g[\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

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- Gauge-invariant interactions can be made out of products of traces. The simplest one is:

$$\mathrm{tr}\, \boldsymbol{F}^{\mu\nu} \boldsymbol{F}_{\mu\nu}$$

  which contains the terms (inside the trace):

$$(\partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu)^2 - 4g\, \partial_\mu \boldsymbol{A}_\nu [\boldsymbol{A}^\mu, \boldsymbol{A}^\nu] + g^2 [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu][\boldsymbol{A}^\mu, \boldsymbol{A}^\nu]$$

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  (ii) Solves the potential unitarity problem associated to these particles,

  (iii) Predicts a relation between the different possible self-interactions allowed by Lorentz invariance:

$$(\partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu)^2 + \alpha\, \partial_\mu \boldsymbol{A}_\nu [\boldsymbol{A}^\mu, \boldsymbol{A}^\nu] + \beta [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu][\boldsymbol{A}^\mu, \boldsymbol{A}^\nu]$$

  namely,

$$\beta = \left(\frac{\alpha}{4}\right)^2$$

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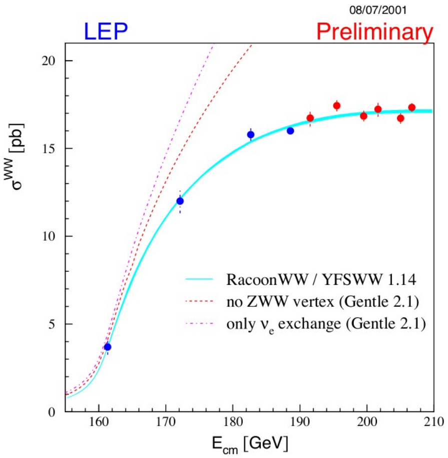

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- However the weak interactions are short-range and therefore the mediating particles must be massive.
- This problem was resolved via the Higgs mechanism, a surprising mechanism in which gauge invariance, though present, appears to be "spontaneously broken".
- Something different ("confinement") takes place for strong interactions.
- We see that physical implementation of the gauge principle can take a long time and may require novel mechanisms.

Outline

Supergravity: a new gauge principle

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- In each case, supersymmetry changes the spin by $\frac{1}{2}$ (in units of $\hbar$).

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- This is the first time in physics that a basic symmetry like translation has been written as a composite of another symmetry!

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- Then a beautiful thing happens. Since two supersymmetries combine into a translation, local supersymmetry implies local translation symmetry.
- This in turn is the same as local Lorentz invariance which, as we have seen, implies the existence of gravity.
- Therefore local supersymmetry gives rise to a supersymmetric extension of gravity called supergravity.

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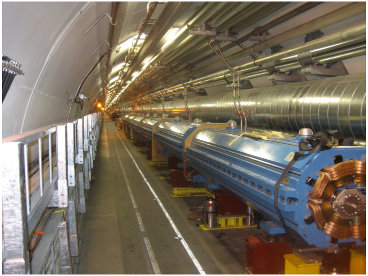

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- If gravitinos are found to exist, it will confirm that at the most fundamental level, Nature chooses to be governed by gauge theories!

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- This is appealing because fermions occur rather abundantly in nature, and these arise automatically once we have supermanifolds.

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String theory

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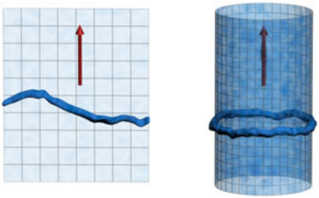

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- But what sort of particle theory? This is entirely dictated by the background in which the string propagates.
- The simplest background is a flat Minkowski space-time.
- Here we understand how to satisfy conformal invariance. All physical states of the string must be annihilated by an infinite set of "Virasoro operators":

$$L_n|\text{phys}\rangle = 0$$

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- The first condition says the field is massless. Taken together, the two conditions give us the Maxwell equations:

$$k^\mu\,(k_\mu \zeta_\nu - k_\nu \zeta_\mu) = 0 \quad \leftrightarrow \quad \partial^\mu F_{\mu\nu} = 0$$

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- This means it is equivalent to zero. Thus for arbitrary $\Lambda(k)$, we have the equivalence of polarisation vectors:

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- Thus open string theory has gauge invariance! We did not require it, rather it emerged upon quantising the theory.

