Gravity, Species and Information

Work with

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hep-th 0812 1940

Gravity Scale and Species:

Natural Gravity scale M_{pe}

"Quantum" information ~~theory~~ meaning of M_{pe}:

$$\boxed{l_{pe} = \frac{1}{M_p} \text{ is the minimum size where we can encode a bit of information.}}$$

Uncertainty Principle:

box of size l $\Rightarrow$ $M \sim \frac{1}{l}$

General Relativity:

$$R_{sch} = \frac{M}{M_{pe}^2} = \frac{1}{l M_{pe}^2}$$

In order to be able to use this information we need

$$R_{sch} \lesssim l$$

$$\Rightarrow \quad \frac{1}{l M_{pe}^2} \lesssim l$$

The bound is saturated for $\boxed{l = l_{pe}}$

Thus:

$$\boxed{\text{Q. Mechanics} + \text{General Relativity} \Rightarrow \text{Bounds on Information encoding}}$$

Holography.

Let us consider now a volume of size L.

Let us denote N the # of bits we can encode in L.

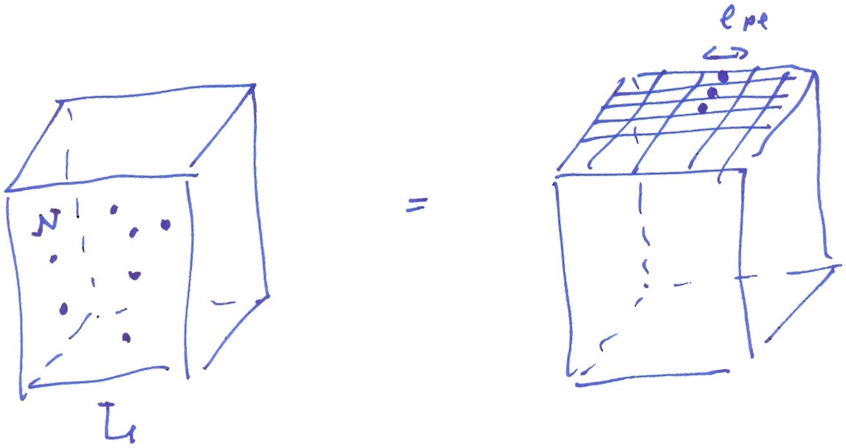
$$M \sim \frac{N}{L}$$

Again: $R_{sch} = \frac{N}{L M_{pe}^2}$ and

$$R_{sch} \leq L \implies$$

$$\frac{N}{L M_{pe}^2} \leq L \implies N_{max} = L^2 M_{pe}^2 = \underbrace{Area/\ell_{pe}^2}_{\text{area in Planck units.}}$$

Holography.



Species.

Assume N different types of particles.

In order to identify these particles we need a detector with at least N bits.

$\Rightarrow$ Minimum size of this detector is

$$\boxed{L_N = \frac{\sqrt{N}}{M_{pe}} = \sqrt{N}\, l_{pe}}$$

In particular this means that at scales $l < L_N$ we can not identify the different types of particles.

What is the physical meaning of L_N?.

We will discuss this question from different points of view.

Entanglement Entropy versus B.H Entropy.

What is entanglement entropy ?.

Consider a system AB . $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$.

Hilbert space of states.

ψ a pure state describing AB.

For ψ entanglement means that $\psi \neq \psi_A \otimes \psi_B$.

The density matrix $\rho_{AB} = |\psi\rangle\langle\psi|$.

We can define now

$$\rho_A = \mathrm{Tr}_{\mathcal{H}_B}\, \rho_{AB} \qquad \rho_B = \mathrm{Tr}_{\mathcal{H}_A}\, \rho_{AB}$$

and define

$$S_A = -\mathrm{Tr}\, \rho_A \ln \rho_A \qquad S_B = -\mathrm{Tr}\, \rho_B \ln \rho_B .$$

<u>Th</u>: $\boxed{S_A = S_B}$ = Entanglement entropy.

