SU(N) Gauge Theory Thermodynamics from Lattice

Saumen Datta
(in collaboration with Sourendu Gupta)

July 21, 2009

Introduction

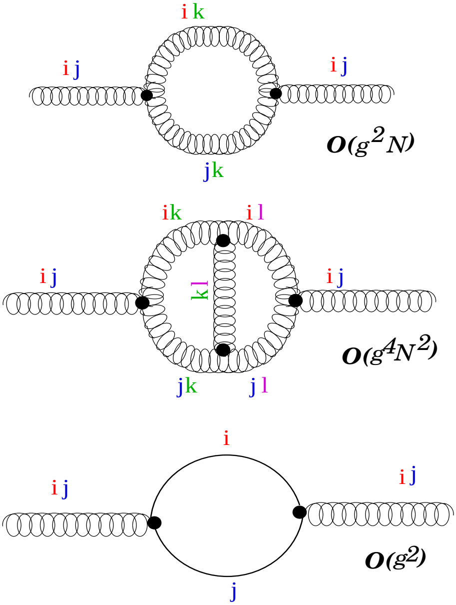
The theory of Strong interactions: SU(3) gauge theory
Quark fields ψ_i : triplet of SU(3), $i = 1, 2, 3$ color

Gluon fields: traceless Hermitian matrices A^{μ}_{ij}

$$\alpha_S(Q^2) = \frac{g^2(Q^2)}{4\pi} = \frac{4\pi}{(11 - \frac{2N_f}{3}) \ln Q^2/\Lambda^2} \quad \textit{in leading order}$$

The interactions become strong at hadron physics scales:
no natural small expansion parameter

G. 't Hooft 1974 Consider SU(N), use 1/N as expansion parameter
Leads to some simplifications and qualitative understandings of some properties of hadrons



$g \sim 1/\sqrt{N}$ for nontrivial large N limit

$\lambda_{tH} = g^2 N$

Quark loops ignored in leading order

unless $N_f \sim N_c$

$$\lambda_{tH}(Q^2) \sim \frac{48\pi^2}{11 \ln Q^2/\Lambda^2}$$

SU(N) Gauge Theory at Large N

- SU(N) gauge theory confines for all N.
  Nonperturbative physics for $q < \Lambda_{SU(N)}$
- Glueballs, mass $\sim\ O(N_c^0)$, decay widths $\sim\ O(1/N_c)$
  Mesons, mass $\sim\ O(N_c^0)$, scattering $\sim\ O(1/N_c)$

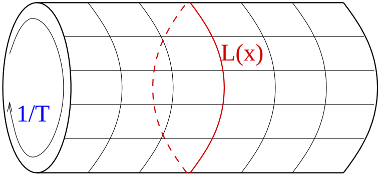
  Lattice study: more expensive. String tension, Glueball spectrum, etc. studied.

  Lucini & Teper; Narayanan & Neuberger; etc.
- At large temperatures, $O(N_c^2)$ gluon degrees of freedom
- 1st order phase transition at $T_c \sim \Lambda_{SU(N)}$ with latent heat $O(N_c^2)$

  SU(3): 1st order phase transition at $T_c \sim 270$ MeV

Equilibrium Thermodynamics on Lattice

$$Z(T) = \int dU \ \exp(-\beta \int_{0,pbc}^{1/T} d\tau \int d^3x \mathcal{L}(U))$$



As lattice spacing $a \to 0$,
$\beta\mathcal{L} \to \mathrm{Tr} F_{\mu\nu}^2$ with
$\beta = 2N/g^2 = 2N^2/\lambda_{tH}$

- $A_\mu(\vec{x}, 0) = A_\mu(\vec{x}, 1/T)$
  Z_N group of aperiodic gauge transformation
  $U_\mu(\vec{x}, 1/T) = e^{2\pi i/N} U_\mu(\vec{x}, 0)$
- Order parameter for deconfinement transition:

$$L = \frac{1}{N} \mathrm{Tr} \prod_{x_0=1}^{N_\tau} U_{(x_0,\mathbf{x}),\hat{0}} \to 1/N \, \mathrm{Tr} \, \mathrm{P} \, \mathrm{e}^{\int_0^{1/T} \mathrm{d}\tau \, A_0(\tilde{x},\tau)}$$

$$Z_N \quad : \quad L = \frac{1}{V} \sum_x L(\vec{x}) \to e^{2\pi i/N} L$$

Z_N Symmetry and Deconfinement

- $| < L > | \sim e^{-F}$, F free energy of a static quark source
- $< L > \neq 0 \ \rightarrow \ Z_N$ broken and deconfinement
- SU(2): 2nd order transition in Z_2 universality class

Engels et al. 1982-90

- For $N > 2$: Z_N: 1st order transition expected
- The QCD transition $N_f = 2 + 1$ is a *crossover*.
  Expected to be first order for all finite mass at SU(∞)
- Chiral limit may be more involved.
- Similarly, complicated phase structures have been predicted for $\mu_B \sim O(N_c)$.