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- This leads one to suspect that at low energies, closed strings describe gravity, including its gauge symmetries. This is true and has by now been confirmed in many different ways.
- Thus in string theory, both the original gauge principles (electromagnetism and gravity) emerge automatically (one from open strings and the other from closed strings).

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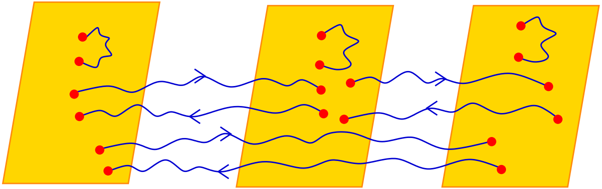

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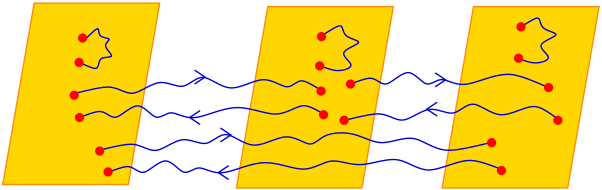

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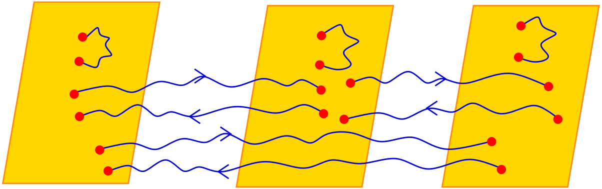

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- Mathematicians associate Lie algebras to a root diagram. The above is a physical realisation of this root diagram!

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- Closed strings inevitably produce massless "tensor fields". An example is the 2nd rank tensor $B_{\mu\nu}$, whose interactions are invariant under:

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- Also in some backgrounds, string theory exhibits an infinite-dimensional W_∞ symmetry.

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- It originated, like many other good things, with Nambu:

- His idea was to generalise the 2d phase space, which comes with variables (p, q), to a 3d space with variables, say, (p, q, r).
- Then the Poisson bracket can be naturally generalised:

$$[G, H] \;\rightarrow\; [F, G, H] = \frac{\partial F}{\partial p}\frac{\partial G}{\partial q}\frac{\partial H}{\partial r} \pm \cdots$$

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- This field theory is supposed to describe membrane excitations of strongly coupled string theory.
- The simplest such theory turns out to be uniquely determined by supersymmetry. It has eight real scalar particles and eight 2-component fermions.
- Now in (2+1) dimensions it is known that scalar fields have a scale dimension $\frac{1}{2}$ and therefore the interaction ϕ^6 is dimensionless – a necessary (though not sufficient) requirement for conformal invariance.

- The novel mathematical structure employed in these works is the concept of a 3-algebra:

$$[\boldsymbol{T}^a, \boldsymbol{T}^b, \boldsymbol{T}^c] = f^{abc}{}_d \, \boldsymbol{T}^d$$

generalising the notion of Lie algebra:

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- If we have a number of scalar fields $\phi^{I\,a}$ and we write:

$$\boldsymbol{\phi}^I = \phi^{I\,a} \boldsymbol{T}^a$$

then it's natural to postulate a 6-th order coupling:

$$\mathrm{tr}[\boldsymbol{\phi}^I, \boldsymbol{\phi}^J, \boldsymbol{\phi}^K]^2 \sim f^{abc}{}_g f^{defg}\, \phi^I_a\, \phi^J_b\, \phi^K_c\, \phi^I_d\, \phi^J_e\, \phi^K_f$$

by analogy with the 4th-order coupling:

$$\mathrm{tr}[\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]^2 \sim f^{abc} f^{de}{}_c\, A_{\mu\,a}\, A_{\nu\,b}\, A^\mu{}_d\, A^\nu{}_e$$

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- This theory has some remarkable properties including the possibility of the non-dynamical gauge field transmuting into a dynamical one. These will be discussed in my subsequent lectures here.
- 3-algebras are the most recent form of gauge symmetry to be introduced in physics. They could be relevant not only in the particle physics/string theory context but also for condensed-matter systems in the context of quantum criticality.

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- This principle endows relativistic quantum field theory with new particles, consistency and predictive power.
- New gauge symmetries like supergravity and 3-algebra symmetries have been proposed and may well be tested.
- String theory naturally embodies the gauge principle and has given us clues about how it could be generalised.