

# **TIFR Annual Report 2001-02**

## **THEORETICAL PHYSICS**

### **Foundations of Quantum Theory**

#### **Bell Inequalities in $2N$ dimensional Phase Space and the $N+2$ Marginal Theorem**

The  $N+2$  marginal problem in  $2N$ -dimensional phase space has been solved. This proves a long standing conjecture: there exist quantum states such that it is impossible to have quantum probabilities of more than  $N+1$  Complete Commuting Sets (CCS) of observables coexisting as marginals of one positive phase space density. To achieve this, “phase space Bell inequalities” were derived and it was then demonstrated that quantum mechanics violates them. The inequalities enable “experimental” tests of the orthodox-versus-hidden variable interpretations of quantum mechanics within the position-momentum sector, analogous to those performed within the spin sector. They are also useful in quantum information processing as quantitative tests of quantum entanglement. To logically complete the problem, the most general positive phase space density which has the maximum number of marginals ( $N+1$ ) coinciding with corresponding quantum probabilities of  $N+1$  different (noncommuting) Complete Commuting Sets of observables has been obtained explicitly. These results will enable the construction of maximally realistic quantum theories. They also open up new applications in signal and image processing. [G. Auberson (University of Montpellier II), G. Mahoux (C.E.N. Saclay), S.M. Roy and V.Singh]

#### **Maximally Classical Realistic Quantum Theory**

A stationary principle which gives a nonperturbative definition of a maximally classical and maximally realistic phase space density has been formulated. It is shown in the cases of coherent states and bound states of an oscillator and Gaussian free particle states, that the maximally classical trajectories are in fact exactly classical in these simple examples. In contrast, it is known that the De Broglie-Bohm realistic theory gives highly nonclassical trajectories. [S.M.Roy]