TIFR Annual Report 2010 - 11

Department of Theoretical Physics

Condensed Matter and Statistical Physics

Highlights

A spin-1 Kitaev model in one dimension was studied, and its eigenstates in different sectors corresponding to different values of Z_2 constants of motion were studied. A new mixed strategy for very efficient utilization of resources in minority games was analysed.

Impurity induced fractional spin excitations were identified as being responsible for the unusual NMR data in the frustrated antiferromagnet SCGO.

In a joint theoretical and experimental study, it was shown that in 2D semiconductor heterostructures delta-doped with Mn, resistivity anomalies, usually associated with the onset of ferromagnetism, can arise even in the absence of long-range order and may exist significantly below the Curie temperature. This is very different from the 3D counterparts where the resistivity anomalies usually appear near the Curie temperature.

Motivated by experimental studies of transport and processing of biomolecules through the Golgi organelle in the cell, a statistical physics model incorporating drive, injection and interconversion was defined and shown to give rise to interesting growing phases.

Text

Spin-1 Kitaev model in one dimension

A one-dimensional version of the Kitaev model has been studied on a ring of size N, in which there is a spin $S > 1/2$ on each site and the Hamiltonian is $\Sigma_n J S^x_n S^y_{n+1}$. The cases where S is integer and half-odd-integer are qualitatively different. It was shown that there is a Z_2 valued conserved quantity W_n for each bond ($n, n+1$) of the system. For integer S, the Hilbert space can be decomposed into 2^N sectors, of unequal sizes. The number of states in most of the sectors grows as d^N, where d depends on the sector. The largest sector contains the ground state. Exact diagonalization was carried out for small systems. The extrapolation of these results to large N indicates that the energy gap remains finite in this limit. In the ground state sector, the system can be mapped to a spin-1/2 model. Variational wave functions were developed to study the lowest energy states in the ground state and other sectors, which give rather precise estimates of the ground state energy (less than 0.1% error). The first excited state of the system is the lowest energy state of a different sector and its excitation energy was estimated. A more general Hamiltonian was considered, adding a term $\lambda \Sigma_n W_n$. It was shown that this has gapless excitations in the range $\lambda^{c1} <= \lambda <= \lambda^{c2}$. The variational wave functions were used to study how the ground state energy and the defect density vary near the two critical points λ^{c1} and λ^{c2}. [K. Ramola, Deepak Dhar with D. Sen (I.I.Sc.) and R. Shankar (I.M.Sc.)]

An efficient mixed strategy for minority games

A variation of the minority game was studied. There are N agents. Each has to choose between one of two alternatives every day, and there is a reward to each member of the smaller group. The agents cannot communicate with each other, but try to guess the choice others will make, based only on the past history of number of people choosing the two alternatives. A simple probabilistic strategy was described using which the agents, acting independently, can still maximize the average number of people benefitting every day. The strategy leads to a very efficient utilization of resources, and the average deviation from the maximum possible can be made $O(N^{\varepsilon})$, for any $\varepsilon > 0$. It was also shown that a single agent does not expect to gain by not following the strategy. The strategy discussed here is an application of the general win-stay-lose-shift strategy. In the latter, the deviation from best possible can be made of order *1*, but the time required grows as $N^{1/2}$. A much faster approach to optimum was obtained by using a shift probability that depends on the current distance from optimum. [Deepak Dhar and V. Sasidevan with B.K. Chakrabarti (S.I.N.P.)]

Continuum percolation of discs with a power-law tail in the distribution of radii

In problems like effective modelling of random media, the continuum models of percolation are more realistic than their lattice counterparts. Much effort has been put into the study of such systems in the recent past. In two dimensions these models involve objects like discs or squares of the same size or of different sizes thrown at random onto a plane. An interesting sub-class of problems is where the discs or cubes have an unbounded size distribution. A continuum percolation model of overlapping discs in 2D was considered where distribution of the radii of the discs has a power-law tail. Using a rigorous lower bound on the two-point correlation function, it was shown that it decays only as a power-law with distance for arbitrarily low coverage densities. Thus, the entire low density phase is critical. An alternate definition of correlation length of the system was considered in terms of the scaling properties of the two-point function, which gives a finite correlation length in the low-density phase. The critical exponent characterizing the decay of the two-point correlation function at the critical point was determined. An approximate renormalization scheme was proposed to calculate the correlation length exponent and the percolation threshold in such models. Satisfactory agreement was found with the results from simulations. [Deepak Dhar]

Bethe approximation for the ordering transition in a system of hard rods on a lattice

The study of the ordering transition in systems of rod-like objects with only excluded volume interactions has a long history. The question whether the lattice model of long rigid rods undergoes a phase transition has remained unsettled for a long time. In an earlier work, the authors had argued that needles of length *k*, called *k*-mers, on a square lattice, for k >= 7, would undergo two phase transitions, and the nematic phase would exist for only an intermediate range of densities $\rho_1^* < \rho < \rho_2^*$, where $\rho_1^* \sim 1/k$. While there have been subsequent studies of the first transition, the second phase transition is still not well understood. Given the paucity of exact results about the second transition, it is worthwhile to investigate this problem using well-known approximation methods like the Bethe approximation. However, there are problems in the application of the conventional Bethe approximation techniques, even to the first isotropic-nematic transition. This is related to the fact that packing *k*-mers on the Bethe lattice with a density close to one is not possible, and the nature of the high-density phase on the Bethe lattice is not very clear. Random graphs of fixed-coordination number, which have been used to study the Bethe approximation earlier, also do not allow full coverage by k-mers. A new lattice was introduced called the random locally tree-like layered lattice. It was shown that there is no difficulty in defining the high-density phase for the *k*-mers and that the Bethe approximation is exact also on this lattice. In fact, there is a finite entropy per site in the fully packed limit. [Deepak Dhar with R. Rajesh (I.M.Sc.) and J. F. Stilck (Niteroi)]

