

Quantum Theory of Materials: Methods and Applications

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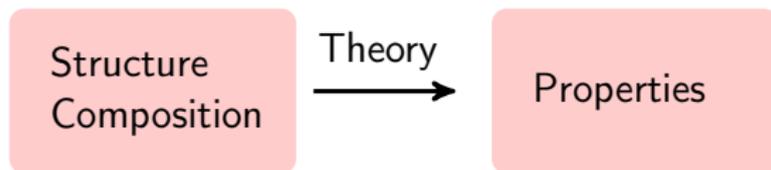


May 16, 2012

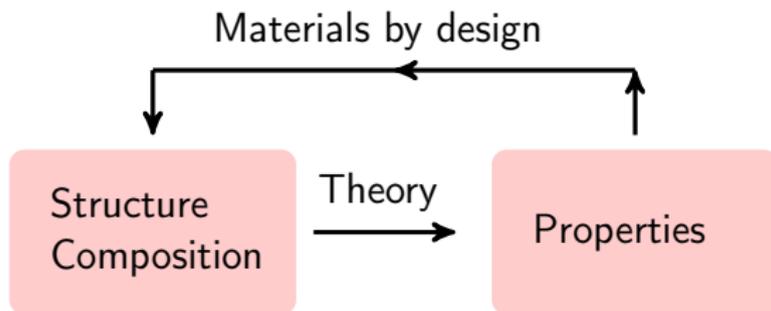


- Introduction.
 - Quantum theory of materials.
 - Methods for calculating material properties.
- Defects in materials.
 - Defects in high- κ dielectrics.
 - Defects for quantum computing.

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- Detailed microscopic study – **understand** relationships between structure, composition and properties.
- **Predict** and **interpret** experiment – structural, dynamical, electronic and optical properties of materials.



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- **Predict** and **interpret** experiment – structural, dynamical, electronic and optical properties of materials.
- Assist in **designing** new materials.

First-principles theoretical methods

Pros:

- **No** adjustable parameters.
- **Reliable**: Well-justified and tested approximations.
- **Predictive**: Can be used to design and understand properties of new materials.

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Cons:

- Computationally **expensive**.

Material properties using *ab initio* methods

Structural properties:

- Lattice parameters, phase transitions, structure factors, pair correlation functions, elastic constants etc.

Dynamical properties:

- Lattice polarizabilities, dielectric constants, Born effective charges, heat capacity, infrared spectra, Raman spectra etc.

Electronic properties:

- Conductivity, density of states etc.

Optical properties:

- Reflectivity, absorption, refractive index, dielectric function etc.

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Electronic

- C

Electronic structure

Optical properties:

- Reflectivity, absorption, refractive index, dielectric function etc.

Solve the Schrödinger's equation for a solid:

$$\hat{H}\Psi = E\Psi$$

where

$$\hat{H} = \hat{T}_e + \hat{T}_z + \hat{U}_e + \hat{U}_z + \hat{U}_{ez} + \hat{V}_{ext}$$

where

\hat{T}_e, \hat{T}_z : Kinetic energy of electrons and nuclei

$\hat{U}_e, \hat{U}_z, \hat{U}_{ez}$: Potential energy of electron, nuclei and
electron-nuclei interactions

\hat{V}_{ext} : External potential

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Equation contains $3*Z*N + 3*N$ variables where Z is the atomic number and N is the number of atoms.

“The fundamental laws necessary for the mathematical treatment of a large part of physics and the whole of chemistry are thus completely known, and the difficulty lies only in the fact that application of these laws leads to equations that are too complex to be solved.”

— P. A. M. Dirac¹

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“It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed ...”

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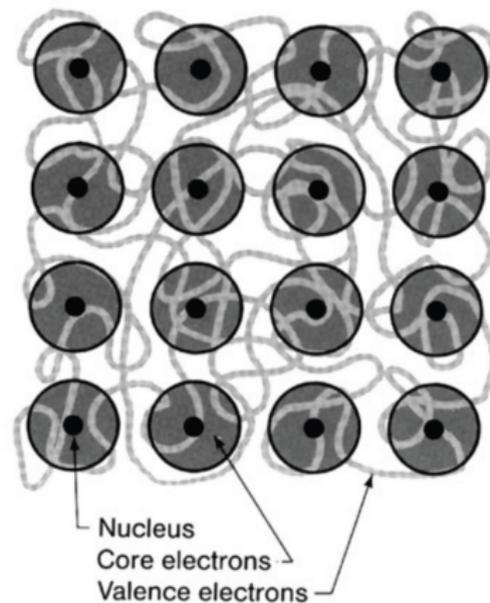
- Electrons move “fast” while nuclei are “slow”.
- Total wavefunction is approximated as:

$$\Psi(\{\vec{r}\}, \{\vec{R}\}) = \Phi(\{\vec{r}\}; \{\vec{R}\}) \chi(\{\vec{R}\})$$

- Dependence of potential on $\{\vec{R}\}$ parametric – electrons move in a field created by static ions.

Pseudopotential Approximation

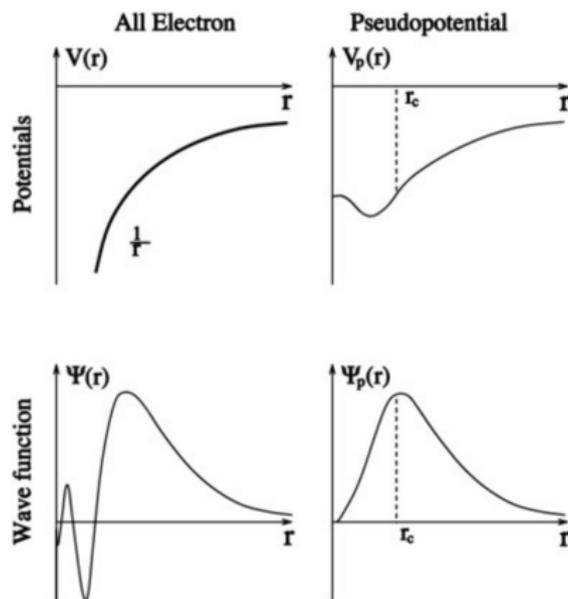
Outer valence electrons determine properties of materials – core electrons do not participate in chemical bonding.



