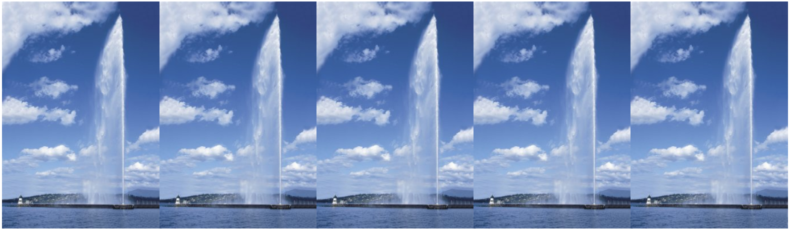
Multiple Membrane Dynamics



Sunil Mukhi
Tata Institute of Fundamental Research, Mumbai

Strings 2008, Geneva, August 19, 2008

- Based on:

  "M2 to D2",
  SM and Costis Papageorgakis,
  arXiv:0803.3218 [hep-th], JHEP 0805:085 (2008).

  " M2-branes on M-folds",
  Jacques Distler, SM, Costis Papageorgakis and Mark van Raamsdonk,
  arXiv:0804.1256 [hep-th], JHEP 0805:038 (2008).

  " D2 to D2",
  Bobby Ezhuthachan, SM and Costis Papageorgakis,
  arXiv:0806.1639 [hep-th], JHEP 0807:041, (2008).

  Mohsen Alishahiha and SM, to appear

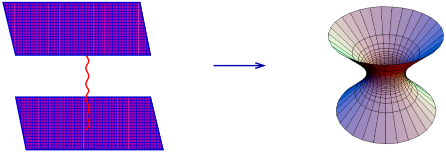
Outline

Motivation

- We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well.

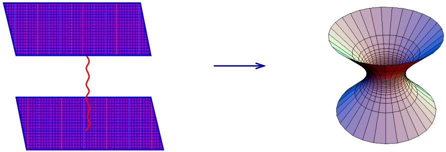
Motivation

- We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well.
- The latter should hold the key to M-theory:



Motivation

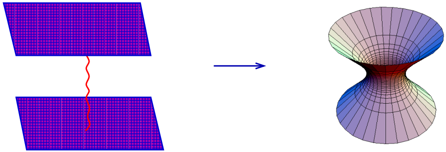
- We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well.
- The latter should hold the key to M-theory:



- While there is an obstacle (due to (anti) self-dual 2-forms) to writing the $M5$-brane field theory, there is no obstacle for $M2$-branes as far as we know.

Motivation

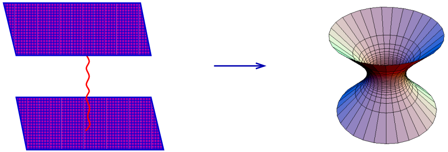
- We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well.
- The latter should hold the key to M-theory:



- While there is an obstacle (due to (anti) self-dual 2-forms) to writing the $M5$-brane field theory, there is no obstacle for $M2$-branes as far as we know.
- And yet, despite ~ 200 recent papers – and two Strings 2008 talks – on the subject, we don't exactly know what the multiple membrane theory is.

Motivation

- We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well.
- The latter should hold the key to M-theory:



- While there is an obstacle (due to (anti) self-dual 2-forms) to writing the $M5$-brane field theory, there is no obstacle for $M2$-branes as far as we know.
- And yet, despite ~ 200 recent papers – and two Strings 2008 talks – on the subject, we don't exactly know what the multiple membrane theory is.
- Even the French aristocracy doesn't seem to know...


2005
Grand Vin du
BARON DE BRANE
MARGAUX
APPELLATION MARGAUX CONTRÔLÉE
HENRI LURTON
SOCIÉTÉ VITICOLE HENRI LURTON - 33460 MARGAUX - FRANCE

- Of course, there is one description that is clearly right and has manifest $\mathcal{N} = 8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM} \to \infty} \frac{1}{g_{YM}^2} \mathcal{L}_{SYM}$$

- Of course, there is one description that is clearly right and has manifest $\mathcal{N}=8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM}\to\infty}\frac{1}{g_{YM}^2}\mathcal{L}_{SYM}$$

- The question is whether this conformal IR fixed point has an explicit Lagrangian description wherein all the symmetries are manifest.

- Of course, there is one description that is clearly right and has manifest $\mathcal{N} = 8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM} \to \infty} \frac{1}{g_{YM}^2} \mathcal{L}_{SYM}$$

- The question is whether this conformal IR fixed point has an explicit Lagrangian description wherein all the symmetries are manifest.
- This includes a global $SO(8)_R$ symmetry describing rotations of the space transverse to the membranes – enhanced from the $SO(7)$ of SYM.

- Of course, there is one description that is clearly right and has manifest $\mathcal{N}=8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM}\to\infty} \frac{1}{g_{YM}^2}\mathcal{L}_{SYM}$$

- The question is whether this conformal IR fixed point has an explicit Lagrangian description wherein all the symmetries are manifest.
- This includes a global $SO(8)_R$ symmetry describing rotations of the space transverse to the membranes – enhanced from the $SO(7)$ of SYM.
- Let us look at the Lagrangians that have been proposed to describe this limit.

- Euclidean 3-algebra [Bagger-Lambert, Gustavsson]: Labelled by integer k. Algebra is $SU(2) \times SU(2)$.

  $\Rightarrow$ Argued to describe a pair of $M2$ branes at Z_k singularity. But no generalisation to > 2 branes.

- Euclidean 3-algebra [Bagger-Lambert, Gustavsson]: Labelled by integer k. Algebra is $SU(2) \times SU(2)$.

  $\Rightarrow$ Argued to describe a pair of $M2$ branes at Z_k singularity. But no generalisation to > 2 branes.

- Lorentzian 3-algebra [Gomis-Milanesi-Russo, Benvenuti-Rodriguez-Gomez-Tonni-Verlinde, Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]: Based on arbitrary Lie algebras, have $\mathcal{N} = 8$ superconformal invariance.

  $\Rightarrow$ Certainly correspond to $D2$-branes, and perhaps to $M2$-branes. Status of latter unclear at the moment.

- Euclidean 3-algebra [Bagger-Lambert, Gustavsson]: Labelled by integer k. Algebra is $SU(2) \times SU(2)$.

  $\Rightarrow$ Argued to describe a pair of $M2$ branes at Z_k singularity. But no generalisation to > 2 branes.

- Lorentzian 3-algebra [Gomis-Milanesi-Russo, Benvenuti-Rodriguez-Gomez-Tonni-Verlinde, Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]: Based on arbitrary Lie algebras, have $\mathcal{N} = 8$ superconformal invariance.

  $\Rightarrow$ Certainly correspond to $D2$-branes, and perhaps to $M2$-branes. Status of latter unclear at the moment.

- ABJM theories [Aharony-Bergman-Jafferis-Maldacena]: Labelled by algebra $G \times G'$ and integer k, with $\mathcal{N} = 6$ superconformal invariance. Is actually a "relaxed" 3-algebra.

