Geometrically frustrated magnets

Arnab Sen, TIFR

Theoretical Physics Colloquium

Collaborators: K. Damle (TIFR), R. Moessner (Dresden), A. Vishwanath (Berkeley) and F. Wang (Berkeley)
Thanks to: Deepak Dhar, T. Senthil

A. Sen, F. Wang, K. Damle and R. Moessner, PRL **102**, 227001 (2009)
A. Sen, K. Damle and A. Vishwanath, PRL **100**, 097202 (2008).

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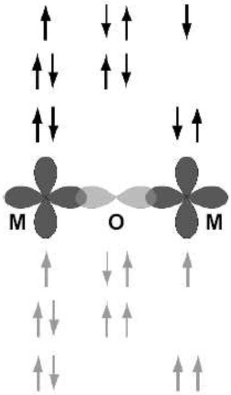
Magnetism in ionic insulators

- Many ionic insulators have magnetic ions with an associated magnetic moment due to incomplete shells.
- Interactions between these localized moments.
- Magnetic dipolar interaction energy (small $\sim 10^{-5}$ eV).
- Exchange energy $E = J\sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$ due to Coulomb interactions and Pauli exclusion.
- How big is J? When is it positive?
  Difficult questions.

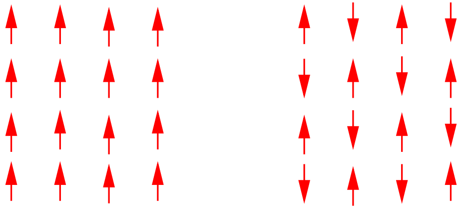
Superexchange interaction



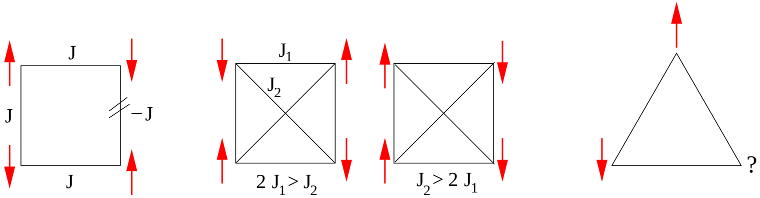
- Consider MnO which is antiferromagnetic ionic solid.
- Mn^{2+} has 5 electrons in its *d* shell being parallel due to Hund's rule.
- O^{2-} has fully occupied *p* orbitals.
- Antiparallel alignment of spins in neighbouring Mn ions has lower kinetic energy due to delocalization of electrons (virtual hoppings).

Unfrustrated magnets

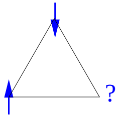
- Usually ordered states at low temperature T ($\mathcal{O}(JS^2)$).
- Ordering at low T largely determined by classical energetics even for small spin length S for unfrustrated magnets.
- E.g. ground state for cubic lattice Heisenberg antiferromagnet is the Neel state and the low energy excitations are long wavelength spin waves.



- Not all the interactions can be simultaneously satisfied.
- Randomness: e.g., Spin glasses
- Multiple interactions: e.g., $J_1 - J_2$ Model
- Geometry



- Arrangement of spins on the lattice such that all interactions cannot be satisfied together.
  Neel state avoided. Simplest case:



- In extreme cases, macroscopic degeneracy of classical minimum energy configurations (highly frustrated magnets).

Schematic of magnetic susceptibility

- No magnetic ordering even for T well below $\mathcal{O}(JS^2)$ unlike unfrustrated magnets, instead in a cooperative paramagnetic regime (J. Villain (1979)).
- $T \rightarrow 0$: Quantum effects+ subdominant interactions like next nearest neighbour interactions, magnetic anisotropies etc.