McLerran & Pisarski '07

Study of SU(N) theory at finite T

Interesting predictions at finite temperature, which may help our understanding of SU(3)

- Symmetry of Polyakov loop $\longrightarrow$ strong 1st order transition?
- How does $T_c/\Lambda_{\overline{MS}}$ scale?
- For $N > 3$ the latent heat $\sim N^2$
- Does one reach the asymptotic state soon after the transition?
- Deviation from conformality in the plasma phase?

Deconfinement transition for SU(4)

Gavai; Ohta & Wingate; 2001

Deconfinement transition for SU(N), and pressure of the high temperature phase from coarse lattices.

Lucini & Teper; Lucini, Teper & Wenger; Bringholtz & Teper 2003–

Need to be careful about discretization errors

Our study

Numerical investigation of SU(4) and SU(6) gauge theories
Study large N theory using results for N=3,4,6

S. Datta & S. Gupta, arXiv:0906.3929 and in preparation

- Study of the cutoff dependence
  Is the lattice spacing small enough for 2-loop running?
- Study of the phase transition in SU(4) and SU(6)
  Finite volume study
- Direct estimation of $T_c/\Lambda_{\overline{MS}}$
- Equation of state for N=4 and 6 theories
  Analyse SU(3) with existing data.

  Engels et al., Nucl. Phys. B 469 (1996) 419

- $\epsilon - 3p$ and conformal symmetry breaking in the deconfined phase

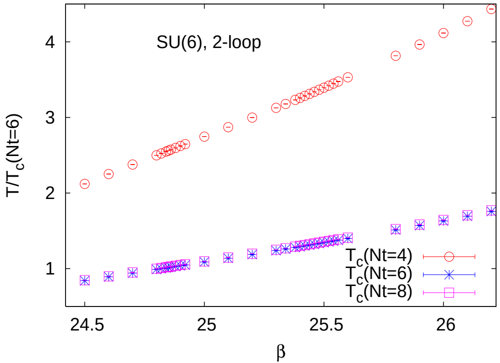
Scaling

$T_c(g^2, N_t) \; = \; \frac{1}{a(g^2)N_t}$

Use $T_c(N_t)$ to predict $T_c(N_t')$

S. Gupta, PRD 64(2001) 034507

we use for coupling $V_{Q,\bar{Q}}(q^2) \; = \; -\frac{N^2-1}{2N}\frac{g^2}{q^2}$ (V-scheme)



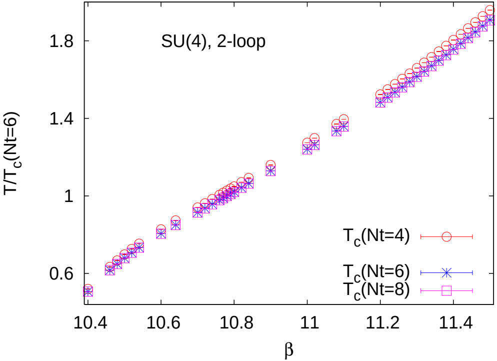
Scaling

$$T_c(g^2, N_t) \;=\; \frac{1}{a(g^2)N_t}$$

Use $T_c(N_t)$ to predict $T_c(N_t')$

S. Gupta, PRD 64(2001) 034507

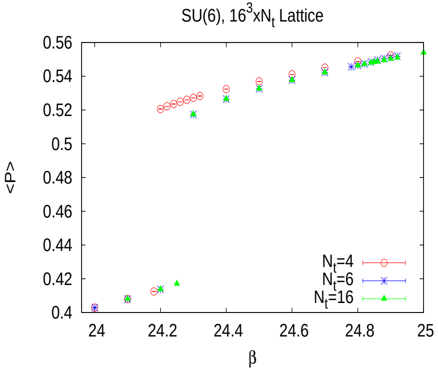
we use for coupling $V_{Q,\bar{Q}}(q^2) \;=\; -\frac{N^2-1}{2N}\frac{g^2}{q^2}$ (V-scheme)

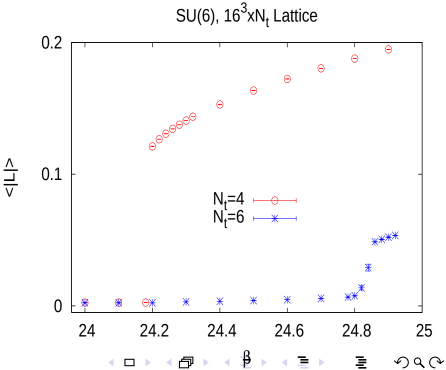


Bulk and Deconfinement Transitions for SU(6)