  $\Rightarrow$ Describe multiple M2-branes at orbifold singularities. But the $k = 1$ theory is missing two manifest supersymmetries and decoupling of CM mode not visible.

- These theories all have 8 scalars and 8 fermions.

- These theories all have 8 scalars and 8 fermions.
- And they have non-dynamical (Chern-Simons-like) gauge fields.

- These theories all have 8 scalars and 8 fermions.
- And they have non-dynamical (Chern-Simons-like) gauge fields.
- Thus the basic classification is:

(i) Euclidean signature 3-algebras, which are $G \times G$ Chern-Simons theories:

$$k \text{ tr}\left(\boldsymbol{A} \wedge d\boldsymbol{A} + \tfrac{2}{3}\boldsymbol{A} \wedge \boldsymbol{A} \wedge \boldsymbol{A} - \tilde{\boldsymbol{A}} \wedge d\tilde{\boldsymbol{A}} - \tfrac{2}{3}\tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}}\right)$$

$$\begin{aligned} \text{BLG} &: G = SU(2) \\ \text{ABJM} &: G = SU(N) \text{ or } U(N), \text{ any } N \text{ (+ other choices)} \\ \text{both} &: \text{scalars, fermions are bi-fundamental, e.g. } X^I_{a\dot{a}} \end{aligned}$$

(ii) Lorentzian signature 3-algebras, which are $\boldsymbol{B} \wedge \boldsymbol{F}$ theories based on any Lie algebra.

$$\text{scalars, fermions are singlet + adjoint, e.g. } X^I_+, \boldsymbol{X}^I$$

- Both classes make use of the triple product X^{IJK}:

$$\text{Euclidean}: \quad X^{IJK} \sim X^I X^{J\dagger} X^K, \quad X^I \text{ bi-fundamental}$$

$$\text{Lorentzian}: \quad X^{IJK} \sim X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + \text{cyclic}$$
$$X_+^I = \text{singlet}, \boldsymbol{X}^J = \text{adjoint}$$

- Both classes make use of the triple product X^{IJK}:

$$\text{Euclidean}: \quad X^{IJK} \sim X^I X^{J\dagger} X^K, \quad X^I \text{ bi-fundamental}$$

$$\text{Lorentzian}: \quad X^{IJK} \sim X^I_+ [\boldsymbol{X}^J, \boldsymbol{X}^K] + \text{cyclic}$$
$$X^I_+ = \text{singlet}, \boldsymbol{X}^J = \text{adjoint}$$

- The potential is:

$$V(X) \sim (\boldsymbol{X}^{IJK})^2$$

therefore sextic.

- One might have expected a simple and unique description for the theory on N M2-branes in flat spacetime.

- One might have expected a simple and unique description for the theory on N M2-branes in flat spacetime.
- It is basically an analogue of 4d $\mathcal{N} = 4$ super-Yang-Mills!

- One might have expected a simple and unique description for the theory on N M2-branes in flat spacetime.
- It is basically an analogue of 4d $\mathcal{N} = 4$ super-Yang-Mills!
- The only "excuse" we have for not doing better is that the theory we seek will be strongly coupled. So it's not even clear what the classical action means.

- One might have expected a simple and unique description for the theory on N M2-branes in flat spacetime.
- It is basically an analogue of 4d $\mathcal{N} = 4$ super-Yang-Mills!
- The only "excuse" we have for not doing better is that the theory we seek will be strongly coupled. So it's not even clear what the classical action means.
- However it's also maximally superconformal, which should give us a lot of power in dealing with it.

- One might have expected a simple and unique description for the theory on N M2-branes in flat spacetime.
- It is basically an analogue of 4d $\mathcal{N} = 4$ super-Yang-Mills!
- The only "excuse" we have for not doing better is that the theory we seek will be strongly coupled. So it's not even clear what the classical action means.
- However it's also maximally superconformal, which should give us a lot of power in dealing with it.
- In this talk I'll deal with some things we have understood about the desired theory.

Outline

The Higgs mechanism

- For the $G \times G$ Chern-Simons class of theories, the following holds true [SM-Papageorgakis].

The Higgs mechanism

- For the $G \times G$ Chern-Simons class of theories, the following holds true [SM-Papageorgakis].
- If we give a vev v to one component of the bi-fundamental fields, then at energies below this vev, the Lagrangian becomes:

$$L_{CS}^{(G\times G)}\Big|_{vev\ \ v} = \frac{1}{v^2} L_{SYM}^{(G)} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

and the G gauge field has become dynamical!

The Higgs mechanism

- For the $G \times G$ Chern-Simons class of theories, the following holds true [SM-Papageorgakis].
- If we give a vev v to one component of the bi-fundamental fields, then at energies below this vev, the Lagrangian becomes:

$$L_{CS}^{(G\times G)}\Big|_{vev\ v} = \frac{1}{v^2} L_{SYM}^{(G)} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

  and the G gauge field has become dynamical!
- This is an unusual result. In SYM with gauge group G, when we give a vev to one component of an adjoint scalar, at low energy the Lagrangian becomes:

$$\frac{1}{g^2_{\mathrm{YM}}} L_{SYM}^{(G)}\Big|_{vev\ v} = \frac{1}{g^2_{\mathrm{YM}}} L_{SYM}^{(G'\subset G)}$$

  where G' is the subgroup that commutes with the vev.

- Let's give a quick derivation of this novel Higgs mechanism, first for $k = 1$:

$$\begin{aligned} L_{CS} &= \mathrm{tr}\left(\boldsymbol{A} \wedge d\boldsymbol{A} + \tfrac{2}{3}\boldsymbol{A} \wedge \boldsymbol{A} \wedge \boldsymbol{A} - \tilde{\boldsymbol{A}} \wedge d\tilde{\boldsymbol{A}} - \tfrac{2}{3}\tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}}\right) \\ &= \mathrm{tr}\left(\boldsymbol{A}_- \wedge \boldsymbol{F}_+ + \tfrac{1}{6}\boldsymbol{A}_- \wedge \boldsymbol{A}_- \wedge \boldsymbol{A}_-\right) \end{aligned}$$

where $\boldsymbol{A}_\pm = \boldsymbol{A} \pm \tilde{\boldsymbol{A}}, \qquad \boldsymbol{F}_+ = d\boldsymbol{A}_+ + \frac{1}{2}\boldsymbol{A}_+ \wedge \boldsymbol{A}_+$.

- Let's give a quick derivation of this novel Higgs mechanism, first for $k = 1$:

$$\begin{aligned} L_{CS} &= \mathrm{tr}\left(\boldsymbol{A} \wedge d\boldsymbol{A} + \tfrac{2}{3}\boldsymbol{A} \wedge \boldsymbol{A} \wedge \boldsymbol{A} - \tilde{\boldsymbol{A}} \wedge d\tilde{\boldsymbol{A}} - \tfrac{2}{3}\tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}}\right) \\ &= \mathrm{tr}\left(\boldsymbol{A}_- \wedge \boldsymbol{F}_+ + \tfrac{1}{6}\boldsymbol{A}_- \wedge \boldsymbol{A}_- \wedge \boldsymbol{A}_-\right) \end{aligned}$$

where $\boldsymbol{A}_\pm = \boldsymbol{A} \pm \tilde{\boldsymbol{A}}, \qquad \boldsymbol{F}_+ = d\boldsymbol{A}_+ + \frac{1}{2}\boldsymbol{A}_+ \wedge \boldsymbol{A}_+$.