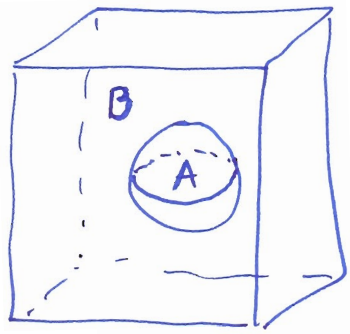
<u>Proof</u>: ⊙ $S_A + S_B \geq S_{AB} \to 0$ for a pure state.

⊙ Schmidt Theorem: $\exists$ orthonormal states $|\tilde{\alpha}_i\rangle$ $|\tilde{\beta}_i\rangle$ of H_A and H_B $\quad i = 1 \ldots \dim \min\{H_A, H_B\}$

such that: $\psi = \sum A_i |\tilde{\alpha}_i\rangle |\tilde{\beta}_i\rangle$

$$\left.\begin{aligned} \rho_A &= \sum A_i A_i^* |\tilde{\alpha}_i\rangle\langle\tilde{\alpha}_i| \\ \rho_B &= \sum A_i A_i^* |\tilde{\beta}_i\rangle\langle\tilde{\beta}_i| \end{aligned}\right\} \Rightarrow S_A = S_B$$

The typical situation:



If $S_A = S_B = S_{ent}$ $\Rightarrow$

$$S_{ent} \sim A(\Sigma_{AB}) \cdot \underbrace{\Lambda^2}_{\text{measured with some cutoff.}}$$

Notice also that entanglement entropy depends on # of species as:

$$\boxed{S_{ent} \sim N \cdot \Lambda^2 \, Area(\Sigma_{AB})}$$

Imagine now that A is a black hole. In this case the natural cutoff would be $\Lambda = M_{pe}$ but then entanglement entropy will go as $N \underbrace{S_{BH}}_{\text{Beckenstein Hawking}}$

However we know that the minimum size needed to identify N species is $\frac{M_{pe}}{\sqrt{N}}$. If we use this cutoff i.e L_N we get

$$\boxed{S_{entanglement} = S_{B.H}}$$

Black Hole Evaporation time and Species.

The rate of energy emission by a black body at temperature T is determined by Stefan Boltzmann law

$$\frac{dE}{dt} \sim \underbrace{A}_{\text{area}} T^4$$

For a B. Hole : $R \sim \frac{M}{M_{pe}^2}$ $T \sim \frac{1}{R}$, $E \sim M$.

$$\frac{dM}{dt} = \frac{1}{R^2} = \frac{M_{pe}^4}{M^2}$$

$$\frac{1}{M_{pe}^4} \int M^2 \, dM = \boxed{\tau \simeq \frac{M^3}{M_{pe}^4} = R^3 M_{pe}^2}$$

If we have N different type of particles the Stefon Boltzmann law changes to

$$\frac{dE}{dt} \sim N A T^4$$

and

$$\boxed{\tau = \frac{R^3 M_{pe}^2}{N}}$$

Now imagine N large and a BH of size $R \gg M_{pe}$ i.e an Einstenian BH but $R < L_N = \sqrt{N}\, l_{pe}$.

$$\tau = \frac{R^2 . R . M_{pe}^2}{N} < \frac{\cancel{l_{pe}^2} \cancel{N} . R \cancel{M_{pe}^2}}{\cancel{N}} = R$$

but that is unconsistent! $\rightarrow$

Black Holes of size $< L_N$ are not Einstenian!!

The information theory meaning of evaporation time.

Page's argument.

1. Consider a system AB with $m = \dim \mathcal{H}_A$, $n = \dim \mathcal{H}_B$

As before we define S_A the entanglement entropy.

Let us define now

$S_{m,n} = \langle S_A \rangle$ the average of S_A on all the states $|\psi\rangle$ for the AB system.