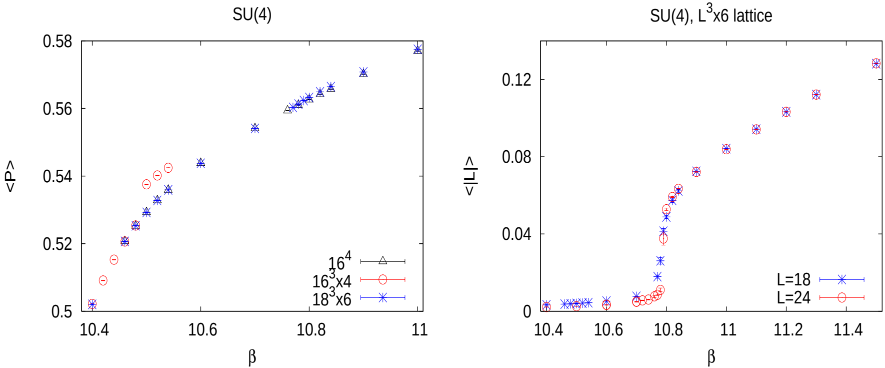
$$P_{\mu,\nu}(x) = \frac{1}{N}\mathrm{Tr}\,U_\mu(x)U_\nu(x+\hat{mu})U^\dagger_\mu(x+\nu)U^\dagger_\nu(x)$$

$$P = \frac{1}{6VT}\sum_{\mu,\nu}\sum_x P_{\mu,\nu}(x)$$





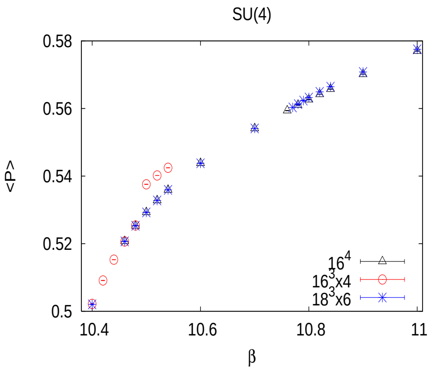
Bulk and Deconfinement Transition for SU(4)

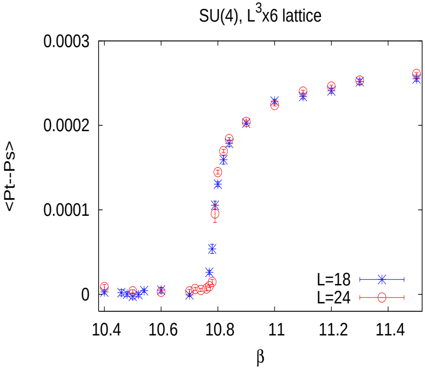


$\langle P_t - P_s \rangle$ associated with deconfinement:

$\frac{\epsilon}{T^4} = 6N^2 N_\tau^4 \frac{P_t - P_s}{\lambda_{tH}} + corrections$

Bulk and Deconfinement Transition for SU(4)

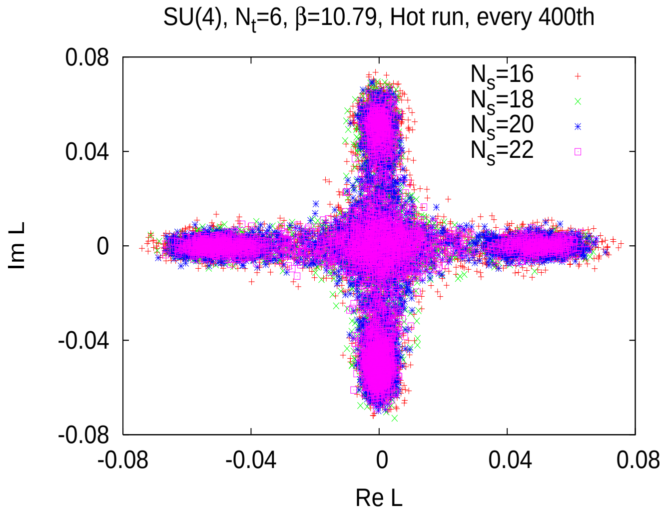




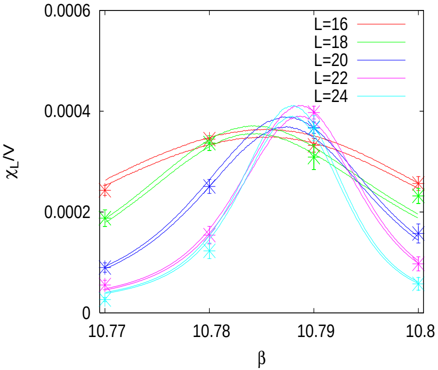
$\langle P_t - P_s \rangle$ associated with deconfinement:

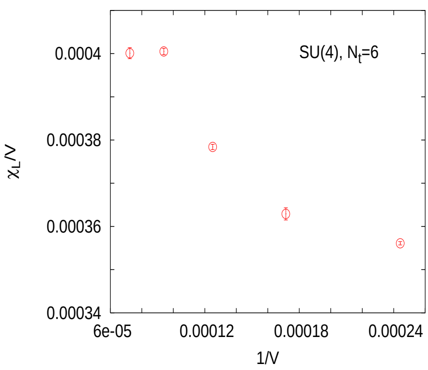
$\frac{\epsilon}{T^4} = 6N^2 N_\tau^4 \frac{P_t - P_s}{\lambda_{tH}} + corrections$