- Also the covariant derivative on a scalar field is:

$$D_\mu X = \partial_\mu X - \boldsymbol{A}_\mu X + X\tilde{\boldsymbol{A}}_\mu$$

- Let's give a quick derivation of this novel Higgs mechanism, first for $k=1$:

$$\begin{aligned} L_{CS} &= \mathrm{tr}\left(\boldsymbol{A}\wedge d\boldsymbol{A}+\tfrac{2}{3}\boldsymbol{A}\wedge\boldsymbol{A}\wedge\boldsymbol{A}-\tilde{\boldsymbol{A}}\wedge d\tilde{\boldsymbol{A}}-\tfrac{2}{3}\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\right) \\ &= \mathrm{tr}\left(\boldsymbol{A}_-\wedge\boldsymbol{F}_+ +\tfrac{1}{6}\boldsymbol{A}_-\wedge\boldsymbol{A}_-\wedge\boldsymbol{A}_-\right)\end{aligned}$$

where $\boldsymbol{A}_\pm=\boldsymbol{A}\pm\tilde{\boldsymbol{A}}, \qquad \boldsymbol{F}_+=d\boldsymbol{A}_+ +\frac{1}{2}\boldsymbol{A}_+\wedge\boldsymbol{A}_+$.

- Also the covariant derivative on a scalar field is:

$$D_\mu X=\partial_\mu X-\boldsymbol{A}_\mu X+X\tilde{\boldsymbol{A}}_\mu$$

- If $\langle X\rangle = v\,\mathbf{1}$ then:

$$-(D_\mu X)^2\sim -v^2(\boldsymbol{A}_-)_\mu(\boldsymbol{A}_-)^\mu+\cdots$$

- Let's give a quick derivation of this novel Higgs mechanism, first for $k=1$:

$$\begin{aligned} L_{CS} &= \operatorname{tr}\left(\boldsymbol{A}\wedge d\boldsymbol{A}+\tfrac{2}{3}\boldsymbol{A}\wedge\boldsymbol{A}\wedge\boldsymbol{A}-\tilde{\boldsymbol{A}}\wedge d\tilde{\boldsymbol{A}}-\tfrac{2}{3}\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\right) \\ &= \operatorname{tr}\left(\boldsymbol{A}_-\wedge\boldsymbol{F}_+ +\tfrac{1}{6}\boldsymbol{A}_-\wedge\boldsymbol{A}_-\wedge\boldsymbol{A}_-\right) \end{aligned}$$

  where $\boldsymbol{A}_\pm=\boldsymbol{A}\pm\tilde{\boldsymbol{A}}$, $\quad \boldsymbol{F}_+=d\boldsymbol{A}_+ +\frac{1}{2}\boldsymbol{A}_+\wedge\boldsymbol{A}_+$.

- Also the covariant derivative on a scalar field is:

$$D_\mu X=\partial_\mu X-\boldsymbol{A}_\mu X+X\tilde{\boldsymbol{A}}_\mu$$

- If $\langle X\rangle = v\,\mathbf{1}$ then:

$$-(D_\mu X)^2\sim -v^2(\boldsymbol{A}_-)_\mu(\boldsymbol{A}_-)^\mu+\cdots$$

- Thus, $\boldsymbol{A}_-$ is massive – but not dynamical. Integrating it out gives us:

$$-\frac{1}{4v^2}(\boldsymbol{F}_+)_{\mu\nu}(\boldsymbol{F}_+)^{\mu\nu}+\mathcal{O}\left(\frac{1}{v^3}\right)$$

  so $\boldsymbol{A}_+$ becomes dynamical.

- One can check that the bi-fundamental X^I reduces to an adjoint under $\boldsymbol{A}_+$. The rest of $\mathcal{N} = 8$ SYM assembles itself correctly.

- One can check that the bi-fundamental X^I reduces to an adjoint under $\boldsymbol{A}_+$. The rest of $\mathcal{N}=8$ SYM assembles itself correctly.
- But how should we physically interpret this?

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{1}{v^2}L_{SYM}^{G} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

- One can check that the bi-fundamental X^I reduces to an adjoint under $\boldsymbol{A}_+$. The rest of $\mathcal{N} = 8$ SYM assembles itself correctly.
- But how should we physically interpret this?

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{1}{v^2} L_{SYM}^{G} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

- It seems like the $M2$ is becoming a $D2$ with YM coupling v.

- One can check that the bi-fundamental X^I reduces to an adjoint under $\boldsymbol{A}_+$. The rest of $\mathcal{N}=8$ SYM assembles itself correctly.
- But how should we physically interpret this?

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{1}{v^2}L_{SYM}^{G} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

- It seems like the $M2$ is becoming a $D2$ with YM coupling v.
- Have we somehow compactified the theory? No.

- One can check that the bi-fundamental X^I reduces to an adjoint under $\boldsymbol{A}_+$. The rest of $\mathcal{N}=8$ SYM assembles itself correctly.
- But how should we physically interpret this?

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{1}{v^2}L_{SYM}^{G} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

- It seems like the $M2$ is becoming a $D2$ with YM coupling v.
- Have we somehow compactified the theory? No.
- For any finite v, there are corrections to the SYM. These decouple only as $v\to\infty$. So at best we can say that:

$$L_{CS}^{G\times G}\Big|_{vev\ v\to\infty} = \lim_{v\to\infty}\frac{1}{v^2}L_{SYM}^{G}$$

- One can check that the bi-fundamental X^I reduces to an adjoint under $\boldsymbol{A}_+$. The rest of $\mathcal{N}=8$ SYM assembles itself correctly.
- But how should we physically interpret this?

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{1}{v^2} L_{SYM}^{G} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

- It seems like the $M2$ is becoming a $D2$ with YM coupling v.
- Have we somehow compactified the theory? No.
- For any finite v, there are corrections to the SYM. These decouple only as $v \to \infty$. So at best we can say that:

$$L_{CS}^{G\times G}\Big|_{vev\ v\to\infty} = \lim_{v\to\infty} \frac{1}{v^2} L_{SYM}^{G}$$

- The RHS is by definition the theory on $M2$-branes! So this is more like a "proof" that the original Chern-Simons theory really is the theory on $M2$-branes.

- However once we introduce the Chern-Simons level k then the analysis is different [Distler-SM-Papageorgakis-van Raamsdonk]:

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{k}{v^2} L_{SYM}^{G} + \mathcal{O}\left(\frac{k}{v^3}\right)$$

- However once we introduce the Chern-Simons level k then the analysis is different [Distler-SM-Papageorgakis-van Raamsdonk]:

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{k}{v^2} L_{SYM}^{G} + \mathcal{O}\left(\frac{k}{v^3}\right)$$

- If we take $k \to \infty, v \to \infty$ with $v^2/k = g_{\mathsf{YM}}$ fixed, then in this limit the RHS actually becomes:

$$\frac{1}{g_{\mathsf{YM}}^2} L_{SYM}^{G}$$

and this is definitely the Lagrangian for $D2$ branes at finite coupling.

