String Theory and Mathematical Physics

Highlights

A successful comparison was made of microscopic and macroscopic entropies, including higher derivative corrections, angular momentum and an extension of these ideas to nonsupersymmetric black holes.

The entropy function for 5D extremal black holes and rings was constructed.

The relationship of the blackhole attractor mechanism to the entropy for both supersymmetric and non-supersymmetric blackholes was elucidated.

The Sen entropy function formalism was generalised to rotating blackholes.

The role of the attractor mechanism in understanding the entropy of non-supersymmetric blackholes was elucidated.

A realistic model of intersecting $D4$-branes and $D8$ and $D8$-bar branes for chiral symmetry breaking in a $QCD$-like theory was studied with the brane-anti-brane tachyon present, which gives rise to fermion masses and hence also masses for the mesons, leading to more realistic $QCD$-like phenomenology.

A more detailed understanding of the AdS/CFT dual gauge theory “resolution” of certain null cosmological singularities was obtained.

The complete evolution of a universe, including both big bang and big crunch singularities, has been studied using matrix string theory.

An exact partition function was derived that counts nonperturbative dyonic states.

S-duality invariance of the degeneracies of dyons in orbifold compactifications of the heterotic string to four dimensions with $N = 4$ supersymmetry was made manifest. The existence of multi-centered particle-like configurations, with subleading entropy in agreement with the microscopic prediction and S-duality invariance, was demonstrated in a situation where there are no big black holes.

Gauge-string duality was demonstrated by the exact computation of some three point correlators using corresponding techniques.

Finite regions of the deconfining phase of a confining gauge theory have been shown to be solitons of the large $N$, long wavelength, effective Lagrangian of the thermal gauge theory.

A simplified matrix model of $IKKT$ type was shown to exhibit spontaneous breakdown of spacetime symmetry above 4-dimensions.
A group theoretic technique was developed to simplify the analysis of the stability of non-supersymmetric attractors obtained in Type II − A compactifications on Calabi Yau manifolds.

It was shown that solitonic configurations called dual giant gravitons exactly count \( BPS \) states of supergravity with four or more supersymmetries.

Matching of the counting of supersymmetric states in certain superconformal field theories with that in their gravity duals has been demonstrated.

Witten-type indices for superconformal field theories in \( d = 3, 5, 6 \) have been constructed.

A field theory realization of black holes as lumps of spinning plasma has been found.

Novel nonperturbative features of topological strings were identified, including a violation of holomorphic factorisation in the partition function due to noncompact branes.

Large-volume analogues of fractional two-branes on resolutions of orbifolds \( \mathbb{C}^3/\mathbb{Z}_n \) were obtained for \( n = 3, 5 \) and a ‘quantum McKay correspondence’ was studied.

An example was found in string theory exhibiting Hitchin’s new notion of a Generalized Hyper Kahler geometry (\( GHK \)). A construction within string theory of a compact \( GHK \) geometry was proposed.

The full phase structure induced by closed string tachyon condensation of unstable nonsupersymmetric conifold-like singularities was explored.

**TEXT**

**Blackholes**

It was argued that the spacetime geometry corresponding to spinning fundamental BPS string states contains a horizon with ring-like topology once higher derivative corrections are included. A general scaling argument was used to show that the entropy of such a horizon is in precise agreement with the microscopic counting including a nontrivial dependence on spin up to overall normalization. [A. Dabholkar and A. Iqubal with N. Iizuka (KITP, USA), A. Sen (HRI) and M. Shigemori(CalTech, USA)]

The 4\( D \) − 5\( D \) lift was used to construct the entropy function for 5\( D \) extremal black holes and black rings, which project down to either static or stationary black holes. This was done in the context of two derivative gravity coupled to abelian gauge fields and neutral scalar fields. [K. Goldstein, R.P. Jena]

Four-dimensional spherically symmetric black hole solutions were investigated in gravity theories with massless, neutral scalars non-minimally coupled to gauge fields. In the non-extremal case, it was explicitly shown that, under the variation of the moduli, the scalar
charges appear in the first law of black hole thermodynamics. It was argued that the attractor mechanism is at the basis of the matching between the microscopic and macroscopic entropies for the extremal non-BPS Kaluza-Klein black hole. [K. Goldstein with D. Astefanesei (HRI), Swapna Mahapatra (Utkal)]