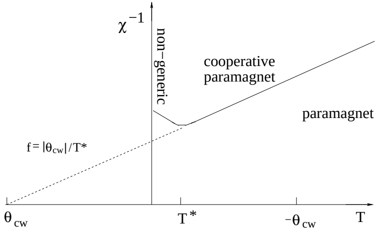
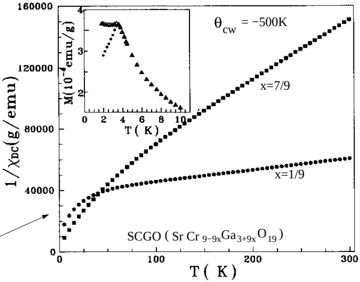
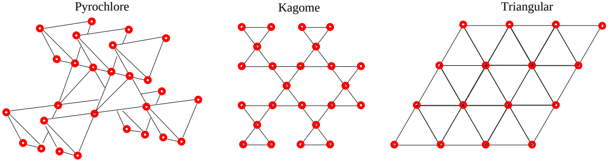
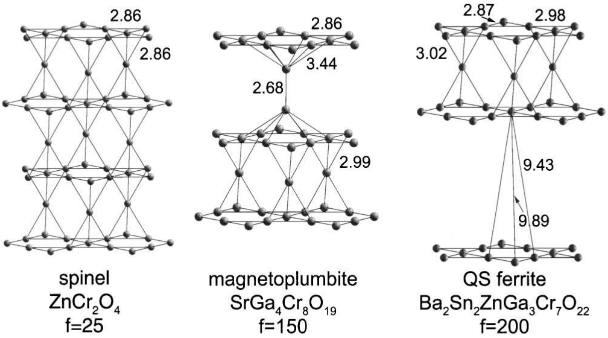


Fig from Martinez et al., PRB 46, 10786 (1992)



Materials





Hagemann et al., PRL 86, 894 (2001).

Interesting questions

Spin liquids

- Description of the system when there is no magnetic ordering? Is the physics trivial?
- Signatures/probes of such correlated phases?

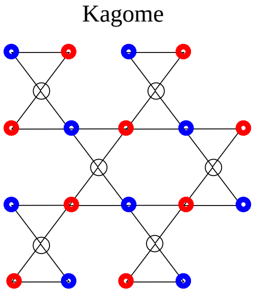
Sensitivity

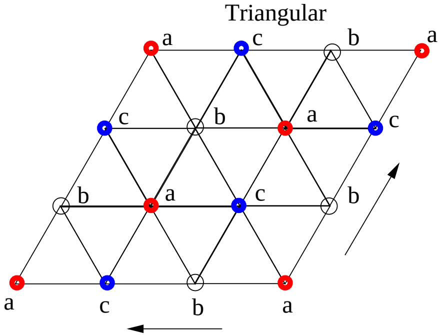
- How can degeneracy be split?
- Can unusual states be obtained in this way?

'Quenching' of leading exchange interactions allow new physics to happen.

Macroscopic degeneracy I

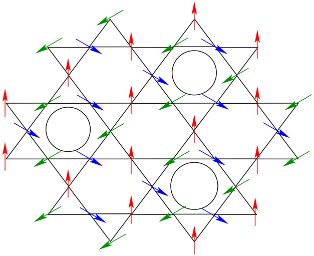
- Macroscopic degeneracy of ground states of the Ising antiferromagnet on both kagome and triangular lattice.
- Disordered at all temperatures.

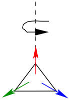


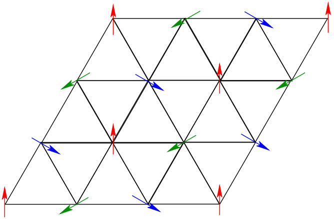


Macroscopic degeneracy II

- Zero modes for the kagome lattice Heisenberg antiferromagnet.
- Triangular lattice Heisenberg antiferromagnet forms an ordered state at low temperatures.

