Towards a complete understanding of black hole entropy

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Black holes

- Black holes are compact astrophysical objects that have been observed.
- They are solutions to the gravitational field equations that are characterized by having an *event horizon*.
- Classically, signals from behind the horizon cannot reach an asymptotic observer.
- Quantum mechanically, they radiate like a black body.
- One can assign thermodynamic quantities like temperature and entropy to a black hole based purely on its gravitational properties.

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What is black hole entropy?

Thermodynamic entropy

- Einstein gravity: Bekenstein-Hawking Area law $S_{BH} = A/4G$.
- A consistent quantum theory of gravity would involve an extension of general relativity. This would generate corrections to the Area law.
- These corrections probe a regime beyond the thermodynamic limit and can be regarded as computing finite size effects.
- If we can compute and understand a formula for the complete black hole entropy in the full quantum theory of gravity (even without understanding all details of the theory), we would have made progress.

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Black holes in string theory

Macroscopic

- String theory (being a theory of quantum gravity) reduces to general relativity coupled to other fields at low energies.
- Find a black hole solution to the effective action which carries charges $\{q_i\}$. Measure its thermodynamic entropy S_{BH}.

Microscopic

- Find a microscopic description of a generic state in the theory with the same charges $\{q_i\}$, perhaps in a different regime of parameter space (weak coupling).
- Count the number of such states $\Omega(q_i)$ and compute the statistical entropy $S_{stat} = \log(\Omega)$.

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Black holes in string theory

PICTURE

Comparison of macroscopic and microscopic entropy

Early success (1996)

Strominger-Vafa, three charge black hole (Q1,Q5,p) in type II string theory.

- On the macroscopic side, find the black hole solution carrying three charges, and measure the area of the horizon.

$$\frac{A}{4G} = 2\pi\sqrt{Q_1 Q_5 p} \ , \tag{1}$$

- On the microscopic side, the generic state carrying the same three charges is a chiral excitation of a two dimensional superconformal field theory. Estimate density of states.

$$\Omega \approx \exp(2\pi\sqrt{Q_1 Q_5 p}) \ . \tag{2}$$

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Non-trivial finite size effects

- Four dimensional black holes with four charges $\sqrt{q_1 q_2 q_3 q_4} \equiv q^2$,

$$\Omega(q_i) \approx \exp\left(2\pi q^2\right) \ , \qquad S_{BH} = \log(\Omega) \ . \tag{3}$$

- Now, corrections to formula in inverse powers of charges;

$$\Omega(q_i) = \exp\left(q^2\, s_0 + s_1 + \frac{1}{q^2}\, s_2 \ldots\right) \ . \tag{4}$$

Here, s_1, s_2.. non trivial functions of the charges.

- Macro: Higher derivative corrections to supergravity action coming from string theory + Wald formula. Cardoso, de Wit, Mohaupt 1999.

- Micro: Exact BPS counting formula involving modular forms + estimation methods. Dijkgraaf, Verlinde, Verlinde 1994.

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Questions and issues

1. How to compute microscopic corrections in more general situations?

2. Why does this approach work?
   Macroscopic *degeneracy* v/s microscopic *index*.

3. Does the microscopic partition function really count the entropy of black holes? What about the other "stuff"?

4. What is the perturbative series an approximation to?
   Non-perturbative effects?

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Some answers in the context of black holes in superstring theory

1. Example: We developed a method to compute the density of states of the D1-D5 SCFT very far from the Cardy regime.
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References

Dijkgraaf,Verlinde,Verlinde; Cardoso,de Wit,Kappeli,Mohaupt; Dabholkar; Pioline; Ooguri,Strominger,Vafa; Gaiotto,Strominger,Yin; Shih,Strominger,Yin; Sen; Denef,Moore; Kraus,Larsen; Castro,Davis,Kraus,Larsen; Jatkar,Sen; David,Sen; Cheng,Verlinde; Banerjee,Sen; Banerjee,Sen,Srivastava; Dabholkar,Nampuri; many, many others . . .