Degenerate Vacuua at T_c

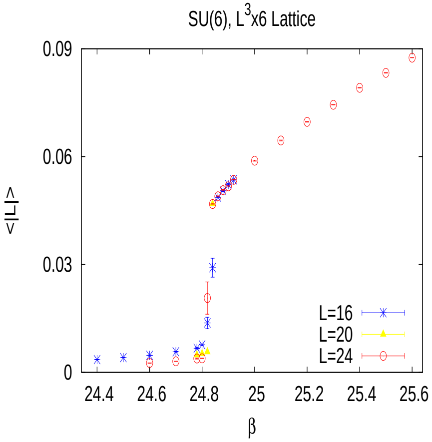


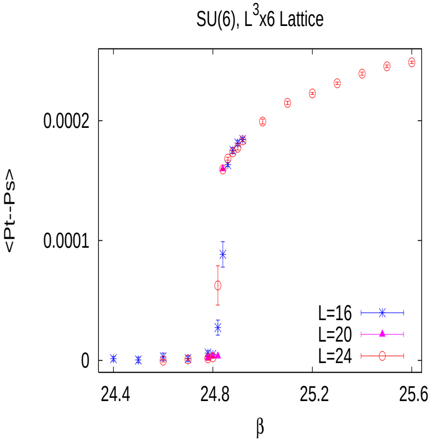
Susceptibility $\chi_L = V(\langle |L|^2\rangle - \langle |L|\rangle^2) \quad \sim V$ for 1st order.





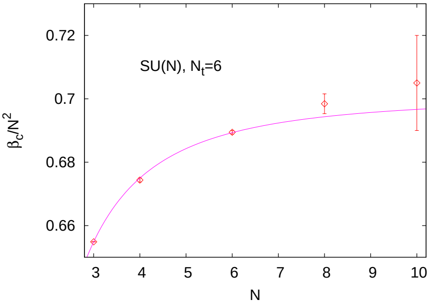
Deconfinement Transition for SU(6)



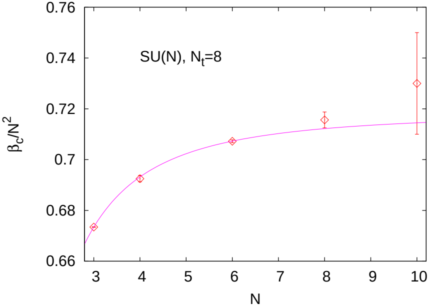


N-dependence of β_c

$$\beta = 2N^2/\lambda_{tH}$$



$$\frac{\beta_c}{N^2} = 0.7008(6) - \frac{0.413(6)}{N^2}$$



$$\frac{\beta_c}{N^2} = 0.7186(5) - \frac{0.406(5)}{N^2}$$

$T_c/\Lambda_{\overline{MS}}$

- Use 2-loop running to directly calculate $T_c/\Lambda_{\overline{MS}}$
- Definitions of g^2 through different schemes
  $g_E^2 \;=\; \frac{8N}{N^2-1}\,(1-P)$ (E-scheme)
  besides g_V^2 and $g^2_{\overline{MS}}$
- For each N, continuum limit of $g^2_{\rm scheme}$
- Scheme-dependence observed to be stronger than cutoff dependence
- N-dependence using $c_0 + \frac{c_1}{N^2}$

$$\boxed{\frac{T_c}{\Lambda_{\overline{MS}}}\Big|_{N\to\infty} \;=\; 1.17(5)}$$

Equation of State from Lattice

Define thermodynamic quantities

$F(T,V) = T \ln Z(T,V)$ Z: grand canonical partition function

$\epsilon = -\frac{1}{V} \frac{\partial F(T,V)/T}{\partial (1/T)}$

$p = \frac{\partial F}{\partial V} \qquad = F/V$ for homogeneous

Equation of State from Lattice

Define thermodynamic quantities

$F(T,V) = T \ln Z(T,V)$ Z: grand canonical partition function

$\epsilon = -\frac{1}{V}\frac{\partial F(T,V)/T}{\partial(1/T)}$

$p = \frac{\partial F}{\partial V} \quad = F/V$ for homogeneous

Trace of energy momentum tensor $\theta^{\mu\mu} = \Delta = \frac{\epsilon - 3p}{T^4}$

measure of breaking of conformal invariance

$$\boxed{\Delta/T^4 = T\frac{\partial}{\partial T}(p/T^4)}$$

Equation of State from Lattice

Define thermodynamic quantities

$F(T,V) = T\ \ln Z(T,V)$ Z: grand canonical partition function

$\epsilon = -\frac{1}{V}\frac{\partial F(T,V)/T}{\partial(1/T)}$

$p = \frac{\partial F}{\partial V} \qquad = F/V$ for homogeneous

Trace of energy momentum tensor $\theta^{\mu\mu} = \Delta = \frac{\epsilon - 3p}{T^4}$

measure of breaking of conformal invariance

$$\boxed{\Delta/T^4 = T\frac{\partial}{\partial T}(p/T^4)}$$

Convenient to calculate this on lattice

$T\frac{\partial}{\partial T} \rightarrow -a\frac{\partial}{\partial a}$

$p/T^4 \rightarrow \frac{N_t^3}{N_s^3}\ln Z$

Equation of State from Lattice

Define thermodynamic quantities

$F(T, V) = T \ln Z(T, V)$ Z: grand canonical partition function

$\epsilon = -\frac{1}{V}\frac{\partial F(T,V)/T}{\partial(1/T)}$

$p = \frac{\partial F}{\partial V} \qquad = F/V$ for homogeneous

Trace of energy momentum tensor $\theta^{\mu\mu} = \Delta = \frac{\epsilon - 3p}{T^4}$

measure of breaking of conformal invariance

$$\boxed{\Delta/T^4 = T\frac{\partial}{\partial T}(p/T^4)}$$

Convenient to calculate this on lattice

$T\frac{\partial}{\partial T} \rightarrow -a\frac{\partial}{\partial a}$

$p/T^4 \rightarrow \frac{N_t^3}{N_s^3} \ln Z$

$$\boxed{\Delta/T^4 = \frac{\partial}{\partial a}\beta\,\frac{N_t^3}{N_s^3}\langle\frac{dS}{d\beta}\rangle}$$

Eqn. of State (Contd.)