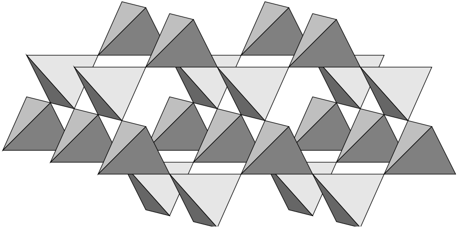


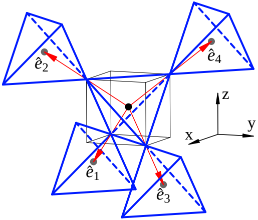




Ground state correlations

- Local constraints can lead to long-ranged correlations.
- Best example $\rightarrow$ dipolar spin liquid on the pyrochlore lattice.
- $\sum_{\boxtimes} S_i^{\alpha} = 0 \rightarrow \nabla \cdot \vec{B}^{\alpha} = 0.$
- Power-law correlators–$\langle B_i^l(0) B_j^m(r) \rangle \propto \delta_{lm} \left(\frac{3 r_i r_j - r^2 \delta_{ij}}{r^5} \right).$

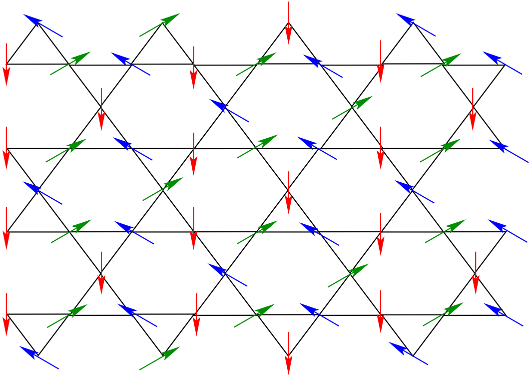




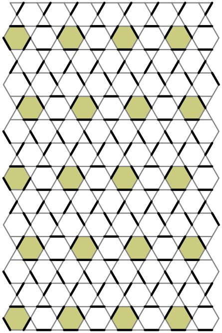
Isakov, Gregor, Moessner and Sondhi, PRL 93, 167204 (2004).

Quantum fluctuations

- Quantum fluctuations can be introduced in a perturbative manner by doing a $1/S$ expansion.
- Large-S calculations on kagome lattice lead to an ordered state (Chubukov, PRL **69**, 832 (1992)).



$S = 1/2$ Case



- Precise nature of ground state remains a subject of debate.
- Exact diagonalization studies find very short ranged spin-spin correlations and a spin gap of $\triangle \approx 0.25J$.
- Recent study finds a state with a 36 site unit cell to be the ground state (Singh and Huse, PRB **76**, 180407(R) (2007)).
- Also obtained by Nikolic and Senthil (PRB, **68**, 214415 (2003), Fig from there) using a Resonating-Valence-Bond picture (Anderson, Science 235, 1196 (1987)) .

Single-ion anisotropy

- Motion of the electrons in the magnetic ions influenced by the crystalline environment.
- Energy depends on the absolute orientation with respect to the crystal axes.
  $H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - D \sum_i (\vec{S}_i \cdot \hat{n}_i)^2$
- Pyrochlore spin ice $Ho_2Ti_2O_7$ (Ho^{3+}, J=8)
  Easy axes $\hat{n}$ point outward from center of each tetrahedron, $D \sim 50K$, $J \sim 1K$.
  Harris *et. al.*, PRL 79, 2554 (1997).
- Kagome Nd-langasite $Nd_3Ga_5SiO_{14}$ (Nd^{3+}, J=9/2)
  Easy axis perpendicular to lattice plane,
  $D \sim 10K$, $J \sim 1.5K$
  A. Zorko *et. al.*, PRL 100, 147201 (2008).

Our work

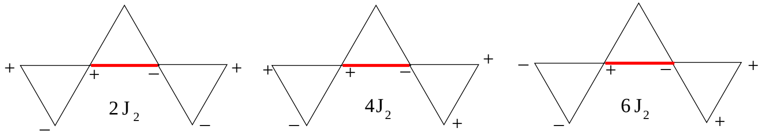
- $S \geq 3/2$ easy axis on the kagome and triangular lattices:

  $H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - D \sum_i (S_i^z)^2$
- Physics when D term forces a collinear state?
- Best answered by working out small J/D expansion for effective Hamiltonian for pseudospin-$1/2$ variables σ_i ($\sigma^z = \pm 1 \leftrightarrow S^z = \pm S$).