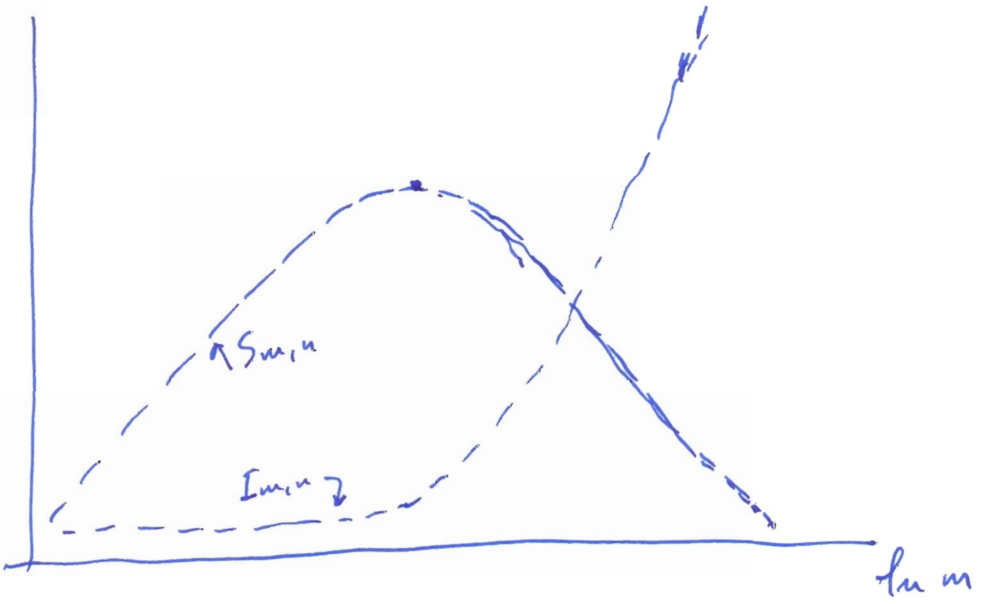
The maximum entropy for the system A is $\log m$.

Let us define Information in the system A as:

$$\boxed{I_{m,n} = \ln m - S_{m,n}}$$

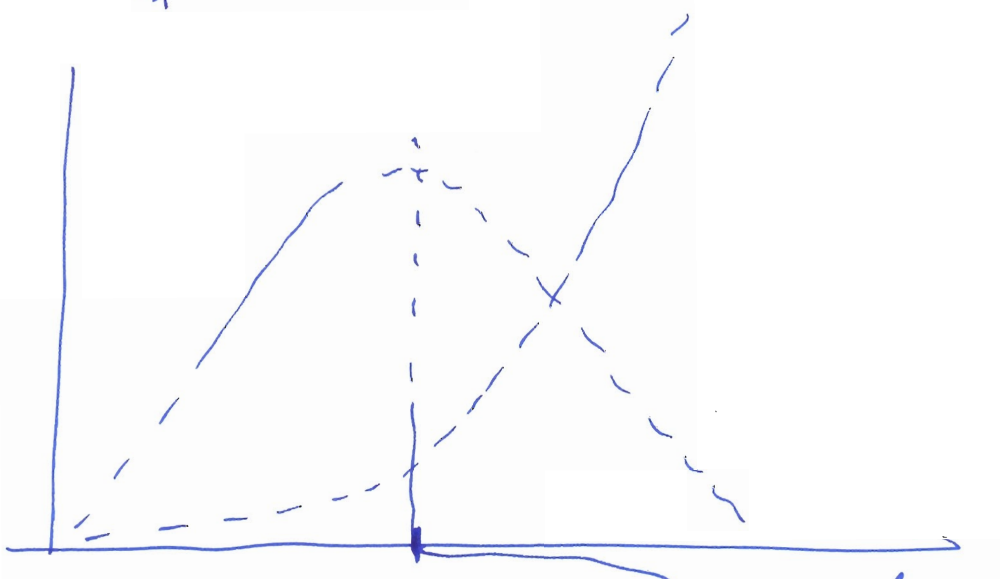
$\exists$ an exact formula for $I_{m,n}$

$$\boxed{I_{m,n} = \ln m + \frac{m-1}{2n} - \sum_{k=n+1}^{m \cdot n} \frac{1}{k}}$$



Consider now $B = B.H$ $\quad n$ $\qquad$ fix $n.m$

$\qquad\qquad\quad A$ = radiation. $\quad m$



m small $\longrightarrow$ more radiation.