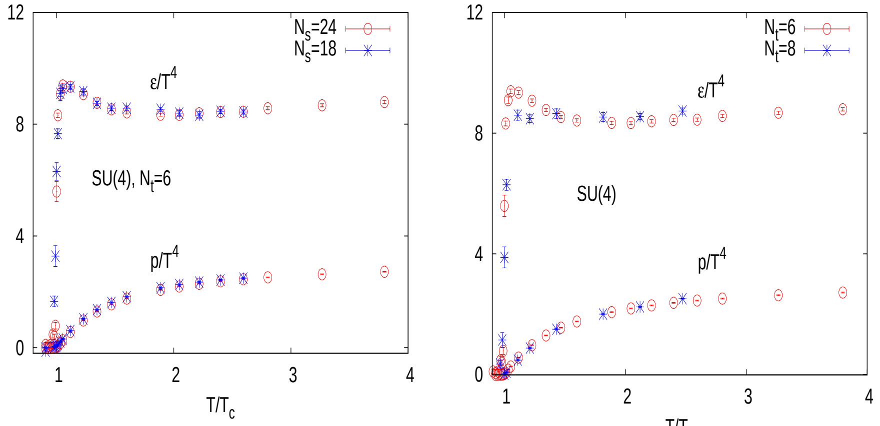
$$\boxed{\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{\beta_0}^{\beta} d\beta \; (P(\beta) - P(\beta_0))}$$

$\frac{\epsilon}{T^4} = 3\frac{p}{T^4} + \frac{\Delta}{T^4}$ $\qquad$ $\frac{s}{T^3} = \frac{\epsilon}{T^4} + \frac{p}{T^4}$

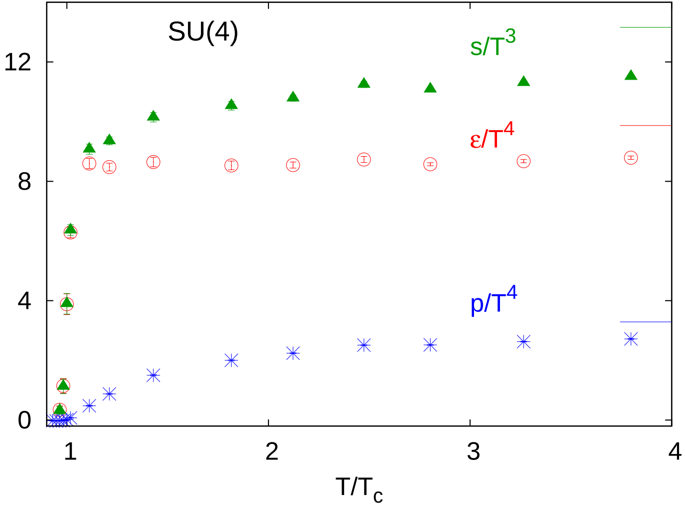
For free gas, Stefan-Boltzmann limit
$\frac{\epsilon}{T^4} = 3\frac{p}{T^4} = (N^2 - 1)\frac{\pi^2}{15} R(N_\tau)$
Here $R(N_\tau)$ discretization error
$= 1 + \frac{10}{21}(\frac{\pi}{N_\tau})^2 + \ldots$

Boyd et al., Nucl.Phys. B 469('96) 419;
Engels et al., Nucl.Phys. B 205('82) 545.

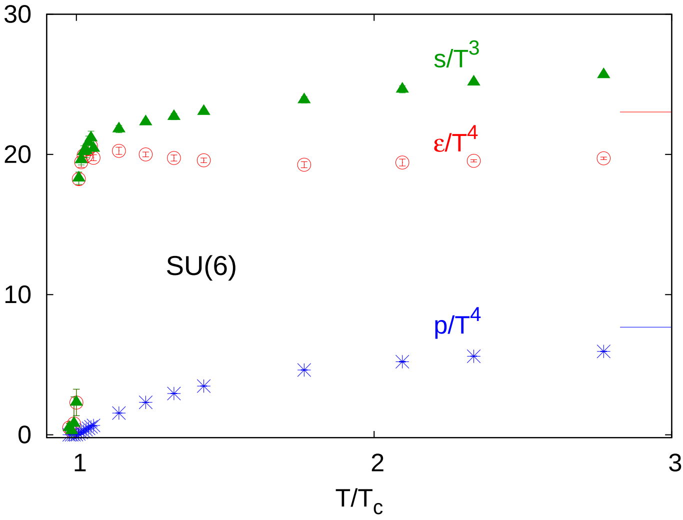
Cutoff and Volume Dependence of EOS for SU(4)