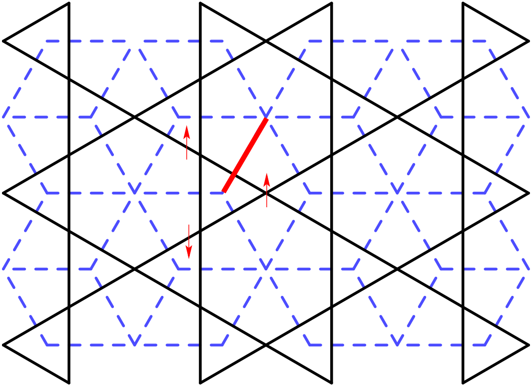
  Assumption: Perturbative results in J/D will be valid as long as collinear states selected by anisotropy.

Zero field

- To $\mathcal{O}(J^3/D^2)$, $\mathcal{H}_{\rm eff}$ is given as
  $\mathcal{H}_{\rm eff} = J_1 \sum_{\langle ij \rangle} \sigma_i \sigma_j - J_2 \sum_{\langle ij \rangle} \frac{1-\sigma_i\sigma_j}{2}(\sigma_i H_i + \sigma_j H_j)$
  where
  $J_1 = JS^2$, $J_2 = (S^3 J^3)/(4D^2(2S-1)^2)$, and exchange field $H_i = \Gamma_{ij}\sigma_j$ with $\Gamma_{ij} = 1$ for nearest neighbours and zero otherwise.
- Additional $\mathcal{O}(J^{2S}/D^{2S-1})$ pseudo-spin exchange term subleading for $S > 3/2$.

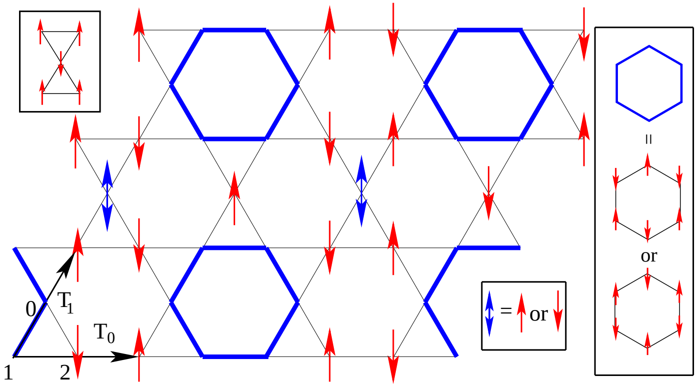


Dice Lattice Dimer Model

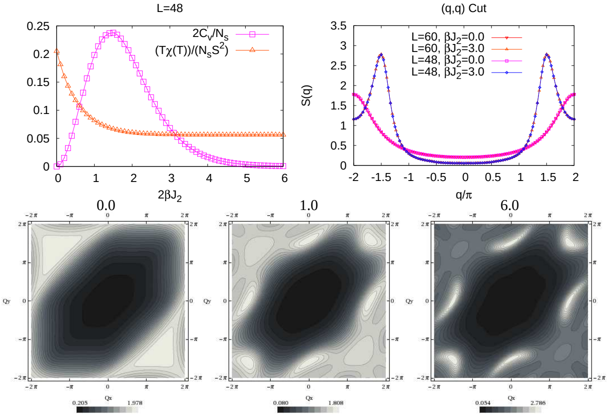


- Can be cast as an interacting dimer model on the Dice lattice.

- $H_D = 2J_2 \sum_P n^2 |nP\rangle\langle nP|$
- Minimizing J_2 for minimally frustrated states $\longrightarrow$ No spin is a minority spin of both the triangles to which it belongs.
- Entropy still macroscopic.