The BH is formed and starts to radiate

Scale of time when the BH starts to emit <u>information</u>.

$\underbrace{m > n}$ (at least half of the BH has been evaporated).

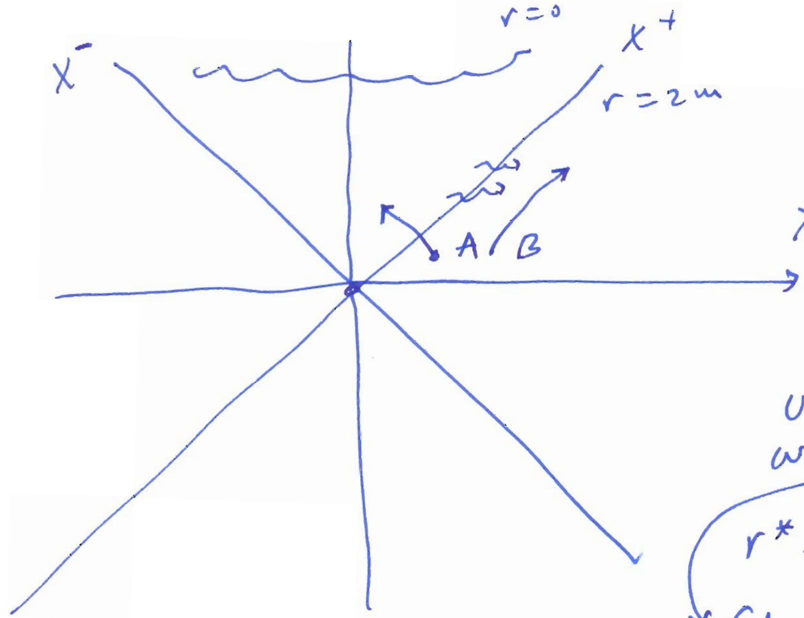
$\Rightarrow$ At evaporation time $\sim T$ the BH starts emitting information by Hawking radiation.

Therefore if $T \sim \frac{R^3 M_{pe}^2}{N}$ we can reduce this time by increasing N. !!

As we will see this is forbiden by the quantum cloning Theorem.

Alice and Bob in near horizon Geometry.

Kruskal coordinates near the B.H horizon



$$X^+ = e^{v/4m} \equiv e^{\tau} \rho$$
$$X^- = -e^{w/4m} \equiv -e^{-\tau} \rho$$

τ - Rindler time.

$$v = t + r^*$$
$$w = t - r^*$$

$$r^* = r + 2m \log(r - 2m)$$

Sch time.

<u>Cloning argument:</u>

i) Alice jumps into the B.H with a quantum state.

ii) After some retrieval time T_{rt} Bob can get the information about q. state from Hawking radiation.

iii) At this moment Bob jumps into the BH. but he will reach the singularity at

$$X^+ X^- = R^2 \quad \text{for} \quad X^+ \approx e^{T_{rt}} \cdot R$$

$$\Rightarrow \quad X^- \sim e^{-T_{rt}} R$$

iv) Alice needs to send to Bob the information in $\Delta X^- \approx R e^{-T_{rt}}$

$$\Rightarrow \quad E \sim \frac{e^{T_{rt}}}{R}$$

v) But $E < E_{BH} = M = R$ (in Planck units)

$$e^{T_{rt}} < R^2 \quad \Rightarrow \quad \boxed{T_{rt} < \log R}$$

Thus if $T_{rt} < \log R \Rightarrow$ Cloning !!.

From Page's result we Know that $\tau_{rt} \sim$ half evaporation time.

$$\tau_{rt} \approx t/M \sim R^3 M_{pe}^2 / R M_{pe}^2 \sim R^2$$

But certainly $R^2 > \log R$ and therefore NO problem with quantum cloning.