- However once we introduce the Chern-Simons level k then the analysis is different [Distler-SM-Papageorgakis-van Raamsdonk]:

$$L_{CS}^{G\times G}\Big|_{vev\ v} = \frac{k}{v^2} L_{SYM}^{G} + \mathcal{O}\left(\frac{k}{v^3}\right)$$

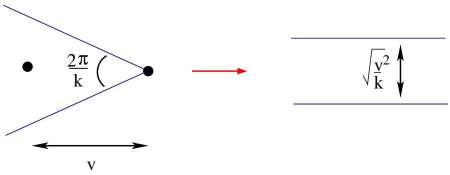
- If we take $k \to \infty, v \to \infty$ with $v^2/k = g_{\text{YM}}$ fixed, then in this limit the RHS actually becomes:

$$\frac{1}{g_{\text{YM}}^2} L_{SYM}^{G}$$

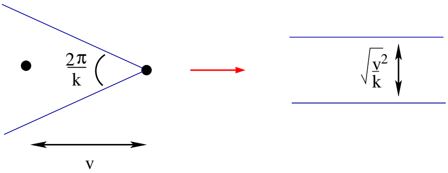
  and this is definitely the Lagrangian for $D2$ branes at finite coupling.

- So this time we have compactified the theory! How can that be?

- We proposed this should be understood as deconstruction for an orbifold C^4/Z_k:

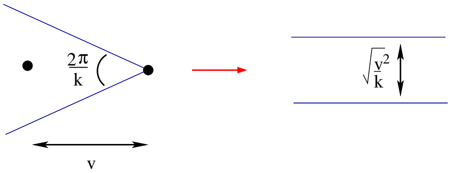


- We proposed this should be understood as deconstruction for an orbifold C^4/Z_k:



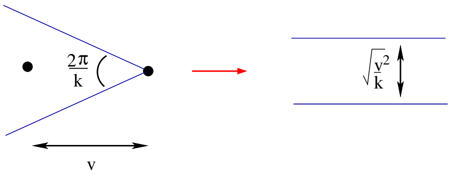
- In our paper we observed that the orbifold C^4/Z_k has $\mathcal{N}=6$ supersymmetry and $SU(4)$ R-symmetry. We thought this might be enhanced to $\mathcal{N}=8$ for some unknown reason.

- We proposed this should be understood as deconstruction for an orbifold C^4/Z_k:



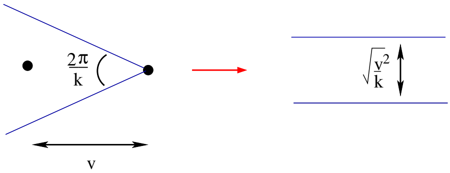
- In our paper we observed that the orbifold C^4/Z_k has $\mathcal{N} = 6$ supersymmetry and $SU(4)$ R-symmetry. We thought this might be enhanced to $\mathcal{N} = 8$ for some unknown reason.
- Instead, as ABJM found, it's the BLG field theory that needs to be modified to have $\mathcal{N} = 6$.

- We proposed this should be understood as deconstruction for an orbifold C^4/Z_k:



- In our paper we observed that the orbifold C^4/Z_k has $\mathcal{N}=6$ supersymmetry and $SU(4)$ R-symmetry. We thought this might be enhanced to $\mathcal{N}=8$ for some unknown reason.
- Instead, as ABJM found, it's the BLG field theory that needs to be modified to have $\mathcal{N}=6$.
- One lesson we learn is that for large k we are in the regime of weakly coupled string theory.

- We proposed this should be understood as deconstruction for an orbifold C^4/Z_k:



- In our paper we observed that the orbifold C^4/Z_k has $\mathcal{N}=6$ supersymmetry and $SU(4)$ R-symmetry. We thought this might be enhanced to $\mathcal{N}=8$ for some unknown reason.
- Instead, as ABJM found, it's the BLG field theory that needs to be modified to have $\mathcal{N}=6$.
- One lesson we learn is that for large k we are in the regime of weakly coupled string theory.
- A lot can be done in that regime, but for understanding the basics of M2-branes, that is not where we want to be.

Outline

Lorentzian 3-algebras

- The Lorentzian 3-algebra theories have the following Lagrangian:

$$
\begin{aligned}
L_{L3A}^{(G)} = \; & \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\
& -\tfrac{1}{12}\left(X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]\right)^2 \Big) \\
& + \; (C^{\mu\, I} - \partial^\mu X_-^I)\partial_\mu X_+^I + L_{\text{gauge fixing}} + L_{\text{fermions}}
\end{aligned}
$$

where

$$
\hat{D}_\mu \boldsymbol{X}^I \equiv \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I
$$

Lorentzian 3-algebras

- The Lorentzian 3-algebra theories have the following Lagrangian:

$$L^{(G)}_{L3A} = \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I$$
$$-\tfrac{1}{12}\left(X^I_+[\boldsymbol{X}^J,\boldsymbol{X}^K] + X^J_+[\boldsymbol{X}^K,\boldsymbol{X}^I] + X^K_+[\boldsymbol{X}^I,\boldsymbol{X}^J]\right)^2\Big)$$
$$+ \ (C^{\mu\, I} - \partial^\mu X^I_-)\partial_\mu X^I_+ + L_{\text{gauge fixing}} + L_{\text{fermions}}$$

where

$$\hat{D}_\mu \boldsymbol{X}^I \equiv \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X^I_+$$

- These theories have no parameter k.

Lorentzian 3-algebras

- The Lorentzian 3-algebra theories have the following Lagrangian:

$$
\begin{aligned}
L_{L3A}^{(G)} &= \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\
&\quad - \tfrac{1}{12}\left(X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J] \right)^2 \Big) \\
&\quad + \ (C^{\mu\, I} - \partial^\mu X_-^I)\partial_\mu X_+^I + L_{\text{gauge fixing}} + L_{\text{fermions}}
\end{aligned}
$$

where

$$
\hat{D}_\mu \boldsymbol{X}^I \equiv \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I
$$

- These theories have no parameter k.
- They have $SO(8)$ global symmetry acting on the indices $I, J, K \in 1, 2, \cdots, 8$.

Lorentzian 3-algebras

- The Lorentzian 3-algebra theories have the following Lagrangian:

$$L^{(G)}_{L3A} = \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda} \boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I$$
$$- \tfrac{1}{12}\left(X^I_+ [\boldsymbol{X}^J, \boldsymbol{X}^K] + X^J_+ [\boldsymbol{X}^K, \boldsymbol{X}^I] + X^K_+ [\boldsymbol{X}^I, \boldsymbol{X}^J]\right)^2 \Big)$$
$$+ \ (C^{\mu\, I} - \partial^\mu X^I_-)\partial_\mu X^I_+ + L_{\text{gauge fixing}} + L_{\text{fermions}}$$

where

$$\hat{D}_\mu \boldsymbol{X}^I \equiv \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X^I_+$$

- These theories have no parameter k.
- They have $SO(8)$ global symmetry acting on the indices $I, J, K \in 1, 2, \cdots, 8$.
- The equation of motion of the auxiliary gauge field C^I_μ implies that $X_+ =$ constant.

- Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]

- Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]
- On giving a vev to the singlet field X_+^I, say:

$$\langle X_+^8 \rangle = v$$

one finds:

$$L_{L3A}^{(G)}\Big|_{vev\ v} = \frac{1}{v^2} L_{SYM}^{(G)} \quad (+\ \ \text{no corrections})$$

- Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]
- On giving a vev to the singlet field X_+^I, say:

$$\langle X_+^8 \rangle = v$$

one finds:

$$L_{L3A}^{(G)}\Big|_{vev\ v} = \frac{1}{v^2} L_{SYM}^{(G)} \quad (+\ \text{no corrections})$$

- This leads one to suspect that the theory is a re-formulation of SYM.

- Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]
- On giving a vev to the singlet field X_+^I, say:

$$\langle X_+^8 \rangle = v$$

one finds:

$$L_{L3A}^{(G)}\Big|_{vev\ v} = \frac{1}{v^2} L_{SYM}^{(G)} \quad (+\ \ \text{no corrections})$$

- This leads one to suspect that the theory is a re-formulation of SYM.
- In fact it can be derived [Ezhuthachan-SM-Papageorgakis] starting from $\mathcal{N} = 8$ SYM.

- The procedure involves a non-Abelian (dNS) duality [deWit-Nicolai-Samtleben] on the (2+1)d gauge field.

- The procedure involves a non-Abelian (dNS) duality [deWit-Nicolai-Samtleben] on the (2+1)d gauge field.
- Start with $\mathcal{N} = 8$ SYM in (2+1)d. Introducing two new adjoint fields $\boldsymbol{B}_\mu, \phi$, the dNS duality transformation is:

$$-\frac{1}{4g_{YM}^2}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} \to \frac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\lambda} - \frac{1}{2}\left(D_\mu\boldsymbol{\phi} - g_{YM}\boldsymbol{B}_\mu\right)^2$$

Note that D_μ is the covariant derivative with respect to the original gauge field $\boldsymbol{A}$.

- The procedure involves a non-Abelian (dNS) duality [deWit-Nicolai-Samtleben] on the (2+1)d gauge field.
- Start with $\mathcal{N}=8$ SYM in (2+1)d. Introducing two new adjoint fields $\boldsymbol{B}_\mu, \phi$, the dNS duality transformation is:

$$-\frac{1}{4g_{YM}^2}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} \to \frac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\lambda} - \frac{1}{2}\left(D_\mu\boldsymbol{\phi} - g_{YM}\boldsymbol{B}_\mu\right)^2$$

  Note that D_μ is the covariant derivative with respect to the original gauge field $\boldsymbol{A}$.
- In addition to the gauge symmetry G, the new action has a noncompact abelian gauge symmetry:

$$\delta\boldsymbol{\phi} = g_{YM}\boldsymbol{M}\ , \qquad \delta\boldsymbol{B}_\mu = D_\mu\boldsymbol{M}$$

  where $\boldsymbol{M}(x)$ is an arbitrary matrix in the adjoint of G.

- The procedure involves a non-Abelian (dNS) duality [deWit-Nicolai-Samtleben] on the (2+1)d gauge field.
- Start with $\mathcal{N} = 8$ SYM in (2+1)d. Introducing two new adjoint fields $\boldsymbol{B}_\mu, \boldsymbol{\phi}$, the dNS duality transformation is:

$$-\frac{1}{4g_{YM}^2}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} \to \frac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\lambda} - \frac{1}{2}\left(D_\mu\boldsymbol{\phi} - g_{YM}\boldsymbol{B}_\mu\right)^2$$

  Note that D_μ is the covariant derivative with respect to the original gauge field $\boldsymbol{A}$.
- In addition to the gauge symmetry G, the new action has a noncompact abelian gauge symmetry:

$$\delta\boldsymbol{\phi} = g_{YM}\boldsymbol{M}\ , \qquad \delta\boldsymbol{B}_\mu = D_\mu\boldsymbol{M}$$

  where $\boldsymbol{M}(x)$ is an arbitrary matrix in the adjoint of G.
- To prove the duality, use this symmetry to set $\phi = 0$. Then integrating out $\boldsymbol{B}_\mu$ gives the usual YM kinetic term for $\boldsymbol{F}_{\mu\nu}$.

- The dNS-duality transformed $\mathcal{N} = 8$ SYM is:

$$
\begin{aligned}
L &= \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\big(D_\mu\boldsymbol{\phi} - g_{\mathsf{YM}}\boldsymbol{B}_\mu\big)^2 \\
&- \tfrac{1}{2}D_\mu \boldsymbol{X}^i D^\mu \boldsymbol{X}^i - \tfrac{g^2_{\mathsf{YM}}}{4}[\boldsymbol{X}^i, \boldsymbol{X}^j]^2 + \text{fermions}\Big)
\end{aligned}
$$

- The dNS-duality transformed $\mathcal{N}=8$ SYM is:

$$
\begin{aligned}
L &= \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\big(D_\mu\boldsymbol{\phi} - g_{\text{YM}}\boldsymbol{B}_\mu\big)^2 \\
&- \tfrac{1}{2}D_\mu \boldsymbol{X}^i D^\mu \boldsymbol{X}^i - \tfrac{g_{\text{YM}}^2}{4}[\boldsymbol{X}^i, \boldsymbol{X}^j]^2 + \text{fermions}\Big)
\end{aligned}
$$

- We can now see the $SO(8)$ invariance appearing.

- The dNS-duality transformed $\mathcal{N} = 8$ SYM is:

$$\begin{aligned} L &= \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\big(D_\mu\boldsymbol{\phi} - g_{YM}\boldsymbol{B}_\mu\big)^2 \\ &\quad - \tfrac{1}{2}D_\mu \boldsymbol{X}^i D^\mu \boldsymbol{X}^i - \tfrac{g_{YM}^2}{4}[\boldsymbol{X}^i, \boldsymbol{X}^j]^2 + \text{fermions}\Big) \end{aligned}$$

- We can now see the $SO(8)$ invariance appearing.
- Rename $\boldsymbol{\phi} \to \boldsymbol{X}^8$. Then the scalar kinetic terms are:

$$-\tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I = -\tfrac{1}{2}\left(\partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - g_{YM}^I \boldsymbol{B}_\mu\right)^2$$

where $g_{YM}^I = (0, \ldots, 0, g_{YM})$.