The entropy function formalism of Sen was generalised to rotating blackholes. It was found that the entropy function can have flat directions so that, while the entropy is independent of the asymptotic moduli, the near horizon geometry is not. Various cases were studied, including the Kerr black hole, Kerr-Newman black hole, black holes in Kaluza-Klein theory, and black holes in toroidally compactified heterotic string theory. [K. Goldstein, R.P. Jena and S.P. Trivedi with A. Sen (HRI)]

It was shown how the attractor mechanism can lead to a better understanding of the entropy for non-supersymmetric extremal black holes. If there is an approximately flat direction of the entropy function, then by going along this direction one can sometimes connect the weak coupling region, where the microscopic counting is valid and the strong coupling region, where the black hole description is valid. Since the entropy cannot change along the flat direction, this explains why the microscopic counting and the Beckenstein Hawking entropy agree in these cases. [A. Dabholkar and S.P. Trivedi with A. Sen (HRI)]

**Brane models and QCD**

The Sakai-Sugimoto model of intersecting $D4$-branes and $D8$ and $D8$-bar branes for chiral symmetry breaking in a $QCD$-like theory is so far the only construction in a string theory setting which realizes non-abelian chiral symmetry and its breaking, with the drawback of zero quark masses leading to massless pions. Studying the model with the tachyon mode of the $D8$ – $D8$-bar branes switched on, however, it has been found that in the holographic dual geometry, near the infrared region where the branes come sufficiently close together, the tachyon condenses, leading to chiral symmetry breaking and pions. The pions are massive if the non-normalizable mode of the tachyon is switched on. This raises the hope of being able to do realistic phenomenology with this model. Further investigations are under way. [A. Dhar and P. Nag]

**Cosmology and Strings**

A more detailed analysis of some earlier work on cosmological singularities, by the same collaboration, was made. Cosmologies which depended on a light-like direction were the focus and it was argued, using AdS/CFT, that in the gauge theory description these singularities were “resolved”. Hence it could be argued that space-time extends past the singularity and
a prescription was found for this extension which is consistent with the gauge theory dual description. [K. Narayan and S.P. Trivedi with S.R. Das (Kentucky) and J. Michelson (Ohio State U.)]

Following work by Craps, Sethi and Verlinde on cosmologies with a Big Bang, an extended background describing a universe, with both big bang and big crunch singularities, was proposed in terms of Matrix String Theory. A simple theory capable of describing the complete evolution of this closed universe was developed. [C. Papageorgakis with J. Bedford and J. Ward (Queen Mary, London) and D. Rodriguez-Gomez (Princeton)]

Counting dyons in orbifold compactifications

The partition function of dyons in $N = 4$ supersymmetric $CHL$ compactification was computed in terms of a Siegel modular form from the genus-two partition function of the heterotic string. [A. Dabholkar with D. Gaiotto (LPTHE, Jussieu)]

S-duality invariance of the degeneracies of dyons in orbifold compactifications of the heterotic string to four dimensions with $N = 4$ supersymmetry was made manifest. Using M-theory it was shown that the genus-two contribution captures the degeneracy, without additional contributions from higher genus Riemann surfaces, only if a specific irreducibility criterion is satisfied by the charges. Even though there are no big black holes in supergravity corresponding to the negative discriminant states which are predicted, it was shown that there are multi-centered particle-like configurations with subleading entropy in agreement with the microscopic prediction and our prescription for S-duality invariance. [A. Dabholkar and S. Nampuri with D. Gaiotto (LPTHE, Jussieu, Paris)]

Gauge-string duality

Exact three point correlators of boundary chiral operators were computed in $AdS_3 \times S^2 \times T^4$ background using conformal field theory techniques and were shown to be in precise agreement with the same quantities computed in the boundary $CFT$ of symmetric product of $T^4$ from totally different means. This provides a nontrivial dynamical test of the gauge-string duality. [A. Dabholkar with A. Pakman (Stony Brook)]
Gauge theories and matrix models

Finite regions of the deconfining phase of a confining gauge theory (plasma balls/kinks) are seen to be solitons of the large $N$, long wavelength, effective Lagrangian of the thermal gauge theory expressed in terms of suitable order parameters. The effective Lagrangian of this class of confining gauge theories is a $1-d$ unitary matrix model, whose dynamics can be studied by an exact mapping to a non-relativistic many fermion problem on a circle. [P. Basu, B. Ezhuthachan and S.R. Wadia]