However we Know that in the presence of N species:

$$\tau_{rt} \to R^2/N$$

But $R^2/N > \log R$ is NOT the case for BH's of size $R \leq L_N = \sqrt{N}\, l_{pe}$.

Therefore:

If BH's of size $\lesssim L_N$ are Einsteinian $\Rightarrow$

Violation of cloning theorem

i.e violation of quantum mechanics.

Comment:

Cloning Theorem $\Rightarrow$ a bound on retrieval time:

$$\boxed{T_{rt} \gtrsim \log R}$$

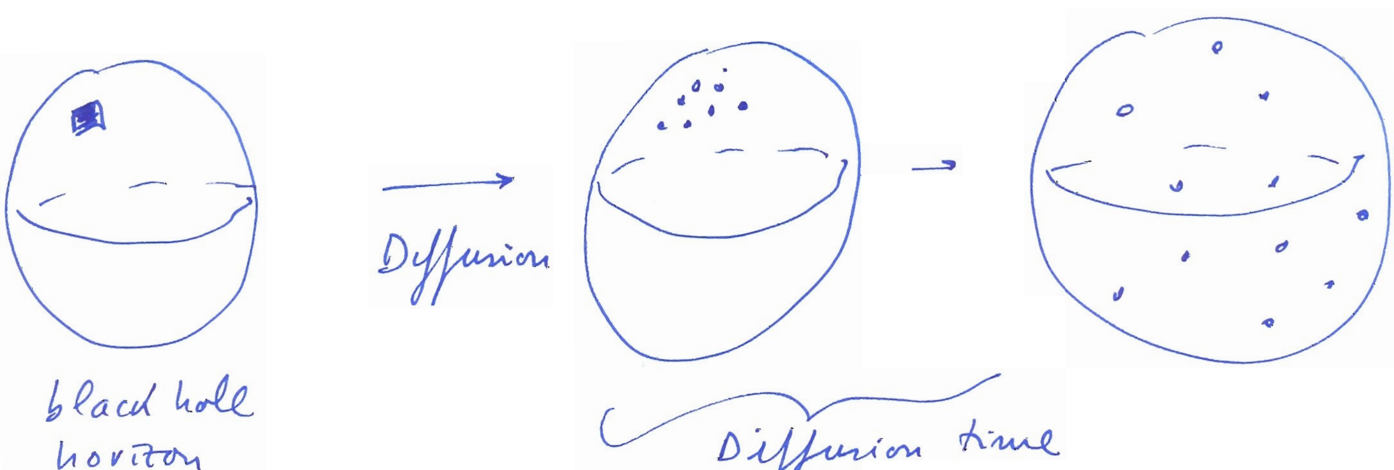
Can we saturate this bound? (Hayden-Preskill)

The role of Scrambling.

To saturate the bound means:

$$T_{rt} \sim \log R$$
$$\Rightarrow \quad t \sim R \log R$$

What is this time? This is thermalization time.



$$e^{\alpha t} \sim R \qquad t \sim \frac{1}{\alpha} \log R$$

$$\frac{1}{\alpha} = D = \frac{R}{ST} = \frac{1}{4\pi} \frac{1}{T}$$

$\Big\}\ t \simeq R \log R$

viscosity / entropy bound.

