1 TIFR Annual Report (2011-12) - Department of Theoretical Physics

2 Highlights

Condensed Matter and Statistical Physics

Simple mathematical models of proportionate growth in biology were studied. The phase transition from the nematic to high-density disordered phase in two-dimensional assemblies of hard needles was studied by Monte Carlo simulations. Stochastic strategies for minority game were studied, and phase transitions in the steady state of the system investigated.

A new method based on measurement of frequency-dependent resistivity noise was proposed for probing magnetization dynamics of ferromagnets. This method was employed in the context of Mn-doped semiconductor heterostructures for addressing a long-standing question of whether the ferromagnetism in Mn-doped semiconductors had an indirect exchange or double exchange origin.

An interesting connection was established between the well-known resonating valence bond wavefunctions introduced by Anderson to model quantum spin liquid phases of Mott insulators, and classical interacting dimer models on the same lattice. This connection allowed for a calculation of the non-trivial long-distance behaviour of the energy correlator in these wavefunctions.

TEXT

2.1 Condensed Matter and Statistical Physics

Growing Sandpiles

An important question in biology is how the relative size of different organs is kept nearly constant during growth of an animal. This property, called proportionate growth, has re-

ceived increased attention in recent years. It was shown that in a model of growing sandpiles, this feature comes out quite naturally from local rules of evolution. The patterns produced are composed of large distinguishable structures with sharp boundaries, all of which grow at the same rate, keeping their overall shapes unchanged. The effect of noise in the initial pattern was also studied. [Deepak Dhar]

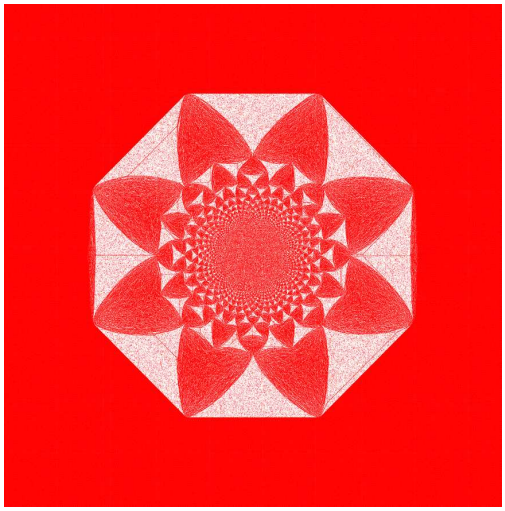

Figure 1: An example of growing pattern produced by adding 896K particles on the F lattice with background chessboard pattern with 10% noise.

Stochastic Minority games

In minority games, a finite number of agents have to repeatedly choose between two alternatives, and the group in the minority wins. These games have been studied a lot as prototypical examples of self-organization in economic systems: markets in which agents interact with each other, and show learning, adaptation, and co-evolution. A variation of this game, in which agents use probabilistic strategies, rather than deterministic ones was studied. It was shown that the steady state of such a system shows phase transitions, even when the number of agents is finite. [Deepak Dhar]

Phase transitions in a system of hard rigid rods

Rigid hard rods on the square and triangular lattices are an interesting system, as they are expected to undergo a second transition from ordered nematic to a high-density disordered

phase, as the density of rods is increased. This has been studied using Monte Carlo techniques, and the existence of the second transition is confirmed, and critical exponents of this transition estimated for the first time. [Deepak Dhar, R. Rajesh and J. Kundu (IMSc)and J. F. Stilck (UFF]

Spherical model on spider-web graphs

An explicit lattice model was studied with effectively infinite dimension, for which the mean-field theory gives the exact critical exponents, but one also gets an essential singularity in the free energy of the spherical model on this lattice near the phase transition, with the singular part varying as $\exp[-K(T-T_c)^{-1/2}]$, for temperatures T near and greater than the critical temperature T_c. [A. C. Balram and Deepak Dhar]

Phase diagram of a strongly disordered s-wave superconductor, NbN, close to the metal-insulator transition

