TIFR Annual Report 2009-10

THEORETICAL PHYSICS

Condensed Matter and Statistical Physics

HIGHLIGHTS

Proportionate growth, of the type that commonly occurs in nature when organisms grow, was successfully modeled by a simple sandpile model in which grains were added at a single site. The rich mathematical structure of this model was elucidated in some detail.

A number of systems composed of hard objects on continuous lines were studied for their tendency to order with increasing density, by mapping to equivalent systems of classical interacting spins. A system of disks centred on either of two close-by parallel lines was shown to map exactly to a 1-D antiferromagnetic model, needle-like objects on a line mapped to Ising-like models, and objects with arrowheads mapped to spin systems with chiral interactions.

Direct second-order quantum phase transitions, forbidden by Landau theory, were observed between the Néel ordered phase and valence bond solid phase in square lattice spin-1/2 antiferromagnets. Logarithmic violations of conventional quantum critical scaling were also detected at these transitions.

A novel Kondo effect was discovered in a study of magnetic impurity effects in the honeycomb lattice $S = 1/2$ Kitaev model which shows a Z_2 spin liquid behaviour over a range of parameters. The scaling of the impurity coupling was found to be independent of its sign, with an intermediate coupling unstable fixed point separating weak and strong coupling sectors. Most interestingly, the unstable fixed point separates topologically different sectors: the strong coupling sector is associated with a finite Z_2 flux (a non-abelian anyon), while the weak coupling sector is flux free.

TEXT

Patterns Formation in growing sandpiles

Adding sandgrains at a single site, on a periodic background,and allowing the configuration to stablize produces interesting and beautiful patterns. These also provide a simple mathematically tractable model of proprtionate growth. If the heights at all sites are sufficiently low, then the pattern produced has diameter varying as $N^{1/d}$ in d-dimensions. If the heights at different sites are sufficiently high, the size of the disturbed region becomes infinite, after a finite number of particles have been added.

It was shown that there is a class of periodic baclgrounds in two dimensions, when the size of the region affected remains finite for any finite number of particles, but the area grows as N^2 for large N. The number of topplings $T(x, y)$ at site (x, y) for a fixed N, were found to be a piece-wise linear function of x and y. This also makes an interesting connection with the recently active field of tropical mathematics. [D. Dhar and T. Sadhu]

Percolation of overlapping discs with two sizes

Continuum percolation of overlapping circular discs of two sizes was studied. A phenomenological scaling equation for the increase in the effective size of the larger discs due to the presence of the smaller discs was proposed. The critical percolation threshold as a function of the ratio of sizes of discs, for different values of the relative areal densities of two discs, can be described in terms of a scaling function of only one variable. The recent accurate Monte Carlo estimates of critical threshold by Quintanilla and Ziff [Phys. Rev. E, 76 051115 (2007)] are in very good agreement with the proposed scaling relation. [D. Dhar with A.C. Balram (IISER, Pune)]

Correlations between hard objects in a continuum

Systems composed of hard objects in a continuum often show a tendency to order as the density is increased, but exact results on such systems are scarce. A system of disks centered on either of two close-by parallel lines, interacting only through mutual exclusion, was, therefore, considered. Each

disk attempts a random displacement on the line, and also exchanges its location between lines, with attempts being successful only if the hard core constraint is respected. An exact mapping was demonstrated of this to an antiferromagnetic Ising model in one dimension, with exchange coupling proportional to the pressure. The system tends to approach close packing and develop a staggered order involving placements in the lines. Other systems with hard core interactions between constituents on a continuous line can also be mapped similarly. Interestingly, systems of needle-like objects map onto Ising-like models, whereas if the objects resemble arrowheads, they map onto spin systems with chiral interactions. The dynamics of these systems have also been investigated by numerical simulation and interesting differences have been observed.
[M. Barma with M. Khandkar (Pillai's Institute of Information Technology)]

Impurity spin texture at a deconfined quantum critical point

The spin texture surrounding a non-magnetic impurity in a quantum antiferromagnet is a sensitive probe of the novel physics of a class of quantum phase transitions between a Néel ordered phase and a valence bond solid phase in square lattice $S = 1/2$ antiferromagnets. Using a newly developed $T = 0$ Quantum Monte Carlo technique, the form of this spin texture was computed at these transitions. It was found that this texture does not obey the universal scaling form expected at a scale invariant quantum critical point. The precise logarithmic form of these scaling violations was also identified. These results are expected to yield important clues regarding the probable theory of these unconventional transitions.
[A. Banerjee and K. Damle with F. Alet (Toulouse)]