Captures the physical content of the periodic table.

Pseudopotential Approximation

Valence electrons experience a repulsive potential near core due to orthogonalization to core states.



Energy and length scales set by the valence states.

Properties that are intrinsic to a system with all its electrons in equilibrium.

- Density functional theory is the “standard model” for understanding **ground-state** properties.



¹P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964).

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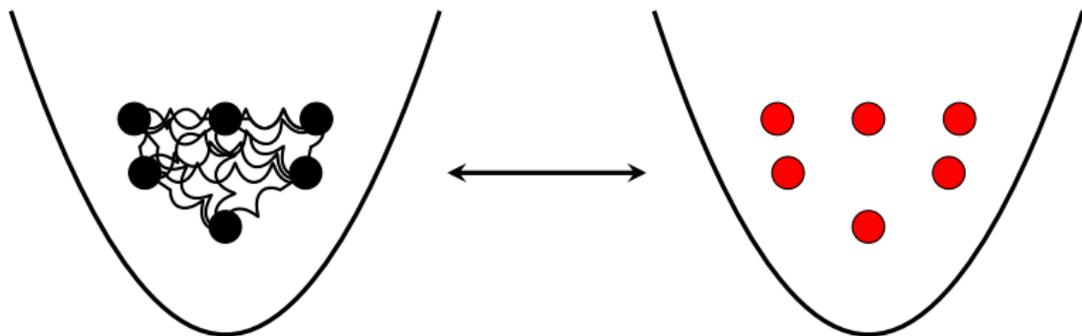
Properties that are intrinsic to a system with all its electrons in equilibrium.



- Density functional theory is the “standard model” for understanding **ground-state** properties.
- Total energy is a functional of the charge density.
- Ground-state properties depend on the total energy of the system.

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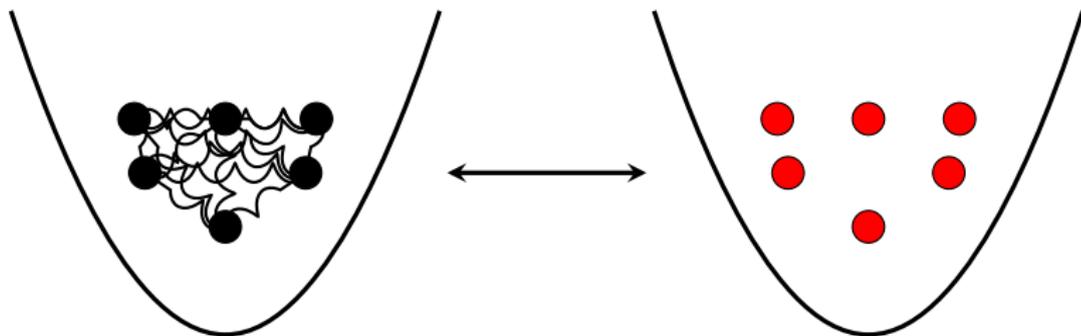
- Kohn-Sham formulation: Map the interacting many-electron problem to non-interacting electrons moving in a self-consistent field.



¹W. Kohn and L. Sham, Phys. Rev. **140**, A1133 (1965).

Methods – Ground-state properties

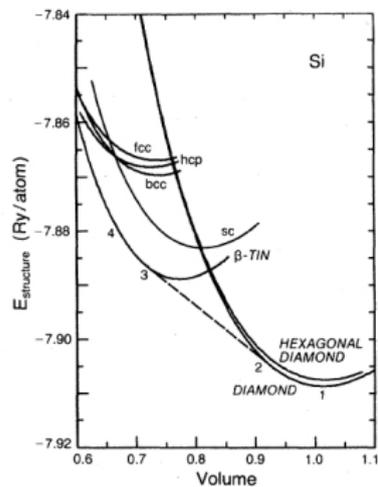
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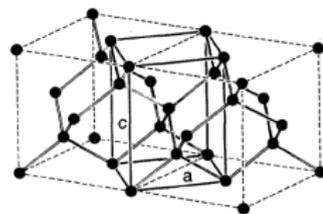
- Not known exactly – local density approximation (LDA) and generalized gradient approximation etc.

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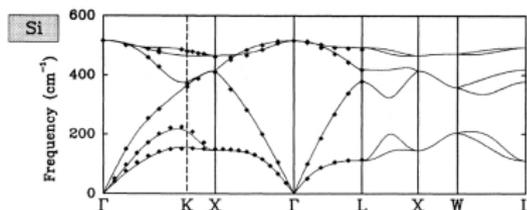
Methods – Ground-state properties



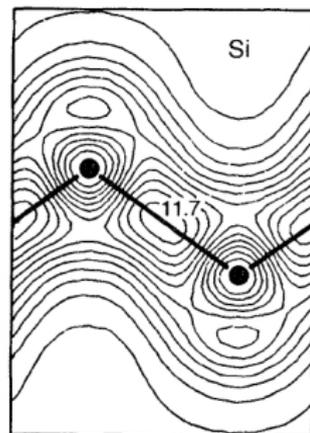
Phase transitions¹



Crystal Structure¹



Phonons²



Charge density¹

¹M. T. Yin and M. L. Cohen, Phys. Rev. B **26**, 5668 (1982).

