Foundations of Quantum Theory

Marginal distributions in $2N$-dimensional phase space and the quantum $N + 1$ marginal theorem

The problem of constructing a probability density in $2N$-dimensional phase space which reproduces a given collection of $n$ joint probability distributions as marginals is solved. Only distributions authorized by quantum mechanics, i.e. depending on a (complete) commuting set of $N$ variables, are considered. The set of “admissible” data, i.e. those types of data for which the problem always admits solutions, are determined exactly. This is done in the case where the joint distributions originate from quantum mechanics as well as in the case where this constraint is not imposed. In particular, it is shown that a necessary (but not sufficient) condition for the existence of solutions is $n \leq N + 1$. In all cases of admissibility, the general solution for phase space density is determined explicitly. [G. Auberson (University of Montpellier II), G. Mahoux (C.E.N. Saclay), S. M. Roy and V. Singh].