- The dNS-duality transformed $\mathcal{N}=8$ SYM is:

$$L = \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\big(D_\mu\boldsymbol{\phi} - g_{\mathrm{YM}}\boldsymbol{B}_\mu\big)^2 - \tfrac{1}{2}D_\mu\boldsymbol{X}^i D^\mu\boldsymbol{X}^i - \tfrac{g_{\mathrm{YM}}^2}{4}[\boldsymbol{X}^i,\boldsymbol{X}^j]^2 + \text{fermions}\Big)$$

- We can now see the $SO(8)$ invariance appearing.
- Rename $\boldsymbol{\phi}\to\boldsymbol{X}^8$. Then the scalar kinetic terms are:

$$-\tfrac{1}{2}\hat{D}_\mu\boldsymbol{X}^I\hat{D}^\mu\boldsymbol{X}^I = -\tfrac{1}{2}\left(\partial_\mu\boldsymbol{X}^I - [\boldsymbol{A}_\mu,\boldsymbol{X}^I] - g_{\mathrm{YM}}^I\boldsymbol{B}_\mu\right)^2$$

  where $g_{\mathrm{YM}}^I = (0,\ldots,0,g_{\mathrm{YM}})$.
- Next, we can allow g_{YM}^I to be an arbitrary 8-vector.

- The action is now $SO(8)$-invariant if we rotate both the fields X^I and the coupling-constant vector g^I_{YM}:

$$\begin{aligned} L &= \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\ &- \tfrac{1}{12}\left(g^I_{\mathsf{YM}}[\boldsymbol{X}^J, \boldsymbol{X}^K] + g^J_{\mathsf{YM}}[\boldsymbol{X}^K, \boldsymbol{X}^I] + g^K_{\mathsf{YM}}[\boldsymbol{X}^I, \boldsymbol{X}^J]\right)^2 \Big) \end{aligned}$$

- The action is now $SO(8)$-invariant if we rotate both the fields X^I and the coupling-constant vector g^I_{YM}:

$$L = \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I - \tfrac{1}{12}\left(g^I_{\mathsf{YM}}[\boldsymbol{X}^J, \boldsymbol{X}^K] + g^J_{\mathsf{YM}}[\boldsymbol{X}^K, \boldsymbol{X}^I] + g^K_{\mathsf{YM}}[\boldsymbol{X}^I, \boldsymbol{X}^J] \right)^2 \Big)$$

- This is not yet a symmetry, since it rotates the coupling constant.

- The action is now $SO(8)$-invariant if we rotate both the fields X^I and the coupling-constant vector g^I_{YM}:

$$
\begin{aligned}
L \;=\; & \operatorname{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\
& - \tfrac{1}{12}\left(g^I_{\text{YM}}[\boldsymbol{X}^J, \boldsymbol{X}^K] + g^J_{\text{YM}}[\boldsymbol{X}^K, \boldsymbol{X}^I] + g^K_{\text{YM}}[\boldsymbol{X}^I, \boldsymbol{X}^J]\right)^2 \Big)
\end{aligned}
$$

- This is not yet a symmetry, since it rotates the coupling constant.
- The final step is to introduce an 8-vector of new (gauge-singlet) scalars X^I_+ and replace:

$$
g^I_{\text{YM}} \longrightarrow X^I_+(x)
$$

- The action is now $SO(8)$-invariant if we rotate both the fields X^I and the coupling-constant vector g^I_{YM}:

$$
\begin{aligned}
L \;=\; & \operatorname{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\
& - \tfrac{1}{12}\left(g^I_{\text{YM}}[\boldsymbol{X}^J,\boldsymbol{X}^K] + g^J_{\text{YM}}[\boldsymbol{X}^K,\boldsymbol{X}^I] + g^K_{\text{YM}}[\boldsymbol{X}^I,\boldsymbol{X}^J]\right)^2\Big)
\end{aligned}
$$

- This is not yet a symmetry, since it rotates the coupling constant.
- The final step is to introduce an 8-vector of new (gauge-singlet) scalars X^I_+ and replace:

$$
g^I_{\text{YM}} \to X^I_+(x)
$$

- This is legitimate if and only if $X^I_+(x)$ has an equation of motion that renders it constant. Then on-shell we can recover the original theory by writing $\langle X^I_+\rangle = g^I_{\text{YM}}$.

- Constancy of X_+^I is imposed by introducing a new set of abelian gauge fields and scalars: C_μ^I, X_-^I and adding the following term:

$$L_C = (C_I^\mu - \partial X_-^I)\partial_\mu X_+^I$$

- Constancy of X_+^I is imposed by introducing a new set of abelian gauge fields and scalars: C_μ^I, X_-^I and adding the following term:

$$L_C = (C_I^\mu - \partial X_-^I)\partial_\mu X_+^I$$

- This has a shift symmetry

$$\delta X_-^I = \lambda^I, \quad \delta C_\mu^I = \partial_\mu \lambda^I$$

which will remove the negative-norm states associated to C_μ^I.

- Constancy of X_+^I is imposed by introducing a new set of abelian gauge fields and scalars: C_μ^I, X_-^I and adding the following term:

$$L_C = (C_I^\mu - \partial X_-^I)\partial_\mu X_+^I$$

- This has a shift symmetry

$$\delta X_-^I = \lambda^I, \quad \delta C_\mu^I = \partial_\mu \lambda^I$$

  which will remove the negative-norm states associated to C_μ^I.
- We have thus ended up with the Lorentzian 3-algebra action [Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]:

$$\begin{aligned} L = \ & \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}_\mu \boldsymbol{X}^I \\ & - \ \tfrac{1}{12}\left(X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]\right)^2\Big) \\ & + \ (C^{\mu\, I} - \partial^\mu X_-^I)\partial_\mu X_+^I + L_{\text{gauge-fixing}} + \mathcal{L}_{\text{fermions}} \end{aligned}$$

- The final action has some remarkable properties.

- The final action has some remarkable properties.
- It has manifest SO(8) invariance as well as $\mathcal{N} = 8$ superconformal invariance.

- The final action has some remarkable properties.
- It has manifest SO(8) invariance as well as $\mathcal{N} = 8$ superconformal invariance.
- However, both are spontaneously broken by giving a vev $\langle X_+^I \rangle = g_{YM}^I$ and the theory reduces to $\mathcal{N} = 8$ SYM with coupling $|g_{YM}|$.

- The final action has some remarkable properties.
- It has manifest SO(8) invariance as well as $\mathcal{N} = 8$ superconformal invariance.
- However, both are spontaneously broken by giving a vev $\langle X_+^I \rangle = g_{YM}^I$ and the theory reduces to $\mathcal{N} = 8$ SYM with coupling $|g_{YM}|$.
- It will certainly describe M2-branes if one can find a way to take $\langle X_+^I \rangle = \infty$. That has not yet been done.

Outline

Higher-order corrections for Lorentzian 3-algebras

- One might ask if the non-Abelian duality that we have just performed works when higher order (in α') corrections are included.

Higher-order corrections for Lorentzian 3-algebras

- One might ask if the non-Abelian duality that we have just performed works when higher order (in α') corrections are included.
- For the Abelian case [Duff, Townsend, Schmidhuber] we know that the analogous duality works for the entire DBI action and that fermions and supersymmetry can also be incorporated [Aganagic-Park-Popescu-Schwarz].