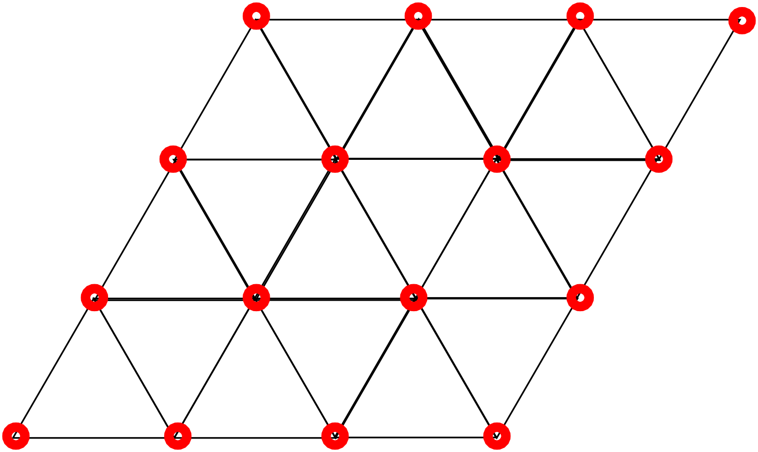


Semiclassical spin liquid

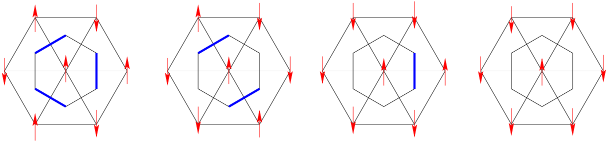


- System remains a spin liquid down to very low temperatures.
- Very different short-ranged correlations from classical Ising model.

Triangular Lattice

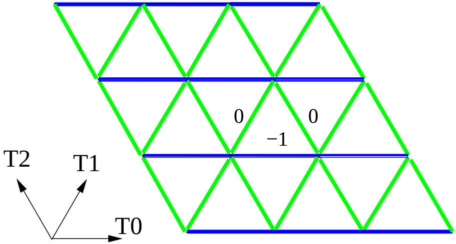


The J_2 term



- H_D can be written in a simplified form as
  $H_D = 2J_2 \sum_{n=0}^{3} n^2 |nP\rangle\langle nP|$
- H_D can be minimized by noting that the average number of dimers on the perimeter of a hexagon is 2 and $\langle x^2\rangle \geq \langle x\rangle^2$
- Minimum potential energy for all configurations with $g_2 = 1$.

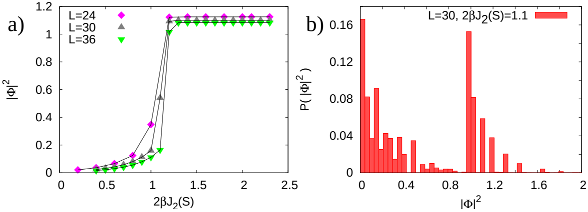
Orientationally ordered state



- Translational symmetry of the triangular lattice intact.
- However, symmetry of $\pi/3$ rotations about a lattice site broken.

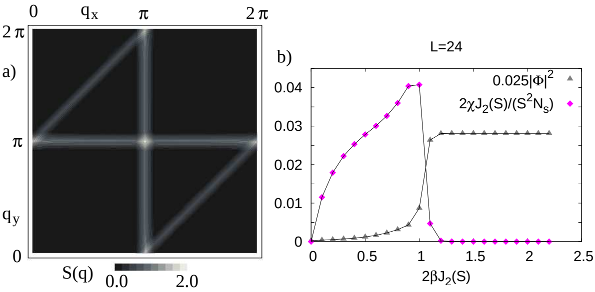
  Orientational OP $\Phi = \sum_P -B_p \exp(i2p\pi i/3)$ where B_p denotes the average of the Ising exchange energy on all links of the p^{th} (p=0,1,2) on the triangular lattice.

Transition



- Below a critical temperature $T_c \approx 1.67 J_2$, the system orders in an orientationally ordered state.
- The transition has a first order nature.

Consequences (I)



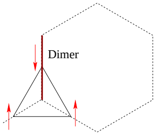
- Bragg lines with enhanced signal are the signature of orientational order.
- Sudden drop in magnetic susceptibility χ when the ordering transition takes place.

Consequences (II)

- Presence of low-temperature zero magnetization plateau that extends for a range of magnetic fields $0 < B < B_c \sim J^3/D^2$.
- Slow glassy dynamics of spins due to extended nature of stripes in this disorder-free system.