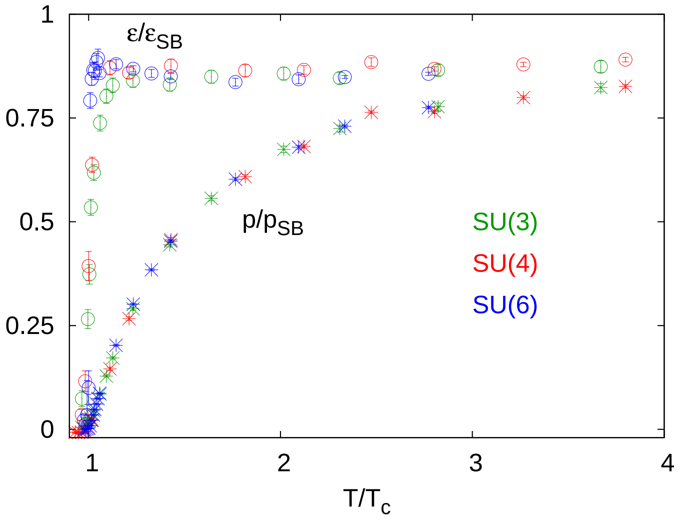
Thermodynamics for SU(4)



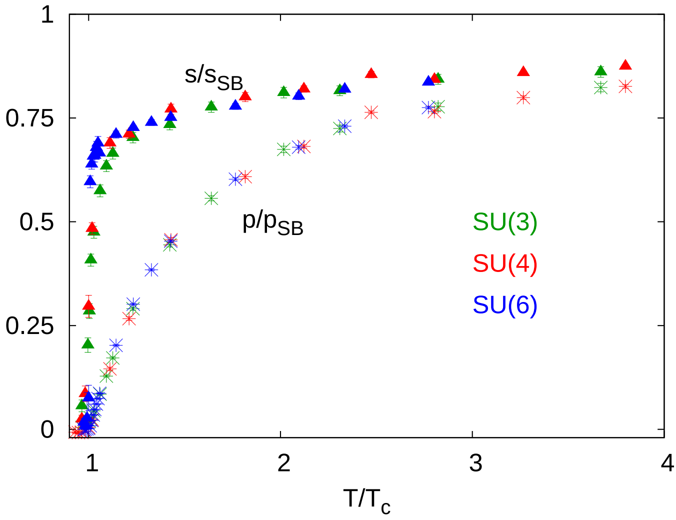
Thermodynamics for SU(6)



Scaling of Thermodynamic Quantities with N



Scaling of Thermodynamic Quantities with N

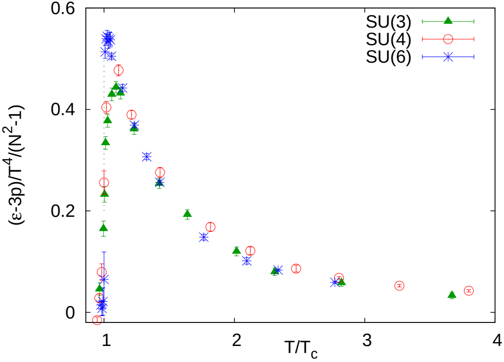


Interaction Measure $\epsilon - 3p$

SU(N) gauge theory classically scale invariant $\rightarrow T^{\mu\mu} = 0$

Quantum theory may break this symmetry

$\epsilon - 3p$ measure of breaking of scale invariance

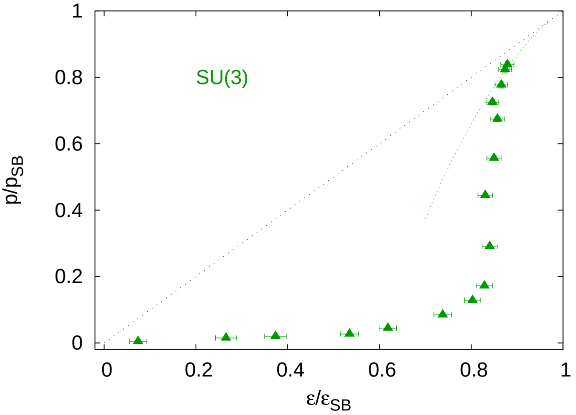


Substantial conformal symmetry breaking in plasma

Peak moves from $\sim 1.09\,T_c$ at N=3 to $\sim 1.025\,T_c$ at N=6

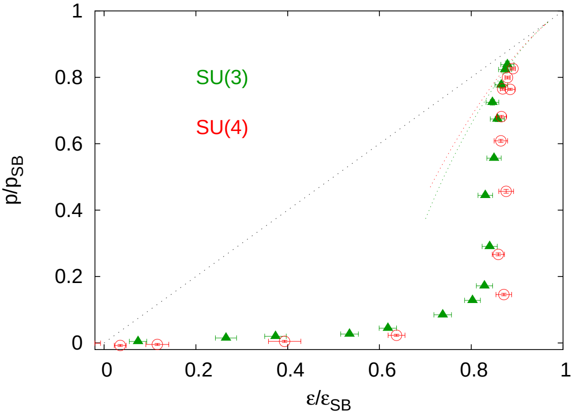
$\sim 1/T^2$ fall-off at intermediate temperature