Outline

1. Finite size corrections from supergravity
2. Finite charge corrections from microscopics
3. Non-perturbative effects
4. Black holes v/s other "stuff"

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The string theory setup

$\mathcal{N} = 4$ string theory in four dimensions

- Heterotic string theory on T^6. Low energy fields are the metric, 28 gauge fields, $132 + 2$ scalar fields and their superpartners.
- Huge symmetry group of the theory under which these fields are organized:
$$G(\mathbb{Z}) = O(22, 6; \mathbb{Z}) \times SL(2, \mathbb{Z}). \tag{5}$$

Dyonic states

- States in the theory carry charges
$$\Gamma^i_\alpha \equiv \begin{bmatrix} Q^i \\ P^i \end{bmatrix} \tag{6}$$
- Charge invariants are Q^2, P^2, and $Q \cdot P$.

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Low energy effective action

- The two-derivative effective action is $\mathcal{N} = 4$ supergravity in four dimensions (16 supercharges). Bosonic part for metric and axion-dilaton is

$$S_{eff} = \int d^4x \, \sqrt{-\det g} \, S \left[R + \frac{1}{S^2} \, g^{\mu\nu} (\partial_\mu S \, \partial_\nu S - \frac{1}{2} \partial_\mu a \, \partial_\nu a) \right] \quad (7)$$

- Stringy effects generate higher dimension operators in the effective action.

$$\begin{aligned} \delta S_{eff} &= \int d^4x \, \sqrt{-\det g} \, \phi(a,S) \left\{ R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right\}, \\ \phi(a,S) &= -\left(12 \ln S + \ln \eta^{24}(a+iS) + \ln \eta^{24}(-a+iS) \right) . \end{aligned} \quad (8)$$

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Black holes in string theory

- Black hole solutions to the effective action which are 1/4 BPS. Extremal Reissner-Nordstrom black holes with non-trivial scalar fields.
- They carry dyonic charges (Q_i, P_i) and carry entropy

$$\frac{A}{4G} = \pi\sqrt{\Delta}, \qquad \Delta = Q^2 P^2 - (Q.P)^2.$$

- Near the horizon, the geometry is $AdS_2 \times S^2$ and the scalars get fixed.

Wald entropy

- On addition of higher derivative corrections, the Bekenstein-Hawking law is no longer correct.

- Replaced by Wald's formula

$$S_{BH} = -\,8\pi \int_H d\theta d\phi \frac{\delta S^{eff}}{\delta R_{rtrt}} \sqrt{-g_{rr} g_{tt}}\ . \tag{9}$$

- The Wald entropy of our black holes can be computed to be

$$S_{Wald} = \pi\sqrt{\Delta} + \phi\left(\frac{Q.P}{Q^2}, \frac{\Delta}{Q^2}\right) + \ldots \tag{10}$$

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A warmup example

Half-BPS states of the heterotic string theory on T^6.

- Can be represented as chiral excitations of a fundamental heterotic string in ten dimensions with winding w and momentum p Dabholkar-Harvey.
- String sees 8 transverse spacetime dimensions + 16 internal oscillators = effectively 24 free fields.
- Each free field has oscillator modes of energy $n = 0, 1, 2, \ldots$
- How many ways of distributing energy pw in 24 types of oscillators?

$$
\begin{aligned}
\mathrm{Z}(\tau) &= e^{-2\pi i\tau} \prod_{n=1}^{\infty} (1 - e^{2\pi i n\tau})^{-24} \equiv \frac{1}{\eta^{24}(\tau)} \, , \\
\Omega(p, w) &= \oint d\tau \; e^{-i\pi p w \tau} \, \mathrm{Z}(\tau) \approx e^{4\pi\sqrt{pw}} \, . \qquad (11)
\end{aligned}
$$

Microscopic counting of the $\frac{1}{4}$-BPS states

The 4d dyon degeneracy formula

$$\Omega(Q,P) = (-1)^{Q.P+1} \oint d\rho\, d\sigma dv\, e^{-i\pi\left(Q^2\rho+P^2\sigma+Q.Pv\right)}\, \mathrm{Z}^{4d}(\rho,\sigma,v)\,, \quad (12)$$

where

$$\mathrm{Z}^{4d}(\rho,\sigma,v) = \frac{1}{\Phi_{10}(\rho,\sigma,v)}\,; \quad (13)$$

Dijkgraaf, Verlinde, Verlinde

Φ_{10} is the *Igusa cusp form*, which is the unique weight 10 Siegel modular form of $Sp(2,\mathbb{Z})$.