Kagome in magnetic field

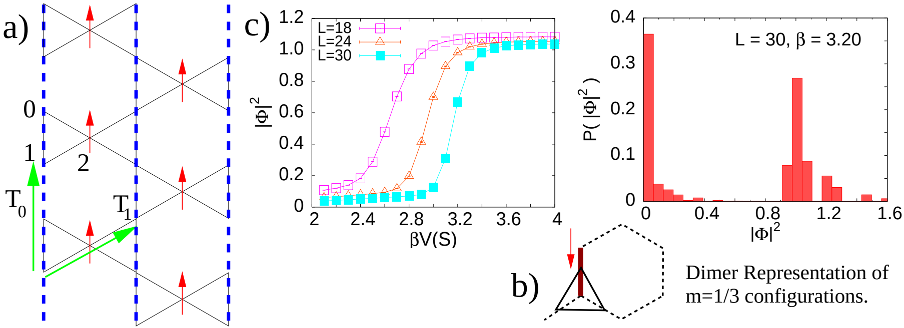
- Include magnetic field along the easy axis
  $H = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j - D\sum_i(S_i^z)^2 - B\sum_i S_i^z$
- For Ising spins, $m = 1/3$ plateau for $0 < |B| < 4JS$, with the Zeeman energy gap largest at $B = 2JS$.
- Ground states characterized by a $2:1$ constraint that requires each triangle to have $(+S, +S, -S)$.
- Can be represented as dimer coverings of honeycomb lattice.



Effective hamiltonian on the $m = 1/3$ plateau

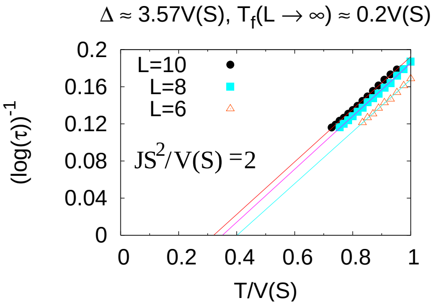
- Because of the strong $2:1$ constraint, the first term (for any $S \geq 3/2$) that breaks degeneracy is a diagonal term at $\mathcal{O}(J^6/D^5)$.
- Leading off-diagonal term $t_{ring} \sim J^{6S-2}/D^{6S-3}$.
- Calculation tricky, result simple
  Hexagons with exactly one dimer on them pay energy penalty $V = \frac{(2S)^6 J^6}{1024(2S-1)^5 D^5}$.

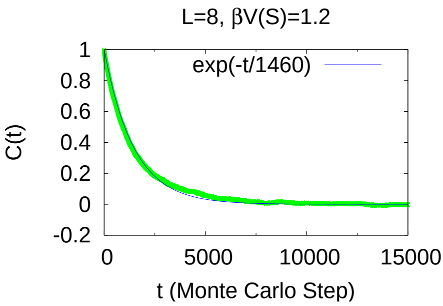
Sublattice symmetry broken



- Ordering can be characterized by a sublattice order parameter $\Phi = \sum_p m_p \exp(2p\pi i/3)$, where m_p denotes the sublattice magnetization of the p^{th} sublattice.

Glassy dynamics without disorder





Vogel-Fulcher law: Relaxation time $\tau = \exp(\Delta/(T - T_f))$

Summary

- $S > 3/2$ kagome and triangular lattice antiferromagnets with strong easy axis anisotropy considered.
- The kagome magnet goes into a semiclassical spin liquid with distinctive and unusual short-range correlations below a crossover temperature $T^* \approx 0.08 J^3 S/D^2$ at zero field.
- The triangular magnet undergoes a first order transition at $T_c \approx 0.1 J^3/D^2$ to an orientationally ordered collinear state that gives rise to a zero magnetization plateau for small magnetic fields along the easy axis.
- On the $m = 1/3$ magnetization plateau on the kagome lattice, the system breaks sublattice rotation symmetry but no translational symmetry.

THANK YOU