Higher-order corrections for Lorentzian 3-algebras

- One might ask if the non-Abelian duality that we have just performed works when higher order (in α') corrections are included.
- For the Abelian case [Duff, Townsend, Schmidhuber] we know that the analogous duality works for the entire DBI action and that fermions and supersymmetry can also be incorporated [Aganagic-Park-Popescu-Schwarz].
- Recently we have shown [Alishahiha-SM] that to lowest nontrivial order (F^4-type corrections) one can indeed dualise the non-Abelian SYM into an $SO(8)$-invariant form.

Higher-order corrections for Lorentzian 3-algebras

- One might ask if the non-Abelian duality that we have just performed works when higher order (in α') corrections are included.
- For the Abelian case [Duff, Townsend, Schmidhuber] we know that the analogous duality works for the entire DBI action and that fermions and supersymmetry can also be incorporated [Aganagic-Park-Popescu-Schwarz].
- Recently we have shown [Alishahiha-SM] that to lowest nontrivial order (F^4-type corrections) one can indeed dualise the non-Abelian SYM into an $SO(8)$-invariant form.
- Here of course one cannot do all orders in α' because a non-Abelian analogue of DBI is still not known.

Higher-order corrections for Lorentzian 3-algebras

- One might ask if the non-Abelian duality that we have just performed works when higher order (in α') corrections are included.
- For the Abelian case [Duff, Townsend, Schmidhuber] we know that the analogous duality works for the entire DBI action and that fermions and supersymmetry can also be incorporated [Aganagic-Park-Popescu-Schwarz].
- Recently we have shown [Alishahiha-SM] that to lowest nontrivial order (F^4-type corrections) one can indeed dualise the non-Abelian SYM into an $SO(8)$-invariant form.
- Here of course one cannot do all orders in α' because a non-Abelian analogue of DBI is still not known.
- However our approach may have a bearing on that unsolved problem.

- Let us see how this works. In (2+1)d, the lowest correction to SYM for D2-branes is the sum of the following contributions (here $\boldsymbol{X}^{ij} = [\boldsymbol{X}^i, \boldsymbol{X}^j]$):

$$
\begin{aligned}
L_1^{(4)} &= \frac{1}{12 g_{YM}^4}\Big[\boldsymbol{F}_{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\mu\rho}\boldsymbol{F}^{\nu\sigma} + \tfrac{1}{2}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\nu\rho}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\sigma\mu} \\
&- \tfrac{1}{4}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\rho\sigma} - \tfrac{1}{8}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}^{\rho\sigma}\Big] \\
L_2^{(4)} &= \frac{1}{12 g_{YM}^2}\Big[\boldsymbol{F}_{\mu\nu}\, D^\mu \boldsymbol{X}^i\, \boldsymbol{F}^{\rho\nu}\, D_\rho \boldsymbol{X}^i + \boldsymbol{F}_{\mu\nu}\, D_\rho \boldsymbol{X}^i\, \boldsymbol{F}^{\mu\rho}\, D^\nu \boldsymbol{X}^i \\
&- 2\boldsymbol{F}_{\mu\rho}\, \boldsymbol{F}^{\rho\nu}\, D^\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^i - 2\boldsymbol{F}_{\mu\rho}\, \boldsymbol{F}^{\rho\nu}\, D_\nu \boldsymbol{X}^i\, D^\mu \boldsymbol{X}^i \\
&- \boldsymbol{F}_{\mu\nu}\, \boldsymbol{F}^{\mu\nu}\, D^\rho \boldsymbol{X}^i\, D_\rho \boldsymbol{X}^i - \tfrac{1}{2}\boldsymbol{F}_{\mu\nu}\, D_\rho \boldsymbol{X}_i\, \boldsymbol{F}_{\mu\nu}\, D_\rho \boldsymbol{X}_i\Big] \\
&- \tfrac{1}{12}\left(\tfrac{1}{2}\boldsymbol{F}_{\mu\nu}\, \boldsymbol{F}^{\mu\nu}\, \boldsymbol{X}^{ij}\, \boldsymbol{X}^{ij} + \tfrac{1}{4}\boldsymbol{F}_{\mu\nu}\, \boldsymbol{X}^{ij}\, \boldsymbol{F}^{\mu\nu}\, \boldsymbol{X}^{ij}\right) \\
L_3^{(4)} &= -\tfrac{1}{6}\Big(D^\mu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j\, \boldsymbol{F}_{\mu\nu} + D^\nu \boldsymbol{X}^j\, \boldsymbol{F}_{\mu\nu}\, D^\mu \boldsymbol{X}^i \\
&+ \boldsymbol{F}_{\mu\nu}\, D^\mu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j\Big)\, \boldsymbol{X}^{ij}
\end{aligned}
$$

$$
\begin{aligned}
L_4^{(4)} = \frac{1}{12}\Big[& D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\nu \boldsymbol{X}^i\, D^\mu \boldsymbol{X}^j + D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^j\, D^\nu \boldsymbol{X}^i \\
+ \; & D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^j - D_\mu \boldsymbol{X}^i\, D^\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\nu \boldsymbol{X}^j \\
- \; & \tfrac{1}{2} D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j \Big] \\
L_5^{(4)} = \frac{g_{YM}^2}{12}\Big[& \boldsymbol{X}^{kj}\, D_\mu \boldsymbol{X}^k\, \boldsymbol{X}^{ij}\, D^\mu \boldsymbol{X}^i + \boldsymbol{X}^{ij}\, D_\mu \boldsymbol{X}^k\, \boldsymbol{X}^{ik}\, D^\mu \boldsymbol{X}^j \\
- \; & 2\boldsymbol{X}^{kj}\, \boldsymbol{X}^{ik}\, D_\mu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^i - 2\boldsymbol{X}^{ki}\, \boldsymbol{X}^{jk}\, D_\mu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^i \\
- \; & \boldsymbol{X}^{ij}\, \boldsymbol{X}^{ij}\, D_\mu \boldsymbol{X}^k\, D^\mu \boldsymbol{X}^k - \tfrac{1}{2}\boldsymbol{X}^{ij}\, D_\mu \boldsymbol{X}^k\, \boldsymbol{X}^{ij}\, D^\mu \boldsymbol{X}^k \Big] \\
L_6^{(4)} = \frac{g_{YM}^4}{12}\Big[& \boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{ik}\boldsymbol{X}^{jl} + \tfrac{1}{2}\boldsymbol{X}^{ij}\boldsymbol{X}^{jk}\boldsymbol{X}^{kl}\boldsymbol{X}^{li} \\
- \; & \tfrac{1}{4}\boldsymbol{X}^{ij}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{kl} - \tfrac{1}{8}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl} \Big]
\end{aligned}
$$