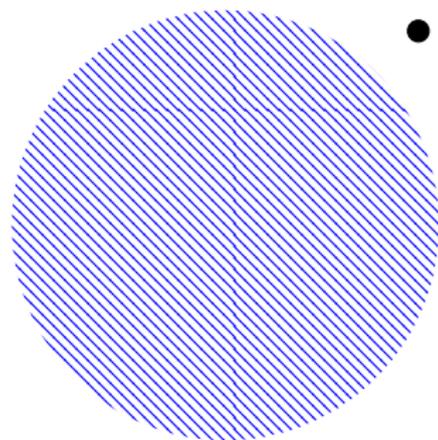
²P. Giannozzi, S. de Gironcoli, P. Pavone, and S. Baroni, Phys. Rev. B, **43**, 7231 (1991).

Spectroscopic properties that involve experiments creating an excited particle above the ground state.

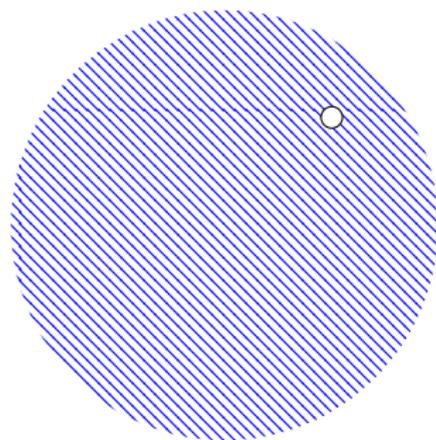
Concept and formalism of the **interacting particle Green's function**.

Methods – Excited-state properties

- $N \pm 1$ particle problem.
- Photoemission, tunneling.
- Quasiparticle approach.



Fermi sea

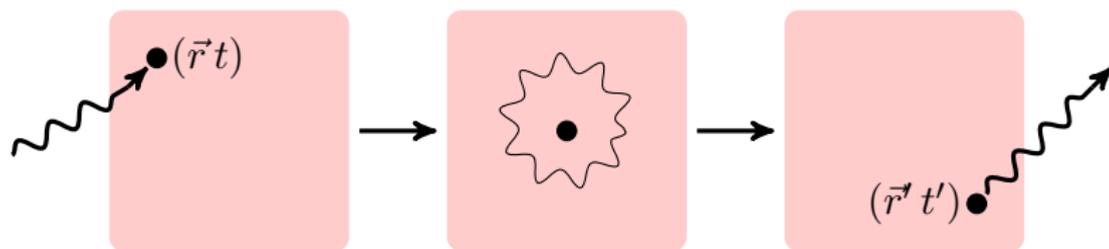


Fermi sea

Methods – Excited-state properties

- $N \pm 1$ particle problem.
- Photoemission, tunneling.
- Quasiparticle approach.

$$G(\vec{r}t, \vec{r}'t') = -i \langle N | T[\hat{\psi}(\vec{r}t) \hat{\psi}^\dagger(\vec{r}'t')] | N \rangle$$



- $N \pm 1$ particle problem.
- Photoemission, tunneling.
- Quasiparticle approach.

$$G(\vec{r}, \vec{r}', \omega) = \sum_s \frac{f_s(\vec{r}) f_s^*(\vec{r}')}{\omega - \omega_s}$$

$$\omega_s = E_N - E_{N-1,s} \quad f_s(\vec{r}) = \langle N-1, s | \hat{\psi}(\vec{r}) | N \rangle \quad \omega_s < \mu$$

$$\omega_s = E_{N+1,s} - E_N \quad f_s(\vec{r}) = \langle N | \hat{\psi}(\vec{r}) | N+1, s \rangle \quad \omega_s \geq \mu$$

- Many-body perturbation theory is the “standard model” for understanding **excited-state** properties.

$$G^{-1} = G_0^{-1} + \Sigma$$

¹L. Hedin and S. Lundqvist, in *Advances in Research and Applications*, edited by F. Seiz, D. Turnbull, and H. Ehrenreich, *Solid State Physics Vol. 23* (Academic Press, New York, 1969) pp. 1-181.

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- GW approximation to the self-energy (Σ).

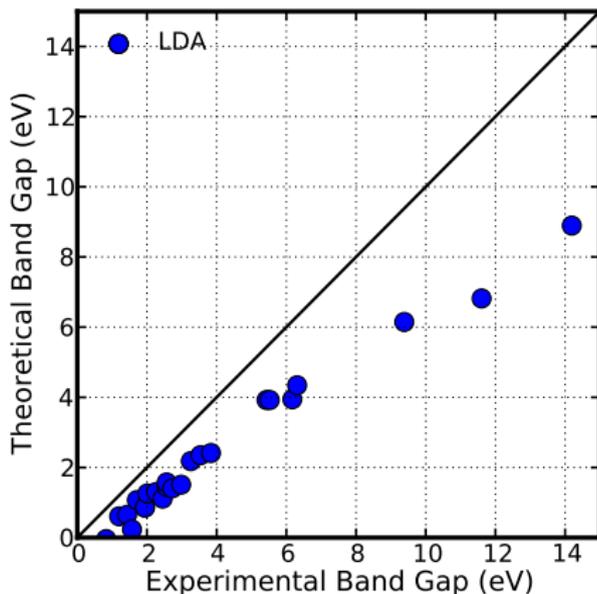

$$\Sigma = \text{---} \leftarrow \text{---} \rightarrow \text{---} = iGW$$
$$W = \varepsilon^{-1} v$$

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Methods – Excited-state properties

Materials: InSb, InAs,
Ge,
GaSb, Si, InP,
GaAs, CdS,
AlSb, AlAs,
CdSe, CdTe,
BP, SiC, C₆₀,
GaP, AlP,
ZnTe, ZnSe,
c-GaN, w-GaN,
InS, w-BN,
c-BN,
diamond, w-AlN,
LiCl,
Fluorite, LiF

