Non-supersymmetric AdS black holes and their entropy.

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Motivation

- Microscopic aspects of AdS black holes
- The situation even for supersymmetric AdS black holes is not as good as for flat black holes

Five dimensional N = 2, $U(1)^3$ gauged supergravity

Bosonic Field Content : metric $g_{\mu\nu}$, three photons, A^I_μ, two unconstrained scalars written in terms of three constrained scalars X^I,

$$X^1 X^2 X^3 = 1.$$

The action is similar to the five dimensional STU model, but there is a potential for the scalars,

$$V \sim \frac{1}{X^1} + \frac{1}{X^2} + \frac{1}{X^3}. \tag{1}$$

Minimum of the potential is the AdS_5 vacuum.

Static Charged Black Hole Solutions.

Behrndt, Cvetic, Sabra, hep-th/9810227.

- $$ds^2 = -(H_1 H_1 H_3)^{-2/3} f\, dt^2 + (H_1 H_1 H_3)^{1/3}(f^{-1} dr^2 + r^2 ds^2_{S^3}) \quad (2)$$

- $$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2} H_1 H_2 H_3, \qquad X^I = H_I^{-1}(H_1 H_2 H_3) \quad (3)$$

- $$A^I = \frac{\tilde{q}_I}{q_I} \frac{1}{L} \left(\frac{1}{H_I} - 1 \right) dt, \qquad H_I = 1 + \frac{q_I}{r^2} \quad (4)$$

- $$\tilde{q}_I = \sqrt{q_I(\mu + q_I)}, \qquad M = \frac{\pi}{4 G_N^{(5)}} \left(\frac{3}{2}\mu + q_1 + q_2 + q_3 + \frac{3L^2}{8} \right), \quad (5)$$

Static Charged Black Hole Solutions......

For $\mu = 0$ solution is a supersymmetric naked singularity.

For $\mu = \mu_c$, solution is an extremal black hole solution, non-supersymmetric.

For $\mu > \mu_c$, solution is a genuine double horizon black hole.

Ten dimensional uplift.

Cvetic + 9, hep-th/9903214

Five dimensional $U(1)^3$ gauged supergravity can be obtained as a consistent truncation of IIB supergravity compactified on $AdS_5 \times S^5$.

What this means is that every solution to $U(1)^3$ supergravity can be lifted to a solution to IIB sugra which is asymptotically $AdS_5 \times S^5$.

Ten dimensional uplift.......

Non-linear Kaluza-Klein reduction formulae

The only fields present are the metric and the five-form flux.

$$ds_{10}^2 = \sqrt{\Delta}\ ds_5^2 + \frac{1}{\sqrt{\Delta}}\ d\Sigma_5^2 \tag{6}$$

$$ds_5^2 = -\frac{f}{H_1 H_2 H_3} dt^2 + \frac{dr^2}{f} + r^2\, d\Omega_3^2 \tag{7}$$

$$d\Sigma_5^2 = \sum_{i=1}^{3} L^2 H_i \left(d\mu_i^2 + \mu_i^2 \left[d\phi_i + A_i\ dt \right]^2 \right) \tag{8}$$

$$\Delta = H_1\, H_2\, H_3 \left[\frac{\mu_1^2}{H_1} + \frac{\mu_2^2}{H_2} + \frac{\mu_3^2}{H_3} \right] \tag{9}$$

$$z_i = r \mu_i e^{i\phi_i} \tag{10}$$

Ten dimensional uplift.......

Myers and Tafjord, hep-th/0109127

These black holes are sourced by non-supersymmetric excitations about the spherical giant graviton branes.

There are three species of giant graviton branes : for example, the first species is a D3 wrapping the $|z_2|^2 + |z_3|^2 = 1$ three-sphere and moving along the ϕ_1 direction ....

The two-charge black hole.

Set $q_1 = 0$. This gives $\mu_c = \frac{q_2 q_3}{L^2}$.

That is, we have a non-supersymmetric extremal black hole for $\mu = \mu_c$, and a non-supersymmetric non-extremal black hole for $\mu > \mu_c$.

$$f = 1 + \frac{q_2 + q_3}{L^2} - \frac{\mu - \mu_c}{r^2} + \frac{r^2}{L^2} \equiv f_0 - \frac{\mu - \mu_c}{r^2} + \frac{r^2}{L^2} \qquad (11)$$

The horizon is at $r_h = 0$ for the extremal case.