Conformal vs. Weak Coupling Limit



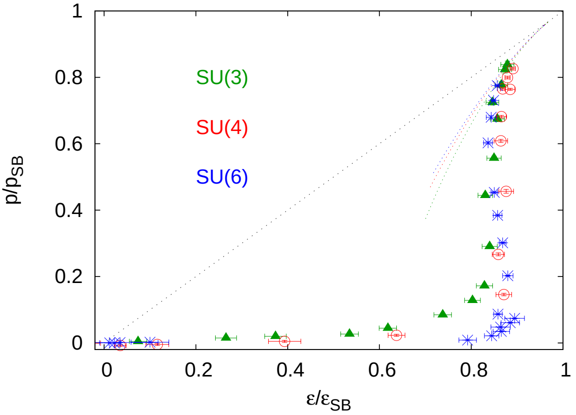
Weak coupling result from Laine et al., Phys.Rev. D 73 (2006) 085009

Conformal vs. Weak Coupling Limit



Weak coupling result from Laine et al., Phys.Rev. D 73 (2006) 085009

Conformal vs. Weak Coupling Limit

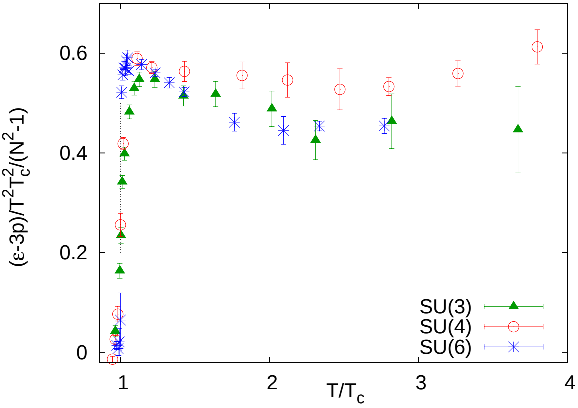


Weak coupling result from Laine et al., Phys.Rev. D 73 (2006) 085009

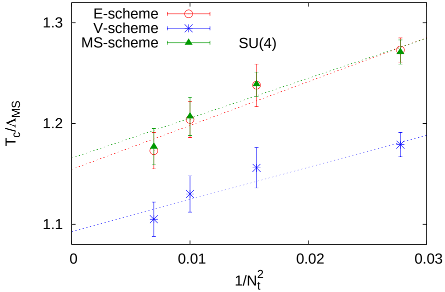
Summary and Conclusions

- SU(N) gauge theories studied at finite temperature for N=3,4,6
- Important to check for discretization errors.
  Under control for $N_\tau \gtrsim 6$.
- First order deconfinement transition confirmed for N=4,6
- $T_c/\Lambda_{\overline{MS}} = 1.17(5)$ with small N dependence
- Nice N^2 scaling for thermodynamic quantities for $T/T_c \gtrsim 1.2$.
- Large deviation from Stefan-Boltzmann limit
- Considerable conformal symmetry breaking till deep in the plasma
  No evidence of a window for strongly coupled conformal theory