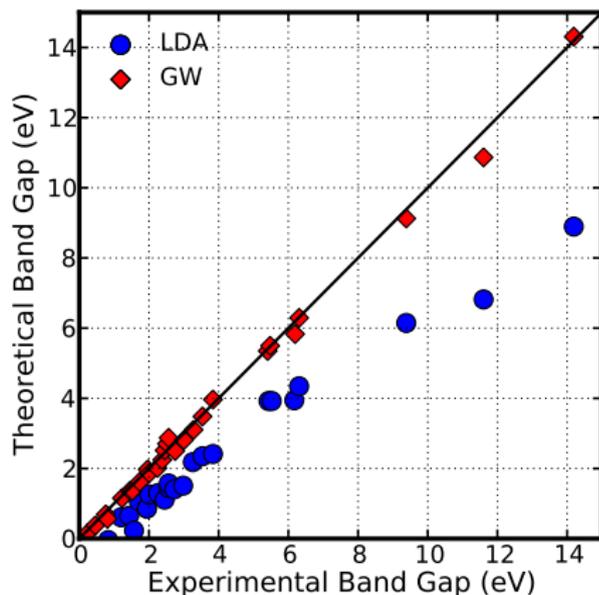


Quasiparticle Gap¹

¹S. G. Louie in Topics in Computational Materials Science, edited by C. Y. Fong (World Scientific, Singapore, 1997).

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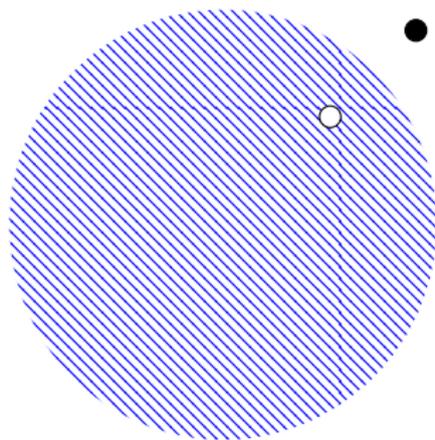


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Methods – Excited-state properties

- $N+2$ particle problem.
- Optical properties – absorption etc.
- Electron-hole interaction (excitons).



Fermi sea

- Coupled electron-hole excitations:

$$|S\rangle = \sum_v \sum_c^{elec\ hole} A_{vc}^S \hat{a}_v^\dagger \hat{b}_c^\dagger |0\rangle$$

where $|0\rangle$ ground-state of many electron system.
 \hat{a}_v^\dagger and \hat{b}_c^\dagger create quasi-particles and quasi-holes.
 A_{vc}^S are coupling constants.

¹M. Rohlfing and S. G. Louie, Phys. Rev. B **62**, 4927 (2000).

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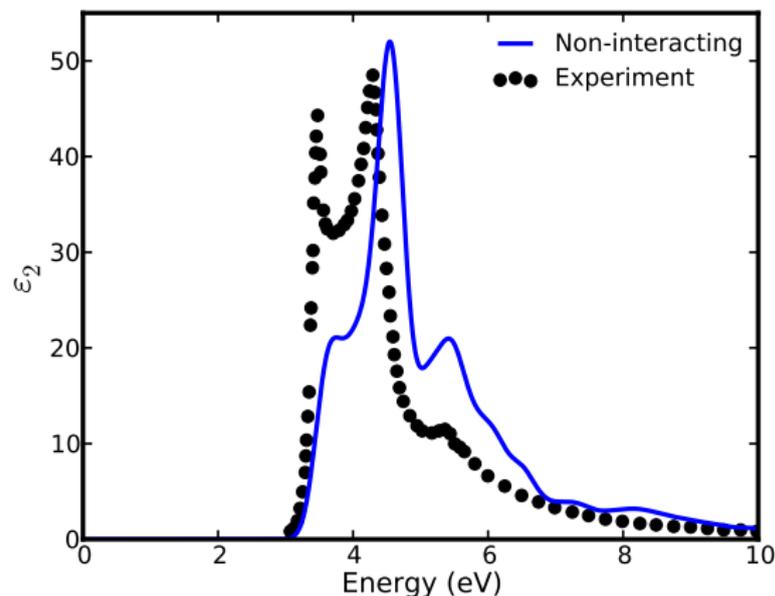
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A_{vc}^S are coupling constants.

- Solve Bethe-Salpeter equation for two particle Green's function.
- Electron-hole interaction kernel:

$$K^{eh} = \begin{array}{c} \text{---} \\ \nearrow \quad \searrow \\ \bullet \quad \bullet \\ \nwarrow \quad \nearrow \\ \text{---} \end{array} + \begin{array}{c} \nearrow \quad \searrow \\ \bullet \quad \bullet \\ \nwarrow \quad \nearrow \\ \text{---} \\ \nwarrow \quad \nearrow \\ \bullet \quad \bullet \\ \nearrow \quad \searrow \end{array}$$

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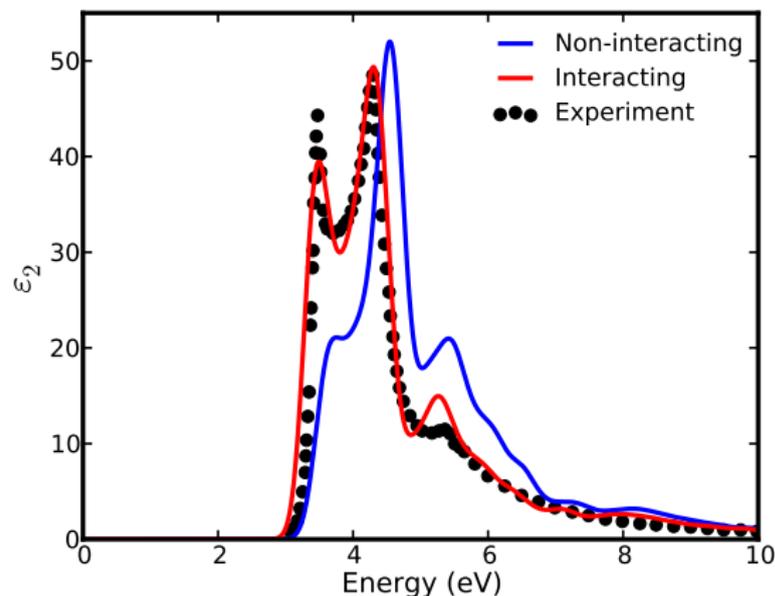
Si Optical absorption¹

¹J. Deslippe, G. Samsonidze, D. A. Strubbe, M. Jain, M. L. Cohen, and S. G. Louie, *Comput. Phys. Commun.* **183**, 1269 (2012).