The phase diagram of three dimensional NbN thin films was obtained as a function of potential disorder in this joint experiment-theory work. For low values of disorder, the superconducting energy gap vanished at the superconducting transition temperature Tc in accordance with Bardeen-Cooper-Schrieffer theory. In this regime, Tc was observed to degrade with increasing disorder, and the effect was understood as arising from the reduction of Cooper pair binding due to increased Coulomb repulsion. In samples with stronger disorder, scanning tunnelling microscopy measurements revealed a pronounced "pseudogap" state where the superconducting energy gap remained large despite the rapid decrease of Tc. This behaviour was understood as arising from increasing fluctuations of the phase of the superconducting order parameter which have the effect of disrupting global phase coherence and thus detrimental to superconductivity. Finally, for very strong disorder, the transition temperature vanished but the local superconducting binding energy was still observed to be finite. This is evidenced from a significant magnetoresistance peak even in the absence of the superconducting transition. [M. Chand (TIFR), G. Saraswat (TIFR), A. Kamlapure (TIFR), M. Mondal (TIFR), S. Kumar (TIFR), J. Jesudasan (TIFR), V. Bagwe (TIFR), L. Benfatto (Sapienza, Rome), V. Tripathi (TIFR) and P. Raychaudhuri (TIFR)]

Resistance noise in a two-dimensional hole gas affected by magnetic impurities

This work concerned a theoretical and experimental study of the frequency and temperature dependence of resistivity noise in semiconductor heterostructures delta-doped by Mn. The resistivity noise was observed to be non-monotonous as a function of frequency. As a function of temperature, the noise increased by two orders of magnitude for a resistivity increase of about 50sources of resistivity noise were studied – dynamic spin fluctuations and charge fluctuations. It was found that dynamic spin fluctuations are more relevant for the observed noise data. The frequency and temperature dependence of resistivity noise provide important information on the nature of the magnetic interactions. In particular, it was shown how noise measurements can help resolve a long standing debate on whether the Mn-doped GaAs is an p-d Zener/RKKY or double exchange ferromagnet. The analysis included the effect of different kinds of disorder such as spin-glass type of interactions and a site-dilution type of disorder.[V. Tripathi (TIFR), Kusum Dhochak (TIFR), B. A. Aronzon (Kurchatov, Moscow), Bertrand Raquet (Toulouse), V. V. Tugushev, K. I. Kugel (ITAE, Moscow)]

Phase transitions in the distribution of the Andreev conductance of superconductor-metal junctions with multiple transverse modes

The full Andreev conductance distribution of a metal-superconductor interface with a large number of transverse channels Nc was studied using a random matrix approach. Near is maximum, the probability distribution of the full Andreev conductance has a Gaussian behaviour with a variance independent of Nc. This is reminiscent of universal conductance fluctuations for disordered non-superconducting systems. The distribution vanishes as a power law at the extreme ends with different indices unlike the symmetric distribution one obtains for the non superconducting case. In addition, there is an intermediate region where the distribution is neither Gaussian nor a power-law. These different regions are separated by weak first order phase transitions in the associated Coulomb gas model. [K. Damle (TIFR), S. Majumdar (Orsay), V. Tripathi (TIFR), P. Vivo (Orsay)]

Vacancy-induced spin texture in a one dimensional $S = 1/2$ Heisenberg antiferromagnet

The effect of a missing spin in a one dimensional $S = 1/2$ antiferromagnet with nearest neighbour Heisenberg exchange J and six-spin coupling $Q = 4qJ$ was studied using Quan-

tum Monte-Carlo (QMC) and bosonization techniques. For $q < q_c \approx 0.04$, the system is in a quasi-long range ordered power-law antiferromagnetic phase, which gives way to a valence-bond solid state that spontaneously breaks lattice translation symmetry for $q > q_c$. A key property of the ground state in such systems is the spin texture $\Phi(r) =< G_\uparrow|S^z(r)|G_\uparrow >$ in the the $S^z_{tot} = 1/2$ ground state $|G_\uparrow >$ of the system with a missing spin. It was found that the QMC results for the alternating part N_z of this texture at $q = q_c$ take on the scaling form expected from bosonization considerations, but violate scaling for $q < q_c$. Within the bosonization approach, such violations of scaling arise from the presence of a marginally irrelevant sine-Gordon interaction. The effects of this marginally irrelevant term were calculated using renormalization group (RG) improved perturbation theory and these field-theoretical predictions were found to agree well with the QMC data for $q < q_c$. [S. Sanyal (TIFR), A. Banerjee (IMSc), K. Damle (TIFR)]