$$T_{ev} \sim \frac{R^2}{N} \qquad \tau_{scr} \sim \log R$$

The condition $T_{ev} > \tau_{scr}$ $\Longrightarrow$

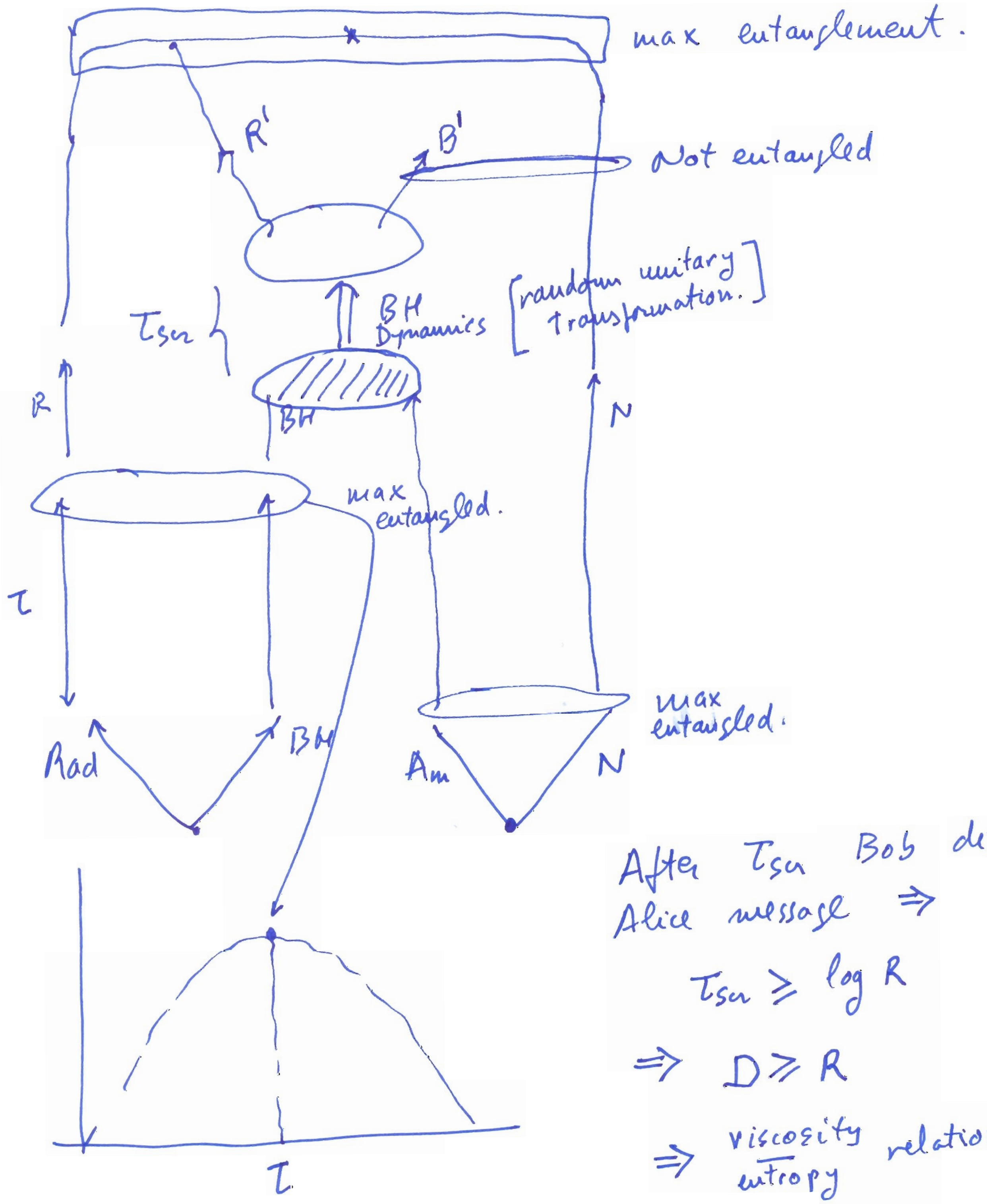
$$\boxed{R > L_N}$$

because due to the viscosity/entropy relation τ_{scr} is not dependent on N.

A question in information theory.

Is scrambling time the minimum retrieval time?.

Information Theory Argument:



After τ_{sc} Bob discover Alice message $\Rightarrow$

$$\tau_{sc} \geq \log R$$

$$\Rightarrow D \geq R$$

$\Rightarrow$ $\frac{\text{viscosity}}{\text{entropy}}$ relation !!.