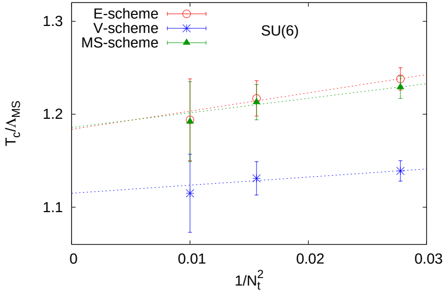
Extra: $(\epsilon - 3p)/T^4$ scaling



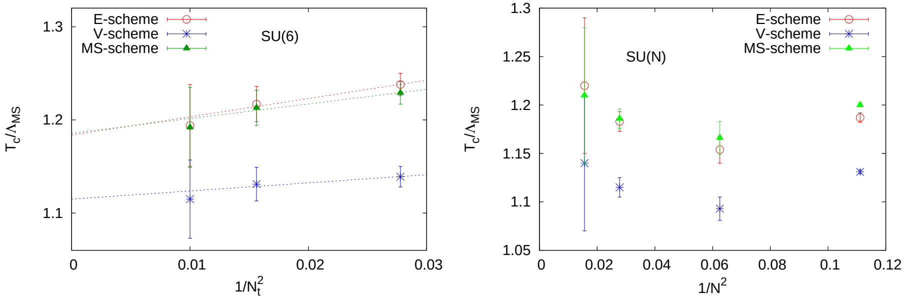
Extra: $T_c/\Lambda_{\overline{MS}}$



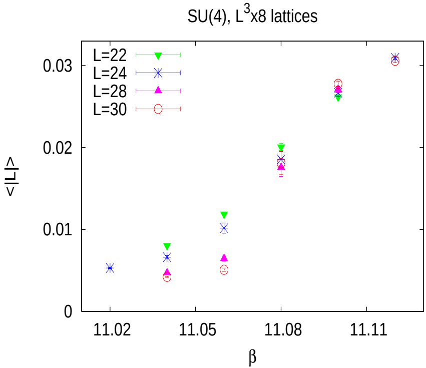
Extra: $T_c/\Lambda_{\overline{MS}}$

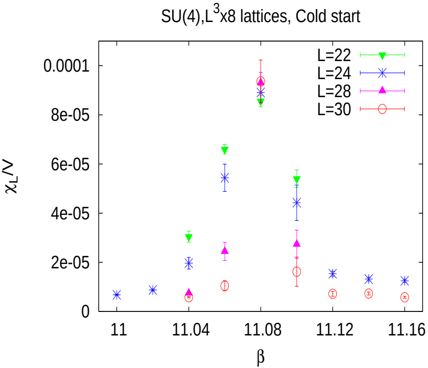


Extra: $T_c/\Lambda_{\overline{MS}}$

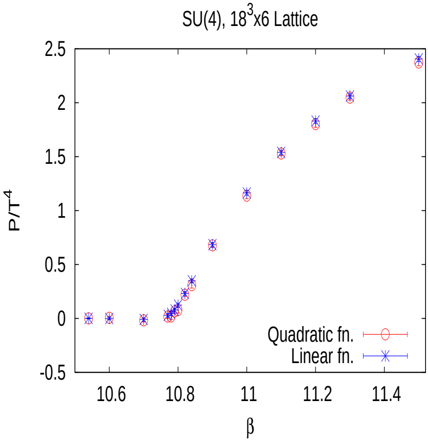


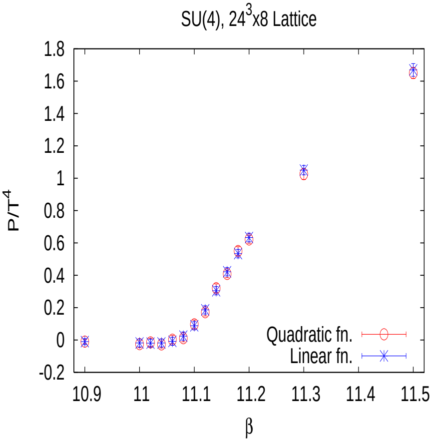
Extra: β_c for $N_\tau = 8$



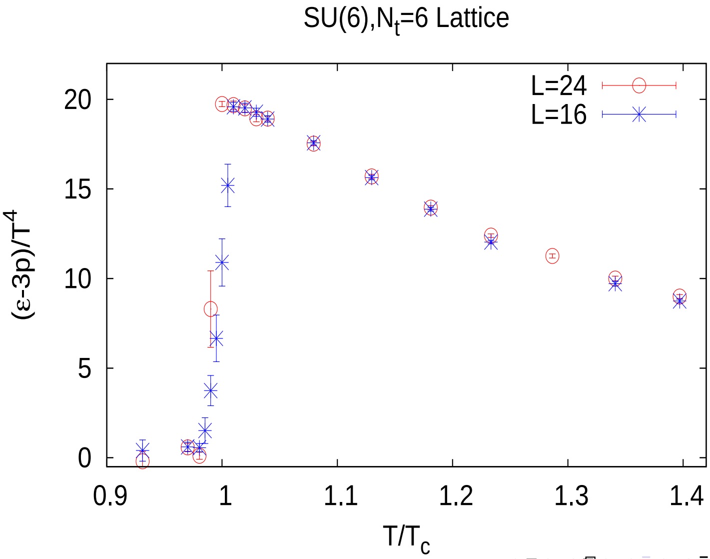


Extra: Integration Method in Pressure

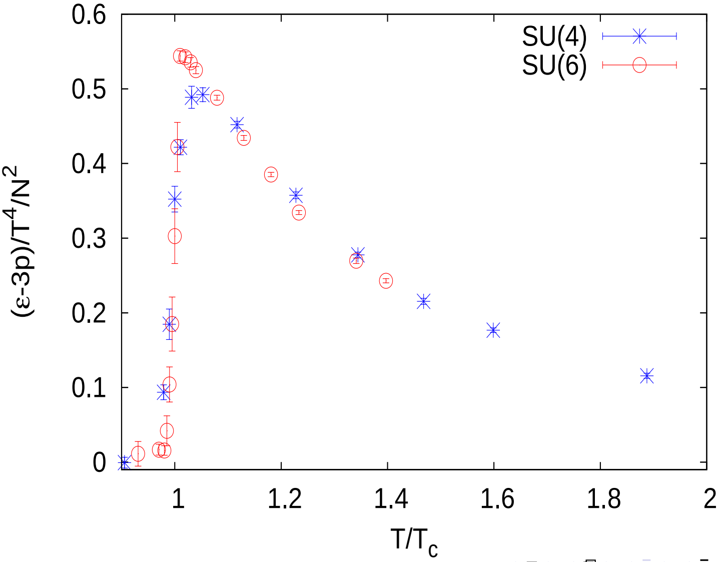




Extra: Volume Dependence of $\epsilon - 3p$



Extra: Volume Dependence of $\epsilon - 3p$



Extra: Latent Heat for N_t=5