²P. Lautenschlager, M. Garriga, L. Vina, and M. Cardona, *Phys. Rev. B* **36**, 4821 (1987).

³G. G. Macfarlane and V. Roberts, *Phys. Rev.* **98**, 1865 (1955).

⁴H. R. Philipp, *J. Appl. Phys.* **43**, 2835 (1972).



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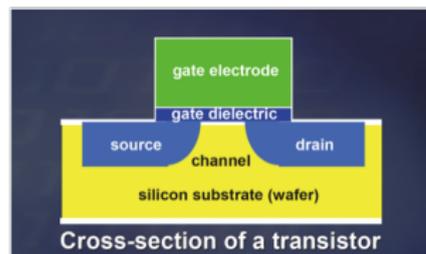
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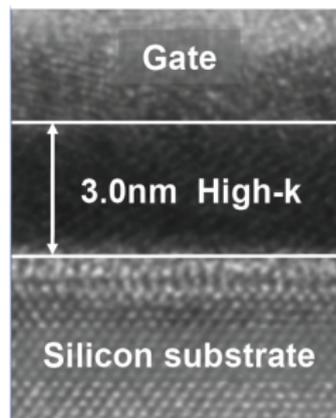
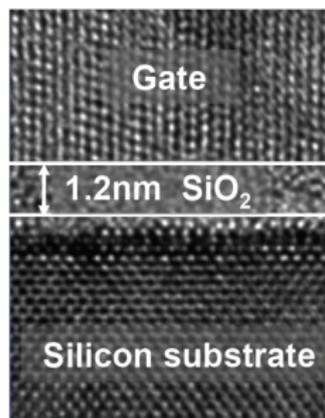
High- κ dielectric materials – Motivation

- Moore's law about size of transistor.
- HfO_2 has replaced SiO_2 in today's MOSFET devices.



$$\kappa_{\text{SiO}_2} \approx 3.9$$

$$\kappa_{\text{HfO}_2} \approx 16 - 17$$



¹<http://www.intel.com/technology/45nm/index.htm>

High- κ dielectric materials – Motivation

- High defect density.
- Charge trapping by defects in the interfacial layer or oxide – threshold voltage instability¹.
- Oxygen-related defects – vacancy² (V_O) or interstitials (I_O) major cause.
- Study the stability of point defects – V_O and I_O .

¹G. Ribes et. al., IEEE Trans. Device Mater. Reliab. **5**, 5 (2005).

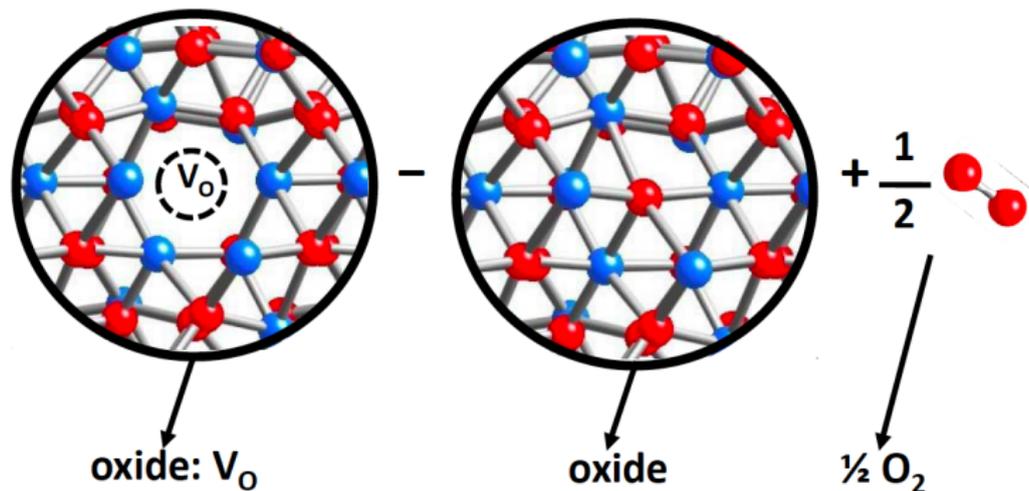
²K. Shiraishi et. al., Jpn. J. Appl. Phys., Part 2 **43** L1413 (2004); H. Park et. al., IEEE Electron Device Lett. **29**, 54 (2008).

High- κ dielectric materials – Definitions

Formation energy of an oxygen-vacancy:

$$E^f[\vec{R}] = E[\vec{R}] - E_{\text{ref}} + \frac{1}{2}E_{\text{O}_2}$$

Example: V_{O}

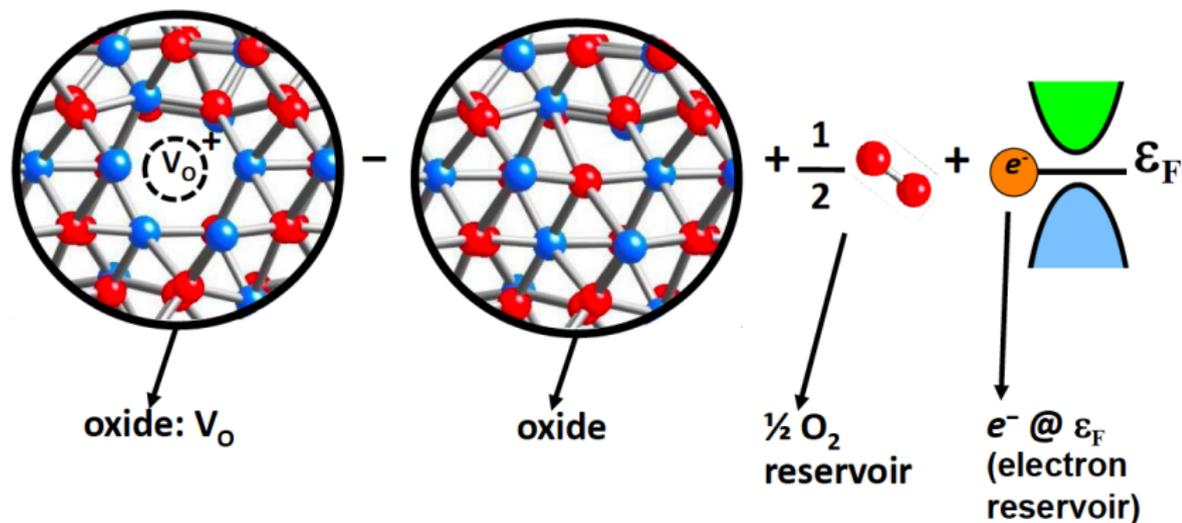


High- κ dielectric materials – Definitions

Formation energy of an oxygen-vacancy:

$$E_q^f[\vec{R}_q](E_F) = E_q[\vec{R}_q] - E_{\text{ref}} + \frac{1}{2}E_{\text{O}_2} + q(E_F + E_v)$$

Example: V_o



Formation energy of an oxygen-related defect in hafnia :

$$E_q^f[\vec{R}_q](E_F) = E_q[\vec{R}_q] - E_{\text{HfO}_2} - n_O \mu_O + q(E_F + E_V)$$

where μ_O is the oxygen chemical potential.

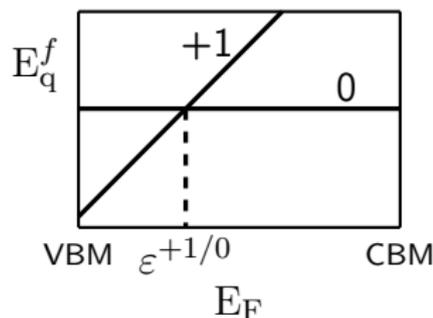
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where μ_O is the oxygen chemical potential.

Charge transition level : $\varepsilon^{q/q-1} = \text{Fermi energy where defect } q \rightarrow q - 1.$



High- κ dielectric materials – Definitions

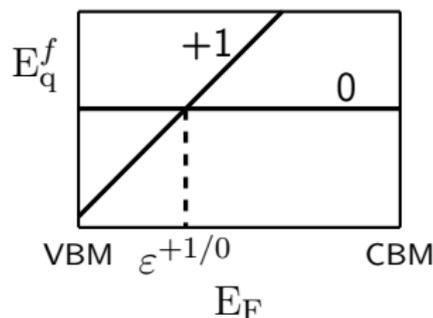
Formation energy of an oxygen-related defect in hafnia :

$$E_q^f[\vec{R}_q](E_F) = E_q[\vec{R}_q] - E_{\text{HfO}_2} - n_O \mu_O + q(E_F + E_V)$$

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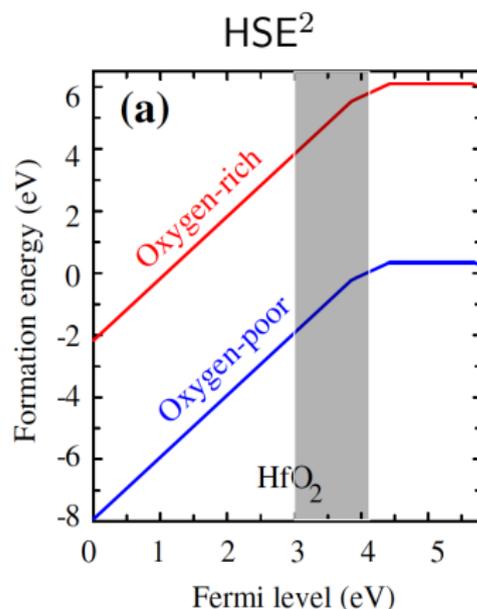
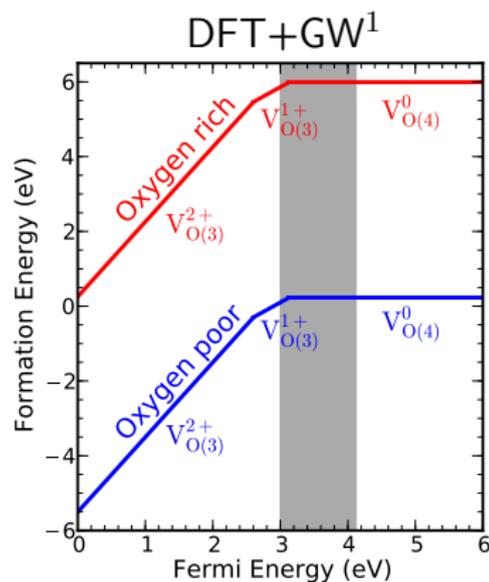
Charge transition level : $\varepsilon^{q/q-1} = \text{Fermi energy where defect } q \rightarrow q - 1.$

$$= E_{q-1}^f[\vec{R}_{q-1}](E_F = 0) - E_q^f[\vec{R}_q](E_F = 0)$$



- Three fold coordinated ($V_{O(3)}$) and four fold coordinated ($V_{O(4)}$) vacancies in charge states $\{0, +1, +2\}$
- Three fold coordinated ($I_{O(3)}$) and four fold coordinated ($I_{O(4)}$) interstitials in charge states $\{0, -1, -2\}$
- 96 atom super cells.
- Bulk structural parameters and band gap (6.00 eV) in good agreement with experiment.