In order to study the emergence of spacetime from the IKKT matrix model a simplified version was studied using the “factorization method”. The simplified model exhibits a spontaneous breakdown of spacetime symmetry above 4-dimensions. [T. Azuma with K.N. Anagnostopoulos (Athens, Tech. U.) and J. Nishimura (KEK)]

Gauge theories at finite temperature

The finite temperature effective action of the $\mathcal{N} = 4$, $SU(N)$ Yang-Mills theory on $S^3$ has been argued to be expressed entirely in terms of the constant Polyakov line on $S^3$. While this can be established in perturbation theory at weak (’t Hooft) coupling and by using the AdS/CFT correspondence at strong coupling, it would be desirable to show this for all values of the coupling constant. In particular this means that we must show that the $SO(6)$ $R$-symmetry of the gauge theory is not spontaneously broken at large $N$, and hence there are no Nambu-Goldstone modes. A program of studying this by numerical methods in a 0 + 1-dimensional model is in progress. It has been shown that the $R$-symmetry is unbroken for massive scalars. [T. Azuma, P. Basu and S.R. Wadia]

Non-Supersymmetric attractors in string theory

Non-Supersymmetric Attractors obtained in Type IIA compactifications on Calabi Yau manifolds were studied. It was shown that the stability analysis can be considerably simplified by using group theoretic techniques. Attractors with $D0 - D4$ and $D0 - D6$ brane charges were analysed. In the latter case it was found that there is a moduli space of solutions and the resulting attractors are stable. The analysis was restricted to the two derivative action. [S. Nampuri and S. Trivedi with P.K. Tripathy (IIT Madras)]
Soliton representation of BPS sectors in AdS/CFT

It was shown that the representation of gravity in terms of half-BPS giant/dual giant gravitons continues to hold in case of fewer, down to four, supersymmetries. Dual giant gravitons preserving four supersymmetries were studied in $AdS_5 \times S^5$ and were quantized semiclassically. It was found that the quantized Hilbert space corresponds to that of up to $N$ 3-dimensional simple harmonic oscillators, where $N$ is given in terms of the radius of curvature of the geometry. The wavefunctions of this theory matched exactly with that of the boundary super Yang-Mills theory in the sector preserving the same supersymmetries. [G. Mandal with N. Suryanarayana (Perimeter Institute, Canada and Imperial College)]

Dual giant gravitons were also studied in $AdS_5 \times Y^5$ and $AdS_4 \times Y^7$ where $Y^n$ represents an $n$-dimensional Sasaki-Einstein manifold. These preserve four supersymmetries. Semiclassical quantization of these configurations were performed using the method of Kaehler quantization. The phase space was found to be symplectically isomorphic to the Kaehler cone over the Sasaki-Einstein manifold and with appropriate identification of the radial coordinate of the cone with the radial coordinate of the $AdS$ space. The dual giant graviton Hilbert space was compared with that of the boundary in the case of $T^{1,1}$ and it was found that the two Hilbert spaces match exactly. [A. Basu and G. Mandal]

The field theory description of $N = 4$ Yang Mills was used to compute the exact, finite $N$ partition function of this theory over $1/8$ BPS supersymmetric states, at weak coupling. Then the gravity description was employed to compute the same object at strong coupling. The two results were the same, suggesting the conjecture that the $1/8$ BPS partition function is non renormalized in $N = 4$ Yang Mills. [S. Minwalla with I. Biswas (TIFR Maths), D. Gaiotto (Harvard) and S. Lahiri (Harvard)]

A computation was performed of the exact, finite $N$ partition function over $1/8$-th BPS states on the world volume of $N$ M2 branes and $N$ M5 branes, using both field theory as well as dual gravity reasoning. These methods yield the same answer. [S. Bhattacharya and S. Minwalla]