Neel to staggered dimer order transition in a generalized honeycomb lattice Heisenberg model

A strongly first-order transition to staggered valence-bond solid order was found to intervene as the

Neel ordered antiferromagnet on the honeycomb lattice gets destabilized by multi-spin interactions that favour staggered valence-bond solid order. This is in contrast to the deconfined second-order transition between Neel order and columnar valence bond solid order, and reinforces the identification of vortex like defects with spinful cores as the key ingredients in deconfined criticality since they are absent in the case studied here. [Argha Banerjee and Kedar Damle with Arun Paramekanti (Toronto)]

Impurity spin texture at a Neel-valence bond solid critical point in d=2 SU(3) quantum antiferromagnets

The impurity spin texture induced by a missing-spin defect was studied at SU(3) versions of the S=1/2 deconfined critical point between Neel-ordered and valence-bond solid ground states. It was found that the SU(3) spin texture obeys standard scaling, suggesting that the logarithmic violations of scaling found in the original SU(2) study were the result of an operator that becomes marginal as the symmetry is reduced to SU(2). [Argha Banerjee and Kedar Damle with Fabien Alet (Toulouse)]

Fractional spin textures in the frustrated magnet $SrCr_{9p}Ga1_{2\text{-}9p}O_{19}$

Fractional spin textures induced by pairs of vacancies were identified as the source of the anomalous NMR line broadening in the frustrated magnet SCGO. These also provide a simple classical example of quasiparticle fractionalization. [Arnab Sen and Kedar Damle with Roderich Moessner (Dresden)]

Generalization of the singlet sector valence-bond loop algorithm to antiferromagnetic ground states with total spin S_{tot}=1/2

The valence-bond basis projector algorithm of Sandvik was generalized to allow for the computation of doublet ground states of antiferromagnets with an odd number of sites. [Argha Banerjee and Kedar Damle]

Charge inhomogeneities and transport in semiconductor heterostructures with a manganese delta-layer

A joint experimental and theoretical study of the effects of disorder, nonlinear screening and magnetism in semiconductor heterostructures containing a delta-layer of Mn was carried out. The charge carriers are confined within a quantum well and hence both ferromagnetism and transport are two-dimensional (2D) and differ qualitatively from their bulk counterparts. Anomalies in the electrical resistance observed in both metallic and insulating structures were interpreted as a signature of significant ferromagnetic correlations. At low charge carrier densities, it was shown how the interplay of disorder and nonlinear screening can result in the organization of the carriers in the 2D transport channel into charge droplets separated by insulating barriers. Based on such a droplet picture and including the effect of magnetic correlations, the transport properties of this set of droplets were analysed and compared with experimental data for insulating samples, and a good agreement found between the model calculations and experiment. The analysis showed that the peak or shoulder-like features observed in the temperature dependence of resistance of 2D heterostructures delta-doped by Mn can appear even in the absence of a phase transition at nonzero temperatures. Furthermore, the anomaly can lie significantly below the mean-field Curie temperature T_c. This is unlike the three-dimensional case, where it lies close to the critical temperature T_0, which is usually not very far from T_c. [V. Tripathi and Kusum Dhochak with B.A. Aronzon , V.V. Rylkov and A. B. Davydov (Kurchatov, Moscow), Bertrand Raquet and Michel Goiran (Toulouse) and K. I. Kugel (ITAE, Moscow)]

Phase fluctuations in a strongly disordered s-wave superconductor close to the metal-insulator transition

Measurements and analysis were reported for the (STM) tunnelling conductance and superfluid density of strongly disordered 3D films of NbN near the metal-insulator transition with the motivation of understanding the role of phase fluctuations in the formation of the pseudogap and the degradation of superconductivity. STM measurements reveal the presence of a pseudogap in the tunnelling conductance at temperatures in excess of the transition temperature T_c at which the superfluid density vanishes. The pseudogap disappears at a sufficiently high temperature T* and the ratio T^*/T_c increases as the disorder in increased. Below (and away from) the transition temperature the superfluid density increases linearly with decreasing temperature, and at very low temperatures it increases quadratically with temperature, ultimately saturating at very low temperatures. This observation is consistent with classical and quantum phase fluctuations respectively and shows that phase fluctuations play a crucial role in this regime. [M. Mondal, A. Kamlapure, M. Chand, G. Saraswat, S. Kumar, J. Jesudasan, V. Tripathi, P. Raychaudhuri with L. Benfatto (Sapienza Univ., Rome)]

Multi-species model with interconversion and injection

Most living cells contain an organelle system known as the Golgi apparatus. Cargo molecules such as proteins and lipids enter through one face of the Golgi, undergo chemical processing as they traverse the organelle and the modified products leave through the other face for different target locations. The organelle system itself is also believed to be formed dynamically by the same traffic processes. A phenomenological model incorporating the elementary processes that may be involved in molecular traffic in the Golgi was studied with the aim of qualitatively addressing some questions of structure formation and transport in the system. The model is a multi-species model with boundary injection of one species of particle, interconversion between the different species of particle, and driven diffusive movement of particles through the system by chipping of a single particle from a site. The model was analysed in one dimension using equations for particle currents. It was found that depending on the rates of various processes and the asymmetry in the hopping, the system exists either in a steady phase, in which the average mass at each site attains a time independent value, or in a 'growing' phase in which the total mass grows indefinitely in time, even in a finite system. The growing phases have interesting spatial structure, with unbounded growth of mass of one or more species of particles occurring only in a limited spatial region of tunable width and location, while steady state is attained in the rest of the system. [Himani Sachdeva and Mustansir Barma with Madan Rao (NCBS)]