The near-horizon near-extremal limit of the two-charge black hole.

Consider the following limit on the ten-dimensional uplift of the two-charge AdS black hole:

$$r = \epsilon\,\rho, \qquad t = \frac{\tau}{\epsilon}, \qquad \mu - \mu_c = \epsilon^2 M \tag{12}$$

$$\phi_1 = \frac{\varphi}{\epsilon}, \qquad \phi_i = \psi_i + \frac{\tilde{q}_i}{q_i L}\frac{\tau}{\epsilon}, \;\; i = 2, 3, \tag{13}$$

keeping $\rho, \tau, \varphi, M, \psi_i$ fixed while $\epsilon \to 0$.

- $r = \epsilon\,\rho, \quad t = \frac{\tau}{\epsilon}$ - near-horizon limit,
- $\mu - \mu_c = \epsilon^2 M$ - near-extremal limit
- $\phi_i = \psi_i + \frac{\tilde{q}_i}{q_i L}\frac{\tau}{\epsilon}$ - near-horizon limit for rotating black holes, similar to the one used in the Kerr-CFT story.

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$$f = f_0 - \frac{M}{\rho^2}, \qquad \Delta = \mu_1^2 \, \frac{q_2 q_3}{\rho^4} \cdot \frac{1}{\epsilon^4}, \qquad H_i = \frac{q_i}{\rho^2} \cdot \frac{1}{\epsilon^2} \, . \tag{14}$$

$$ds_{10}^2 = \mu_1 \, [R_{AdS_3}^2 \, ds_3^2 + R_S^2 \, d\Omega_3^2] + \frac{1}{\mu_1} ds_{\mathcal{M}_4}^2 \tag{15}$$

A *warped* product of a six-metric and a four-metric, the six-metric being a product metric of a static BTZ black hole and a round S^3 and the four-metric is locally flat. This is *a solution to IIB supergravity.*

$$ds_3^2 = -(\rho^2 - \rho_0^2) d\tau^2 + \frac{d\rho^2}{\rho^2 - \rho_0^2} + \rho^2 d\varphi^2, \tag{16}$$

$$ds_{\mathcal{M}_4}^2 = \frac{L^2}{R_S^2} \left[q_2 \, (d\mu_2^2 + \mu_2^2 \, d\psi_2^2) \, + \, q_3 \, (d\mu_3^2 + \mu_3^2 \, d\psi_3^2) \right] . \tag{17}$$

The near-horizon near-extremal limit of the two-charge black hole.......

The AdS_3 and the S^3 factors have *unequal* radii.

$$R^2_{S^3} = \sqrt{q_2\, q_3}, \qquad R^2_{AdS^3} = \frac{\sqrt{q_2\, q_3}}{f_0}, \tag{18}$$

$$M_{BTZ} = \frac{M}{\mu_c}. \tag{19}$$

Compare to the asymptotically flat two-charge black hole, which on a near-horizon decoupling limit gave a *product* metric $BTZ \times S^3 \times T^4$ with *equal* sphere and AdS_3 radii.

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Entropy of the BTZ black hole

The spatial sections of the horizon of the two-charge black hole before and after taking the near-horizon near-extremal limit are *completely different*. One of them is a S^1 and the other is a S^3. But it turns out that the entropies before and after taking the limits are *exactly the same*.

This means that we have not lost any of the degrees of freedom that contribute to the entropy of the black hole in the process of taking the limit.

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Entropy of the BTZ black hole.........