Species, Scales and the AdS / CFT correspondence

Consider N D-3 branes compactified on S^3 of radius R.
From 7D point of view we have

$$M = N \cdot T_{D3} \cdot R^3 =$$

$$= \frac{N}{l_s^4 g_s} R^3 \qquad T_{D3} = \frac{1}{l_s^4 g_s}$$

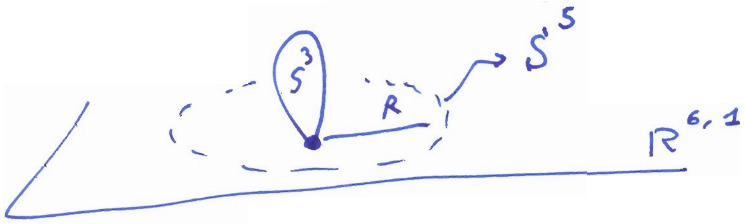
The Schwarzschild radius for this mass is:

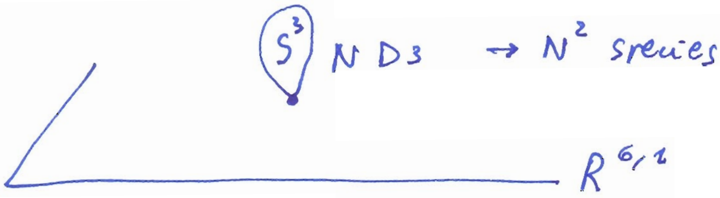
$$R_{Sch} = \left(\frac{N R^3}{l_s^4 g_s}\right)^{1/4} \frac{1}{\left[M_{Pl}^{(7)}\right]^{5/4}}$$

Using $\left[M_{Pl}^{(7)}\right]^5 = R^3 \frac{1}{\left[l_s^8 g_s^2\right]}$

$$\boxed{R_{Sch} = l_s (g_s N)^{1/4}}$$

AdS/CFT correspondence.





$$L \sim \left(\frac{N^2}{L}\right)^{1/4} \frac{1}{\left[M_{pe}^{(7)}\right]^{5/4}} \quad ; \quad L^5 = N^2 \frac{1}{\left[M_{pe}^{(7)}\right]^5}$$

$$\boxed{L_N = N^{2/5} \frac{1}{\left[M_{pe}^{(7)}\right]}}$$

$$L_N = N^{2/5} \frac{l_s^{8/5} g_s^{2/5}}{R^{3/5}}$$

Notice that if $L_N = R$

(UV/IR)

$$\boxed{L_N = l_s (g_s N)^{1/4}}$$

again the AdS/CFT correspondence.

Species and Cosmological Constant.

Consider N species and UV cutoff M_{pe}.

$$\Rightarrow \qquad cc \sim N M_{pe}^4 = \Lambda$$

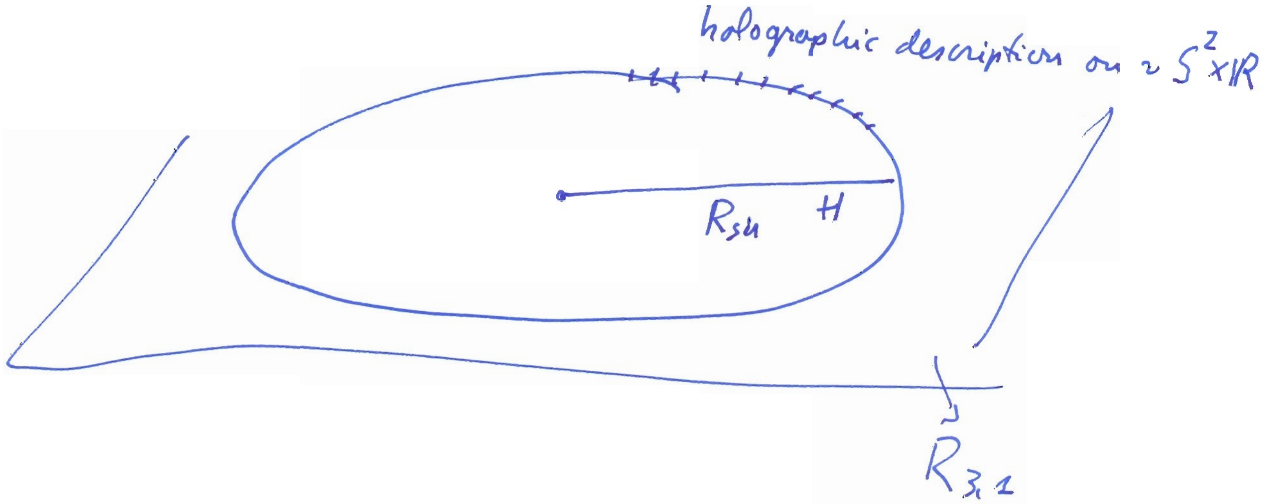
$R_{sch} = \frac{l_{pe}}{\sqrt{N}}$ $\Rightarrow$ B.H smaller than l_{pe} !!.