Stationary solutions have been found to the equations of fluid mechanics that governs the long distance behaviour of the deconfined fluid of $N = 4$ Yang Mills on a Scherk- Schwarz $S^1$ and higher dimensional generalizations thereof. The fluid mechanical solutions found are dual to black holes and black rings in dual $AdS$ spaces. The solutions share all the (well studied) thermodynamical properties of these objects in flat space. The duality between fluid mechanics and black hole physics - derived here from the $AdS/CFT$ correspondence - is reminiscent of the Membrane Paradigm of black hole physics. [S. Minwalla and S. Lahiri (Harvard)]
The half-supersymmetric mesonic excitation spectrum of $N = 1$ superconformal $AdS/CFT$ dual gauge theories obtained from $D3$-branes stacked at nontrivial conifold-like singularities were studied. Mapping gauge invariant operators in the quiver theory to invariant monomials in the corresponding linear sigma model, a partition function over these BPS states was written down which agrees with their bulk interpretation in terms of (mesonic) giant gravitons. [K. Narayan with L. Grant (Harvard)]

**Sine-Liouville CFT and 2d-blackhole**

The duality between the Sine-Liouville conformal field theory and the two dimensional black hole was revisited by considering the two possible Sine-Liouville dressings together. The role of higher winding perturbations was examined in the context of $c = 1$ strings, and it was proposed that they are related to higher-spin discrete states that generalize the 2d black hole operator. [A. Mukherjee and S. Mukhi with A. Pakman (Stony Brook)]

**Stringy geometry**

Large-volume analogues of fractional two-branes on resolutions of orbifolds $\mathbb{C}^3/\mathbb{Z}_n$ were obtained for $n = 3, 5$ and a ‘quantum McKay correspondence’ was studied. Explicit duals to the fractional two branes were constructed. This is related to the discussion of the “missing branes” in the context of the non supersymmetric branes on orbifolds of $\mathbb{C}^2$. The branes dual to the ones corresponding to the missing branes of the $\mathbb{C}^2$ story (the coloumb branch branes) cannot be written as line bundles with support only on the non compact divisors. [B. Ezhuthachan with T. Jayaraman and S. Govindarajan (I.M.Sc., Chennai)]

Hitchin’s notion of Generalized Hyper Kahler geometry ($GHK$) was studied and an example of $GHK$ from string theory provided. The example was the near horizon limit of NS5 brane geometry, which was reformulated as a $GHK$ geometry. A construction within string theory of a compact $GHK$ geometry was proposed by using the $SL(2, \mathbb{Z})$ symmetry of IIB string theory to combine any $SL(2, \mathbb{Z})$ transform of the $B$-fields with the metric to get a generalized complex struture. [B. Ezhuthachan with D. Ghoshal (HRI)]

The full phase structure induced by closed string tachyon condensation of toric nonsupersymmetric unstable conifolds was explored, continuing work initiated and reported on in the previous year. Techniques of toric geometry and the linear sigma model were used. A cascade-like phase structure was found, containing decays to lower order conifold-like singularities consistent with the Type $II$ GSO projection obtained previously. Transitions between various phases include flips and flops. [K. Narayan]
Superconformal field theories

Witten-type indices have been constructed for superconformal field theories in \( d = 3, 5, 6 \). A detailed study has also been performed of the pairing up of short representations into long representations at the unitarity bound, in order to demonstrate that our index is the most general superconformal index in these theories. These indices are conceptually analogous to the elliptic genus and are likely to have a plethora of applications to the study of higher dimensional superconformal field theories. [J. Bhattacharya, S. Bhattacharya and S. Minwalla with S. Raju (Harvard)]

Topological strings

The duality between type 0 noncritical strings and topological \( B \)-model strings was studied, with special emphasis on the flux dependence. The \( B \)-model dual was precisely defined to include both compact and noncompact \( B \)-branes. It was argued that the latter violate holomorphic factorisation and contribute a disc term to the partition function. [A. Mukherjee, S. Mukhi and R. Nigam]

Mathematical Physics

The phenomenon of level rearrangement, which occurs when an attractive long-range potential is perturbed by a short-range attractive potential of variable strength, was investigated through various solvable models. This was first investigated by Zeldovich in a problem of condensed matter physics, but can also occur in exotic atoms and quantum dots. [A.K. Raina with M. Combescure and J-M Richard (Lyon, France), C. Weydert (Grenoble, France), A. Khare (IOP, Bhubaneswar)]

A study was made extending earlier work on \( W \)-geometry to the symplectic group: symplectically equivariant immersions of a curve into the space of Lagrangian subspaces of \( \mathbb{C}^{2n} \) were shown to be in bijective correspondence with a given projective structure and an \( O(n, \mathbb{C}) \)-connection on the curve. [A.K. Raina with I. Biswas (School of Maths, TIFR)]