Computing the three-dimensional Newton's constant.

$$g^{(10)}(x,y) = \mu_1\, g^{(6)}(x) + \frac{1}{\mu_1}\, ds^2_{\mathcal{M}_4}, \qquad (20)$$

$$R^{(10)}_{i\,j} \;=\; \frac{1}{\mu_1}\,\frac{f_0 - 1}{\sqrt{q_2\, q_3}}\, g^{(10)}_{i\,j} + R^{(6)}_{i\,j}, \qquad (21)$$

$$R^{(10)} \;=\; \frac{6}{\mu_1}\,\frac{f_0 - 1}{\sqrt{q_2\, q_3}} + \frac{1}{\mu_1} R^{(6)}, \qquad (22)$$

$$\frac{1}{G_{10}} \int d^{10}x \sqrt{-g^{(10)}}\, R^{(10)} \qquad (23)$$

$$\sim \frac{Vol(\mathcal{M}_4)}{G_{10}} \int d^6x \sqrt{-g^{(6)}} \left(R^{(6)} + \frac{6 f_0 - 6}{\sqrt{q_2\, q_3}} \right) \qquad (24)$$

Entropy of the BTZ black hole.........

In fact, there is a consistent truncation of IIB theory on $\mathcal{M}_4$ to a six-dimensional gravity theory, details of which is there in the paper. Here, we will only need the six-dimensional Newton's constant which on a further compactification on the S^3 yields the three-dimensional Newton's constant.

At the end of the day, the entropy of the BTZ black hole matches exactly with the entropy of the five-dimensional near-extremal black hole as computed from just it's five-dimensional geometry.

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The third charge

"Add" the third charge "perturbatively": $q_1 = \epsilon^4 \, \hat{q}_1$

$$f = f_0 - \frac{M}{\rho^2} + \frac{J^2}{4\rho^4}, \qquad J^2 \equiv 4 \frac{\hat{q}_1 \, q_2 \, q_3}{L^2} = 4\mu_c \hat{q}_1 \tag{25}$$

$$ds^2 = \mu_1 \left[R^2_{AdS} \; ds^2_{BTZ} + R^2_S \; d\Omega^2_3 \right] + \frac{1}{\mu_1} d\mathcal{M}^2_4 \tag{26}$$

$$ds^2_{BTZ} = -N(\rho) d\tau^2 + \frac{d\rho^2}{N(\rho)} + \rho^2 (d\varphi - N_\varphi d\tau)^2 \tag{27}$$

$$N(\rho) = \rho^2 - M_{BTZ} + \frac{J^2_{BTZ}}{4\rho^2}, \qquad N_\varphi = \frac{J_{BTZ}}{2\rho^2} \; , \tag{28}$$

$$M_{BTZ} = \frac{M}{\mu_c}, \qquad J_{BTZ} = 2 \sqrt{\frac{f_0 \, \hat{q}_1}{\mu_c}} \; . \tag{29}$$

The third charge.........

The entropy of the rotating BTZ again matches with the entropy of the five-dimensional near-extremal three-charge black hole as computed from the five-dimensional geometry and setting the third charge to be “perturbative” in comparison to the other charges.

The message is that at the heart of these non-supersymmetric AdS black holes is just a BTZ black hole.

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Four dimensional black holes and M-theory

- Four-charge static black hole solutions to four-dimensional $\mathcal{N} = 2$ $U(1)^4$ guaged supergravity .
- These solutions can be lifted to asymptotically $AdS_4 \times S^7$ solutions to M-theory, using uplift formulae.
- A similar near-horizon near-extremal limit on the eleven dimensional uplift of the *three* charge solution gives a static BTZ black hole, more precisely

$$ds_2 = \mu_1^{4/3} \left(R_A^2 ds_{BTZ}^2 + R_S^2 d\Omega_2^2\right) + \frac{1}{\mu_1^{2/3}} ds_{\mathcal{M}_6}^2 \tag{30}$$

- Adding the fourth charge perturbatively again yields a rotating BTZ black hole.
- The entropy of the near-extremal four-dimensional black hole exactly agrees with the entropy of the BTZ black hole.

Further studies

The states that contribute to the entropy of these black holes, both in IIB and M theory. They are clearly described by a CFT. See papers for some related issues.

For the five-dimensional black holes, there is the $\mathcal{N} = 4$ SYM description. What does the scaling limit on the geometry correspond to there? See papers for some related issues.

There is a body of work by Balasubramanian, de Boer and collaborators who study these black hole in scaling limits where they scale the charges so that the near- extremal black hole is also near-BPS. In this setting, you can use BPS objects, the giant graviton branes and young tableux operators to study black holes.

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