$\Rightarrow$ M_{pe} is not the right cutoff !.

$$C.C \simeq N \frac{M_{pe}^4}{N^2} = \frac{M_{pe}^4}{N}$$

$$\Rightarrow \quad R_{sh} = l_{pe}\sqrt{N}$$

for large N: C.C small !!
R_{sch} Hubble radius.



Large N - Gravity.

1:- $D = 4+n$ space - time dimension

$$G_{(10)} = \underbrace{g_s^2 \, l_s^8}_{\text{string length}} = \underbrace{l_{Pe}^8}_{\text{Planck length in 10 D}}$$

N species

$$\boxed{L_N(D) = N^{\frac{1}{2+n}} \cdot l_p(D)}$$

Species scale in dimension D

$$l_p(D)^{D-2} = g_s^2 \, l_s^8 \, \underbrace{R^{D-10}}_{\text{KK compactification scale.}}$$

$$\boxed{L_N(D)^{(2+n)} = N \, g_s^2 \, l_s^8 \, R^{n-6}}$$

Comment: In 10 D

$$L_N(10) = N^{1/8} \, g_s^{2/8} \, l_s$$

if $N = N_c^2$ i.e $SU(N_c)$ Yang Mills

$$\boxed{L_N(10) = (g_s N_c)^{1/4} \, l_s}$$

Notice that this is the expression of AdS radius in CFT/AdS correspondence. !!

In AdS/CFT you define the "double scaling" limit ($l_s \to 0$ + near horizon)

Let us define a different "gravity" decoupling limit using large # of species.

Large N gravity decoupling limit:

$$N \to \infty$$
$$g_s \to 0 \quad \text{(i.e } G_N \to 0 \text{)}$$

decoupling of gravity

with l_s = finite

$$\boxed{g_s^2 N = \text{finite}} \quad (1)$$

Notice that if $N = N_c^2$ (1) is 't Hooft large N $g_s N_c = g_{YM}^2 N_c$ finite.

Comment: In this decoupling limit the species scale is finite.

$$\boxed{L_N(D) = \lambda^{2/(2+n)} \, l_s^{8/(2+n)} \, R^{\frac{n-6}{2+n}}} \qquad \boxed{\lambda^2 \equiv g_s^2 N}$$

In 10 D

$$\boxed{L_N(10) = \lambda^{1/4} \, l_s}$$

What is the meaning of this species scale?

For scales $< L_N$ $\approx>$ string

For scales $> L_N$ $\approx>$ Q.F.T

But since L_N depends on 't Hooft coupling λ we get:

weak 't Hooft coupling $\approx>$ Q.F.T

Strong 't Hooft coupling $\approx>$ string.

i.e For any theory containing Gravity and N species In the limit $N \to \infty$ $G_N \to 0$ with $g_s^2 N$ finite we get a sort of "Maldacena's Duality".

Degravitation of Cosmological Constant.

If the UV cutoff is $L_N(D)$ for N species the C.C $\approx$

$$\boxed{\Lambda_D = N\, L_N(D)^{-D}}$$

In $N \to \infty$ $g_s \to 0$ limit $\Lambda_D \longrightarrow \infty$ However the Hubble radius for this C.C is finite

$$\boxed{R = L_N(D)}$$

The "M.D" $\overset{?}{=}$ dS Holography