Evaluation of microscopic degeneracy

- Expanding around the dominant saddle point agrees with the answer from supergravity, including higher derivative corrections.

$$\Omega \approx \exp\left(\pi\sqrt{\Delta} + \phi\left(\frac{Q.P}{Q^2}, \frac{\Delta}{Q^2}\right) + \ldots\right) \ , \tag{14}$$

$$\Delta \equiv Q^2 P^2 - (Q.P)^2 \ . \tag{15}$$

- Using the known structure and symmetries of Φ_{10}, we computed a series of exponential corrections to the entropy formula

B. Pioline, S.M.

$$\Omega(Q,P) = \sum_{N=1}^{\infty} \exp\left(\frac{\pi\sqrt{\Delta}}{N} + s_1^{(N)} + \frac{1}{\sqrt{\Delta}} s_2^{(N)} \ldots\right) \ . \tag{16}$$

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Quantum entropy function

- Wald's formalism applies for any local theory of gravity, and computes power law corrections to the classical entropy formula.
- To understand the contributions $\exp(S_0/N)$, $N = 2, 3..$, we need a formalism which goes beyond a local theory of gravity.
- For extremal black holes, there has been such a proposal called the *quantum entropy function*. Sen, arXiv:0809.3304.
- This proposal relies on the near horizon geometry of an extremal black hole being AdS_2.

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Quantum entropy function

- The quantum entropy function $\Omega(Q_i)$ is a Euclidean path integral over asymptotically AdS_2 field configurations with fixed electric charge Q_i, fixed value of the scalar fields at infinity (this includes magnetic fluxes P_i), and a Wilson line insertion.
- The functional integral runs over all fields in the dimensionally reduced two-dimensional field theory.
- The Euclidean path integral is dominated by the field configuration corresponding to pure AdS_2.
- In general, there could be other saddle points approaching AdS_2 asymptotically.

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Semiclassical interpretation

- We find a family of *smooth* solutions to the semiclassical theory labelled by $N \geq 1$ acting as an orbifold on the original AdS_2 geometry.
- They have degeneracy $\exp(\pi\sqrt{\Delta}/N)$, they are all asymptotically AdS_2, but differ in the interior. S.M., B. Pioline; Banerjee, Jatkar, Sen.
- This construction is very general, it applies whenever there is an effective black string in the background.

Outline

Single centered black holes?

- Formulation of microscopic partition functions in flat space at weak coupling involves a representation of a generic charged state as a collection of strings, branes, momentum...
- Assumption – at strong coupling, this configuration gravitates and forms a black hole.
- However, there may exist other solutions in gravity with same charges (Multi-centered black hole bound states).
- The microscopic partition function should count all these configurations.

Symmetries of the partition function

- From far, notion that the black hole is really a wound black string with a modular parameter τ, and an $SL(2,\mathbb{Z})$ action on τ.
- Indeed the full partition function is a modular form, and the Fourier coefficients agree with the black hole degeneracy to good approximation.
- However, when one zooms in, not all of the excitations of the string form the black hole. Some of the excitations form multi-centered black hole bound states.
- The spectrum of the multi-center black hole solutions differ in different regimes of parameter space, while the single-center black hole exists everywhere in parameter space.

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Symmetries of single centered black holes

- What are the modular properties of the single centered black hole partition function?

- *[I have found new] functions [which I shall call mock modular forms] which have asymptotic expansions at every rational point of the same type as those of modular forms, but that there is no single modular form whose asymptotic expansion agrees at all rational points with that of the function itself.*
  – Srinivasa Ramanujan, in his last letter to Hardy, 1920 [slightly paraphrased].

- The single centered black hole partition function is a *mock* modular form. A. Dabholkar, S.M., Don Zagier.

- A mock modular form is a holomorphic function which transforms under modular transformations almost but not quite as a modular form. It can be completed into a modular form by adding a specific non-holomorphic function.

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Some lessons learnt

1. Going beyond the thermodynamic approximation for black holes makes sense.

2. Mathematical structure of the generating function.

3. General structure of quantum gravity path integral – Universal series of exponentially suppressed corrections to the degeneracy of extremal black holes $\exp(S_0/N)$, $N = 2, 3...$

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But much work to do before a complete understanding

- Can one understand all the microscopic structure of an extremal black hole from gravity?
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