High- κ dielectric materials – Formation energy of vacancies



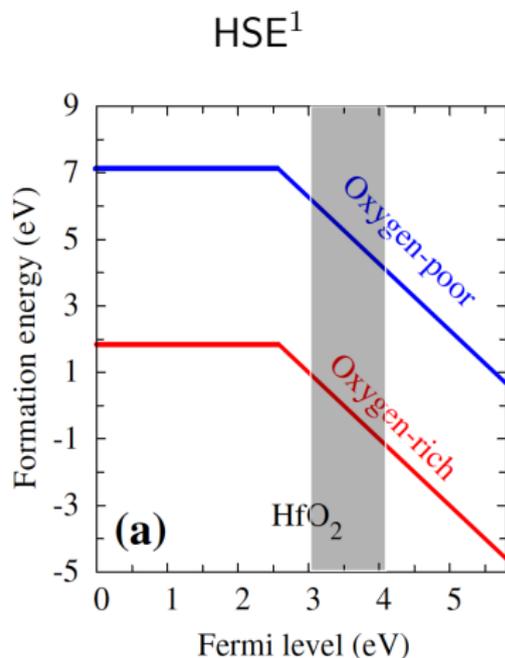
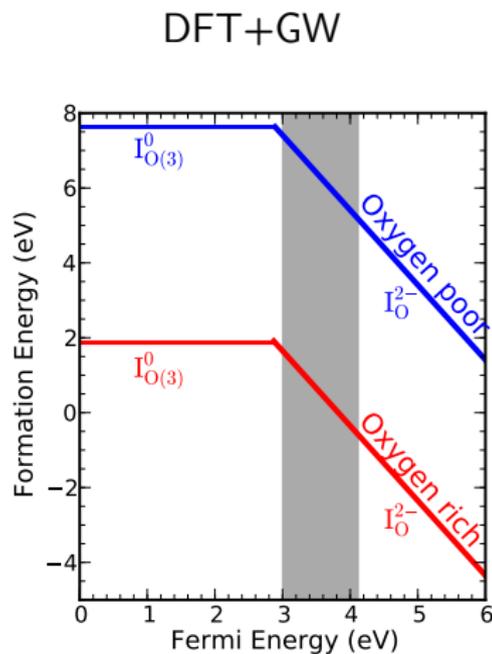
- Qualitative agreement with PBE0³, HSE².
- Quantitatively, charge transition levels are different and which defects are stable when placed next to Si¹.

¹M. Jain, J. R. Chelikowsky and S. G. Louie, Phys. Rev. Lett. **107**, 216803 (2011).

²J.L. Lyons, A. Janotti and C.G. Van de Walle, Microelectronic Engineering **88**, 1452 (2011).

³P. Broqvist and A. Pasquarello, Appl. Phys. Lett. **89**, 262904 (2006).

High- κ dielectric materials – Formation energy of interstitials



- Qualitative agreement with recent HSE¹ calculations.

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High- κ dielectric materials – Conclusions

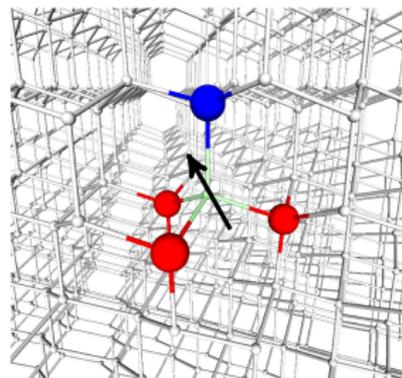
- DFT+GW method for calculating the stability of oxygen vacancies and interstitials.
- *Qualitative* agreement with previous hybrid functional calculations on the vacancies and interstitials.
- *Quantitative* disagreement with previous hybrid functional calculations - vacancies near a Si/HfO₂ interface.

	DFT+GW	HSE
V _O	+1,0	+2,+1
I _O	+2	+2

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Defects for spin-qubit applications

- Room temperature individually addressable spin systems in the solid-state for quantum computing.
- NV^- center in diamond is the leading candidate.



- Possible room-temperature qubit with long coherence time ($\sim 1\text{ms}$) for quantum computing.
- High sensitivity, high-spatial resolution magnetometry.

NV⁻ center in diamond – Motivation

Optical initialization at room-temperature provides initial spin pure state for spin-qubit operation.

Notation : $^{2S+1} \Lambda$

Λ : Irreducible representation of the orbital symmetry

S : Total Spin

NV⁻ center in diamond – Motivation

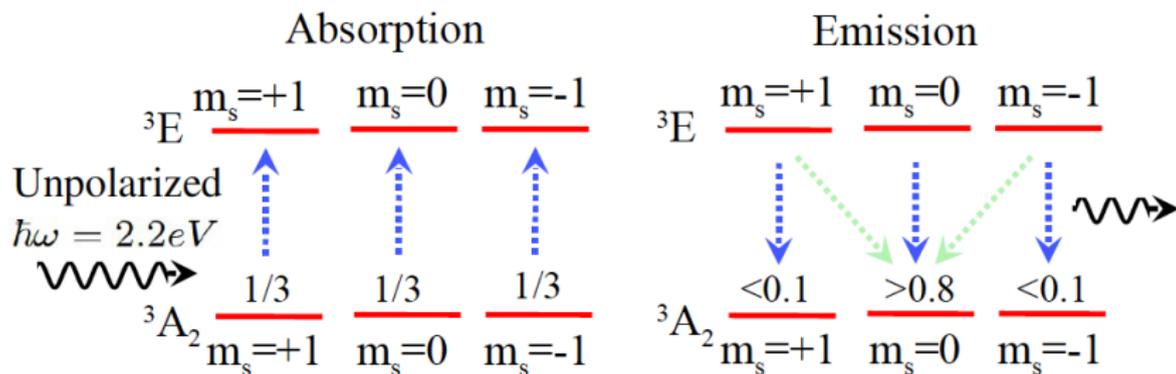
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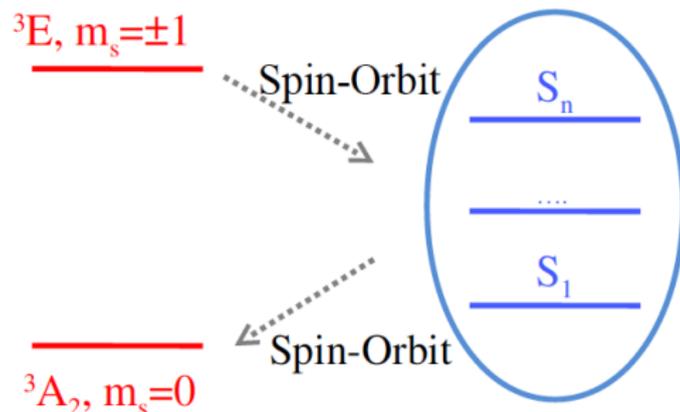
S : Total Spin