Resonating valence bond wavefunctions and classical interacting dimer models

A connection between properties of nearest-neighbour resonating valence bond (nnRVB) wavefunctions for $SU(g)$ spin systems on two dimensional bipartite lattices and those of fully-packed classical dimer models with potential energy V on the same lattice was established. A cluster expansion of V in terms of n-body potentials V_n, which are recursively determined from the nnRVB wavefunction on *finite sub graphs* of the original lattice, was developed. The magnitude of the n-body interaction V_n $(n > 1)$ is of order $\mathcal{O}(g^{-(n-1)})$ for small g^{-1}, while V_1 reduces to a constant due to the fully-packed nature of the model. At leading non-trivial order on the square lattice, the interacting dimer model only has two-body interactions $V_2(g)$ that favour two parallel dimers on elementary plaquettes. Setting $g = 2$ and using the results of earlier work on this interacting dimer model, it was found that the long-distance behaviour of the bond-energy correlation function is dominated by an oscillatory term that decays as $1/|\vec{r}|^\alpha$ with $\alpha \approx 1.22$ for SU(2) spins. This result is in remarkable quantitative agreement with earlier direct numerical studies of the corresponding wavefunction, which give $\alpha \approx 1.20$. [K. Damle (TIFR), D. Dhar (TIFR), K. Ramola (TIFR)]

Antiferromagnetic order in systems with doublet $S_{\rm tot} = 1/2$ ground states

Projector Quantum Monte-Carlo methods were used to study the $S_{\rm tot} = 1/2$ doublet ground states of two dimensional $S = 1/2$ antiferromagnets on a $L \times L$ square lattice with an odd number of sites $N_{\rm tot} = L^2$. The ground state spin texture $\Phi^z(\vec{r}) =< S^z(\vec{r}) >_\uparrow$ in $|G >_\uparrow$,

the $S^z_{\rm tot} = 1/2$ component of this doublet, was computed to investigate the relationship between n^z, the thermodynamic limit of the staggered component of this ground state spin texture, and m, the thermodynamic limit of the magnitude of the staggered magnetization vector of the same system in the singlet ground state that obtains for even $N_{\rm tot}$. A universal relationship between the two, that is independent of the microscopic details of the lattice level Hamiltonian and can be well approximated by a polynomial interpolation formula: $n^z \approx (1/3 - \frac{a}{2} - \frac{b}{4})m + am^2 + bm^3$, with $a \approx 0.288$ and $b \approx -0.306$, was found. It was also found that the full spin texture $\Phi^z(\vec{r})$ is itself dominated by Fourier modes near the antiferromagnetic wavevector in a universal way. On the analytical side, this question was explored using spin-wave theory, a simple mean field model written in terms of the total spin of each sublattice, and a rotor model for the dynamics of $\vec{n}$. It was found that spin-wave theory reproduces this universality of $\Phi^z(\vec{r})$ and gives $n^z = (1 - \alpha - \beta/S)m + (\alpha/S)m^2 + \mathcal{O}(S^{-2})$ with $\alpha \approx 0.013$ and $\beta \approx 1.003$ for spin-S antiferromagnets, while the sublattice-spin mean field theory and the rotor model both give $n^z = 1/3m$ for $S = 1/2$ antiferromagnets. It was argued that this latter relationship becomes asymptotically exact in the limit of infinitely long-range *unfrustrated* exchange interactions. [S. Sanyal (TIFR), A. Banerjee (IMSc), K. Damle (TIFR), A. W. Sandvik (Boston Univ.)]