Magnetic impurities and non-abelian anyons in the honeycomb Kitaev model

Magnetic impurities effects in strongly correlated electron systems are interesting both as probes for the underlying electronic state as well as for their nontrivial effects on the thermodynamic and transport properties of the system. It is in such a context that the $S = 1/2$ Kitaev model on the honeycomb lattice was studied. The Kitaev model is a rare example of a two-dimensional quantum spin system that is integrable by various schemes

of fermionization. Over a range of parameters, one has a peculiar spin liquid phase where each spin fractionalizes into a localized Majorana fermion and a linearly dispersing massless Majorana fermion, and each plaquette is associated with a conserved Z_2 flux. The ground state sector is flux free. Coupling an impurity quantum spins to one of the Kitaev spins results in an unusual Kondo effect. The scaling of the impurity coupling turns out to be independent of the sign of the coupling unlike the Kondo effect in metals, and an intermediate coupling fixed point separates weak and strong coupling sectors. Remarkably, the strong coupling sector was found to be associated with a finite Z_2 flux pinned to the impurity unlike the weak coupling sector which is flux free. In fact this Z_2 vortex is a non-abelian anyon. The authors believe this is the first reported instance of a Kondo effect with a topological transition. When more than one magnetic impurity is present, the RKKY interaction of the impurities (mediated by the delocalized Majoranas) was also found to be very unusual. For spin half impurities, no RKKY interaction is possible. For larger values of the impurity spins, the delocalized Majorana fermions mediate a nematic interaction that falls off as a power law of their separation only when each impurity is magnetically coupled to more than one Kitaev spin. In all other previously studied systems, the RKKY interaction is dipolar and is finite for spin half impurities.

[K. Dhochak and V. Tripathi with R. Shankar (IMSc)]

Temperature dependence of resistivity and Hall-coefficient in a strongly disordered metal: NbN

In a normal disordered metal at low temperatures, two mechanisms that affect the resistivity R and the Hall resistance R_H in very different ways are well-known in the literature: (a) Weak localization corrections that arise from self-interference of the (noninteracting) electron wavefunction affect the resistivity but not the Hall resistance; and (b) Altshuler-Aronov corrections arising from the interplay of disorder and electron interaction are associated with a characteristic signature; $\delta R_H/R_H = 2\delta R/R$, where δR is the correction to resistivity, etc. These two mechanisms are the theoretical limiting cases of noninteracting electrons and interacting electrons respectively. Experimentally, interpolation between the two limits has been engineered in silicon MOSFETS - this is achieved by tuning their dimensionless conductance which changes the relative importance of quantum

interference and interaction effects. In this work, measurements on disordered NbN films have thrown up a very unexpected result. It was observed that $\delta R_H/R_H$ was approximately 0.6 $\delta R/R$ over a wide range of temperatures and the ratio, 0.6, remained the same for different values of the dimensionless conductance. The same ratio was also observed in a single disordered Be film (data provided by P. W. Adams, Louisiana, USA) implying that this was not an effect that was specific to NbN. The reason for this mysterious relation between the resistivity and Hall resistance is yet to be found.
[V. Tripathi with M. Chand (TIFR), A. Mishra (TIFR), Y. M. Xiong (Louisiana), A. Kamlapure (TIFR), S. P. Chockalingam (TIFR), J. Jesudasan (TIFR), V. Bagwe (TIFR), M. Mondal (TIFR), P. W. Adams (Louisiana) and P. Raychaudhuri (TIFR)]

Penetration depth and tunnelling studies in very thin epitaxial NbN films

The magnetic penetration depth $\lambda(T)$ and superconducting gap $\Delta(T)$ of NbN films were directly measured as a function of temperature T over a range of film thicknesses. With the exception of the thinnest films the measured gap Δ and penetration depth λ were found to be consistent with the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity. For the thinnest films, the Berezinski-Kosterlitz-Thouless (BKT) transition was observed instead indicating the two-dimensional (2D) nature of these thin films. The BKT transition is associated with vortex and antivortex excitations forming bound states below a transition temperature T_{BKT}, and being unbound above. The onset of the transition is characterised by a jump in superfluid density n_s, which is inversely proportional to the square of the penetration depth. However it was also observed that the jump in superfluid density at T_{BKT} was many times greater than the theoretical prediction for the simplest model for the 2D superconductor - a 2D ferromagnetic XY model. This mystery was resolved by allowing the core energies of the vortex excitations to be an adjustable parameter. After reasonably accounting for disorder effects, the authors were able to obtain excellent agreement with the data.
[V. Tripathi with A. Kamlapure (TIFR), M. Mondal (TIFR), M. Chand (TIFR), A. Mishra (TIFR), J. Jesudasan (TIFR), V. Bagwe (TIFR), L. Benfatto (Rome) and P. Raychaudhuri (TIFR)]