At Room Temperature



Degenerate mixed ground state un-entangled spin pure state

NV⁻ center in diamond – Unsolved problem



- Identification of singlet-level structure.
- Effective optical initialization path between the two triplet levels.

NV⁻ center in diamond – Unsolved problem

NV⁻ is a deep level center in a band gap of diamond with multiple localized, interacting electrons.

- Strong electron-electron correlation
 - Not appropriate for mean-field type calculation
- Screening from diamond
 - mimicking this system by a small isolated-diamond cluster with a NV⁻ defect without additional input incorrect.
- Large structural relaxation in the excited state.

NV⁻ center in diamond – Extended Hubbard model

$$\hat{H} = \sum_{i,\sigma} E_i n_{i\sigma} + \sum_{i \neq j, \sigma} t_{i,j} c_{i,\sigma}^\dagger c_{j,\sigma} \\ + \sum_i U n_{i,\uparrow} n_{i,\downarrow} + \sum_{i \neq j, \sigma, \sigma'} V n_{i,\sigma} n_{j,\sigma'}$$

i, j : Atomic sites (C or N)

σ, σ' : Spin direction

E_i : On-site energy

$t_{i,j}$: Hopping integral

U : On-site Coulomb repulsion

V : Nearest neighbor

Coulomb repulsion

Effective Coulomb
interaction parameters

Strong electron-electron
correlation

Screening from diamond

Geometry dependent
parameter sets

Large structural relaxation

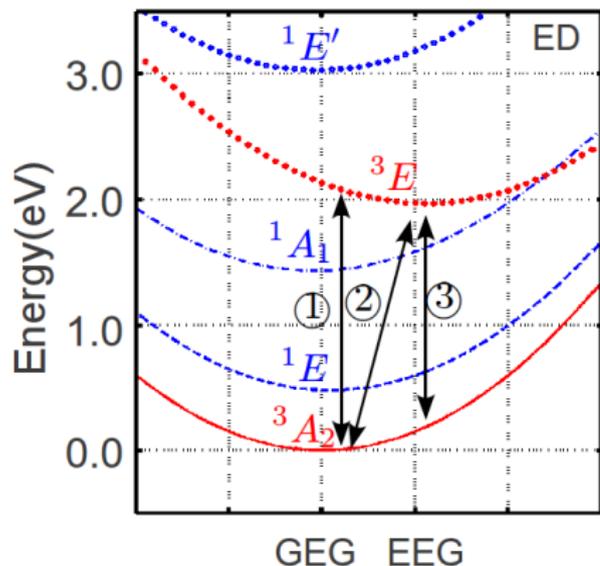
Hindered by the difficulty in getting physically grounded model parameters.

NV⁻ center in diamond – Level diagram

Use *ab initio* GW to get model parameters, incorporating realistic electron-electron interactions.

Level diagram¹ from exact diagonalization of model Hamiltonian

All energies in eV.



	Theory	Exp
①	2.1	2.2
②	2.0	1.945
③	1.8	1.8
${}^3E \tau_{rad}$	20ns	13ns
${}^3E \tau_{nonrad}$	30ns	50ns

¹S. Choi, M. Jain and S. G. Louie, submitted to Phys. Rev. Lett.

- Constructed extended Hubbard Hamiltonian from *ab initio* GW calculations.
- Through exact diagonalization, many-electron effects strongly affect the energy level diagram qualitatively and quantitatively.
- Computed ground- and excited-state energy surfaces and transition rates between them provided a consistent picture with experiments.
- Proposed an optical initialization pathway in which inter-system crossing plays a crucial role.

- *Ab initio* methods to understand and predict properties of materials.
- Methods – accurate, reliable and applicable to *real* materials.
- Oxygen-related defects in high- κ material – HfO₂.
- NV⁻ center in diamond for quantum computing application.

Improvement of methods

- Using many-body perturbation theory, systematically improve approximations via the linearized Sham-Schlüter equation which would lead to an optimized effective potential:

$$\iiint d\epsilon d\mathbf{r}' d\mathbf{r}'' G_{KS}(\mathbf{r}, \mathbf{r}'; \epsilon) [\Sigma_{xc}[G_{KS}](\mathbf{r}', \mathbf{r}''; \epsilon) - v_{xc}(\mathbf{r})\delta(\mathbf{r}' - \mathbf{r}'')] G_{KS}(\mathbf{r}'', \mathbf{r}; \epsilon) = 0$$

- Finite temperature calculations within the GW approximation including electron-phonon interactions.
- Extending GW approximation to calculate quasiparticle and optical properties of open-shell systems.
- Use higher order Feynman diagrams in the self energy (systematically) to go beyond the GW approximation.

Y. M. Niquet, M. Fuchs, and X. Gonze, Phys. Rev. A **68**, 032507 (2003).

F. Giustino, S. G. Louie, and M. L. Cohen, Phys. Rev. Lett. **105**, 265501 (2010).

J. Lischner, J. Deslippe, M. Jain, and S. G. Louie, submitted to Phys. Rev. Lett.

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