- We have been able to show that this is dual, under the dNS transformation, to:

$$
\begin{aligned}
L =\ & \mathrm{tr}\Big[\tfrac{1}{2}\epsilon^{\mu\nu\rho}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\rho} - \tfrac{1}{2}\hat{D}_\mu\boldsymbol{X}^I\hat{D}^\mu\boldsymbol{X}^I \\
& + \tfrac{1}{12}\Big(\hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^I\,\hat{D}^\mu\boldsymbol{X}^J + \hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\mu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^I \\
& \qquad + \hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^I\,\hat{D}^\nu\boldsymbol{X}^J\,\hat{D}^\mu\boldsymbol{X}^J - \hat{D}_\mu\boldsymbol{X}^I\,\hat{D}^\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^J \\
& \qquad - \tfrac{1}{2}\hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\mu\boldsymbol{X}^I\,\hat{D}^\nu\boldsymbol{X}^J\Big) \\
& + \tfrac{1}{12}\Big(\tfrac{1}{2}\boldsymbol{X}^{LKJ}\hat{D}_\mu\boldsymbol{X}^K\boldsymbol{X}^{LIJ}\hat{D}^\mu\boldsymbol{X}^I + \tfrac{1}{2}\boldsymbol{X}^{LIJ}\hat{D}_\mu\boldsymbol{X}^K\boldsymbol{X}^{LIK}\hat{D}^\mu\boldsymbol{X}^J \\
& \qquad - \boldsymbol{X}^{LKJ}\boldsymbol{X}^{LIK}\hat{D}_\mu\boldsymbol{X}^J\hat{D}^\mu\boldsymbol{X}^I - \boldsymbol{X}^{LKI}\boldsymbol{X}^{LJK}\hat{D}_\mu\boldsymbol{X}^J\hat{D}^\mu\boldsymbol{X}^I \\
& \qquad - \tfrac{1}{3}\boldsymbol{X}^{LIJ}\boldsymbol{X}^{LIJ}\hat{D}_\mu\boldsymbol{X}^K\hat{D}^\mu\boldsymbol{X}^K - \tfrac{1}{6}\boldsymbol{X}^{LIJ}\,\hat{D}_\mu\boldsymbol{X}^K\boldsymbol{X}^{LIJ}\hat{D}^\mu\boldsymbol{X}^K\Big) \\
& - \tfrac{1}{6}\epsilon_{\rho\mu\nu}\hat{D}^\rho\boldsymbol{X}^I\,\hat{D}^\mu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^K\boldsymbol{X}^{IJK} - V(\boldsymbol{X})\Big]
\end{aligned}
$$

- In the previous expression,

$$\hat{D}_\mu \boldsymbol{X}^I = \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I$$

$$\boldsymbol{X}^{IJK} = X_+^I [\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J [\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K [\boldsymbol{X}^I, \boldsymbol{X}^J]$$

- In the previous expression,

$$
\begin{aligned}
\hat{D}_\mu \boldsymbol{X}^I &= \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I \\
\boldsymbol{X}^{IJK} &= X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]
\end{aligned}
$$

- Here $V(X)$ is the potential:

$$
\begin{aligned}
V(X) = \tfrac{1}{12}\boldsymbol{X}^{IJK}\boldsymbol{X}^{IJK} + \tfrac{1}{108}\Big[&\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MIK}\boldsymbol{X}^{MJL} \\
&+ \tfrac{1}{2}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MJK}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MLI} \\
&- \tfrac{1}{4}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{MKL} \\
&- \tfrac{1}{8}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\Big]
\end{aligned}
$$

- In the previous expression,

$$\begin{aligned} \hat{D}_\mu \boldsymbol{X}^I &= \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I \\ \boldsymbol{X}^{IJK} &= X_+^I [\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J [\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K [\boldsymbol{X}^I, \boldsymbol{X}^J] \end{aligned}$$

- Here $V(X)$ is the potential:

$$\begin{aligned} V(X) = \tfrac{1}{12} \boldsymbol{X}^{IJK} \boldsymbol{X}^{IJK} + \tfrac{1}{108} \Big[& \boldsymbol{X}^{NIJ} \boldsymbol{X}^{NKL} \boldsymbol{X}^{MIK} \boldsymbol{X}^{MJL} \\ & + \tfrac{1}{2} \boldsymbol{X}^{NIJ} \boldsymbol{X}^{MJK} \boldsymbol{X}^{NKL} \boldsymbol{X}^{MLI} \\ & - \tfrac{1}{4} \boldsymbol{X}^{NIJ} \boldsymbol{X}^{NIJ} \boldsymbol{X}^{MKL} \boldsymbol{X}^{MKL} \\ & - \tfrac{1}{8} \boldsymbol{X}^{NIJ} \boldsymbol{X}^{MKL} \boldsymbol{X}^{NIJ} \boldsymbol{X}^{MKL} \Big] \end{aligned}$$

- We see that the dual Lagrangian is $SO(8)$ invariant.

- In the previous expression,

$$\begin{aligned} \hat{D}_\mu \boldsymbol{X}^I &= \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I \\ \boldsymbol{X}^{IJK} &= X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J] \end{aligned}$$

- Here $V(X)$ is the potential:

$$\begin{aligned} V(X) = \tfrac{1}{12}\boldsymbol{X}^{IJK}\boldsymbol{X}^{IJK} + \tfrac{1}{108}\Big[&\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MIK}\boldsymbol{X}^{MJL} \\ &+ \tfrac{1}{2}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MJK}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MLI} \\ &- \tfrac{1}{4}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{MKL} \\ &- \tfrac{1}{8}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\Big] \end{aligned}$$

- We see that the dual Lagrangian is $SO(8)$ invariant.
- It's worth noting that this depends crucially on the relative coefficients of various terms in the original Lagrangian.

- We see from this that the 3-algebra structure remains intact when higher-derivative corrections are taken into account.

- We see from this that the 3-algebra structure remains intact when higher-derivative corrections are taken into account.
- We conjecture that SO(8) enhancement holds to all orders in α'.

- We see from this that the 3-algebra structure remains intact when higher-derivative corrections are taken into account.
- We conjecture that SO(8) enhancement holds to all orders in α'.
- Unfortunately the all-orders corrections are not known for SYM, so we don't have a starting point from which to check this.

Outline

Summary

- Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.

Summary

- Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.
- But we don't seem to be there yet.

Summary

- Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.
- But we don't seem to be there yet.
- The existence of a large-order orbifold (deconstruction) limit provides a way (the only one so far) to relate the membrane theory to $D2$-branes. One would like to understand compactification of transverse or longitudinal directions, as we do for D-branes.

Summary

- Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.
- But we don't seem to be there yet.
- The existence of a large-order orbifold (deconstruction) limit provides a way (the only one so far) to relate the membrane theory to $D2$-branes. One would like to understand compactification of transverse or longitudinal directions, as we do for D-branes.
- An interesting mechanism has been identified to dualise the $D2$-brane action into a superconformal, $SO(8)$ invariant one. The result is a Lorentzian 3-algebra and this structure is preserved by α' corrections.

- A detailed understanding of multiple membranes should open a new window to M-theory and 11 dimensions.

- A detailed understanding of multiple membranes should open a new window to M-theory and 11 dimensions.
-

...if you were as tiny as a graviton
You could enter these dimensions and go wandering on

And they'd find you...