Remarks on Scaling in gravity and in YM Theory

Two characters in search of an author...

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General Considerations

More than forty years have elapsed since the general theorems on singularities in GR and BH formation have been studied in detail

Apart from BH-Thermodynamics, we have learned that once QM is brought to bare, rather puzzling problems arise

Hawking went futher, and argued that thermal production of radiations in BH emission implied the fundamental loss of coherence in Quantum Mechanics. We lose information and also unitarity

Most people (including Hawking) believe that the problem is basically solved within the AdS/CFT correspondence. The question is how?, what when wrong in Hawking's original argument?

Is string non-locality enough?

General philosophy

One of the most important lessons we have learned from the Maldacena conjecture is that the QCD string, is the fundamental string in some higher dimensional geometry. Accumulated evidence points to the relevance of BH's in the holographic description

An interesting perturbative regime where stringy aspects of QCD naturally appear is the Regge limit of scattering amplitudes s>> -t. The amplitudes are dominated by Log s, making a resummation of terms of the form

$$(\alpha_s \, N \log(s/Q^2))^n$$

mandatory. The re-summation can be done using the BFKL formalism which predicts a Regge behavior for the total cross section. When the vacuum quantum numbers are exchanged in the t-channel, the dominant trajectory is that of the hard pomeron. The LLA breaks unitarity, which is restored by gluon saturation effects, with interesting scaling properties. Is there a holographic description of this behavior?

Basic aim

Scaling phenomena

Gravity ⟷ Yang-Mills

Holography

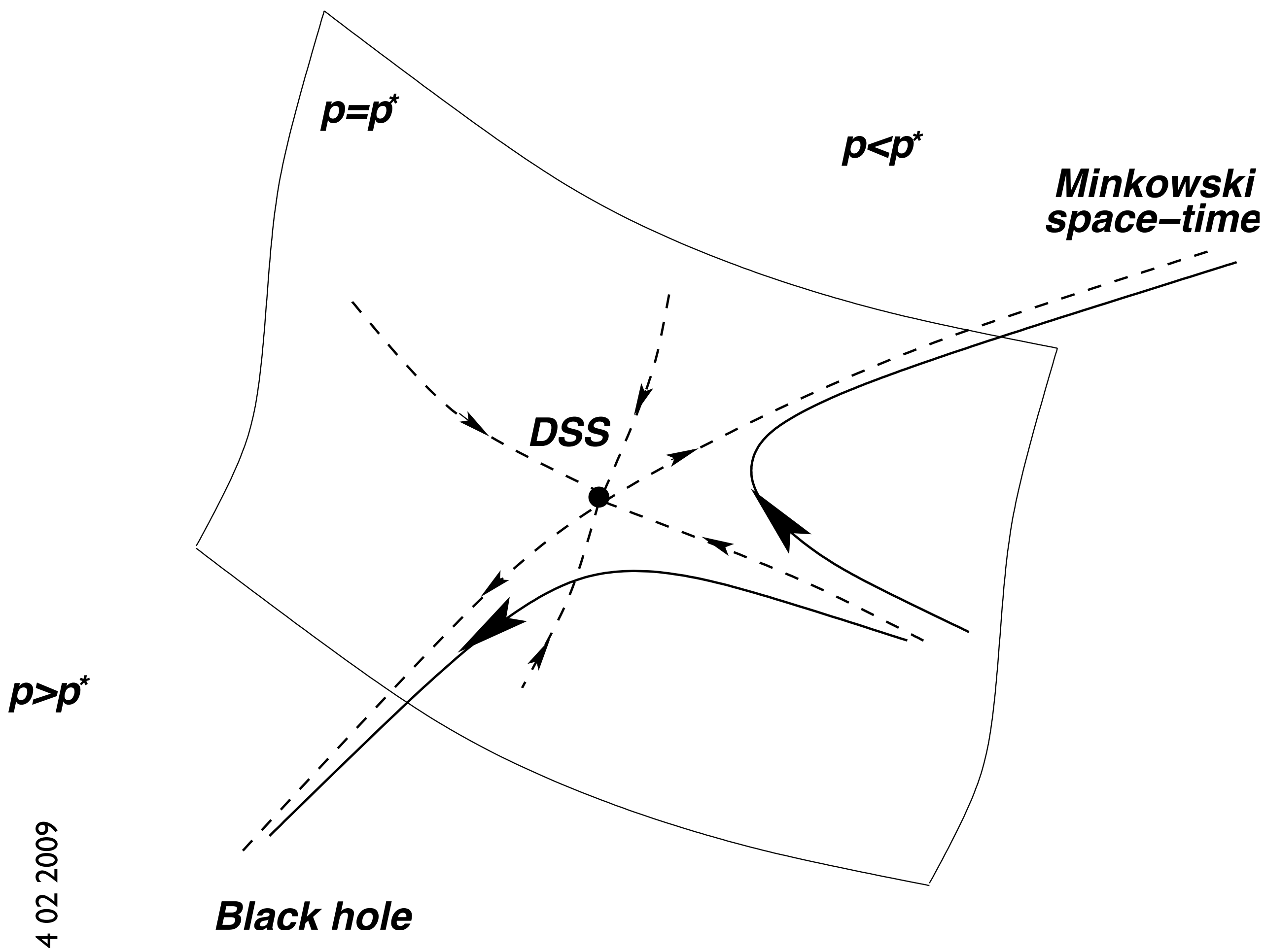
We believe that the Maldacena conjecture should be extended beyond supersymmetry

So far most scenarios explored have been static, i.e. thermal HP...

We want to find dynamical phenomena that could be related and also observable. We also considered criticality in the formation of trapped surfaces

Weakly coupled gauge theories vs strong curvature gravity

Critical behavior in phase space



Choptuik's (93) showed the existence of a co-dimension one critical surface.

For generic one parameter families of initial data, parameterized by p, there is a critical value p* where it crosses the critical surface.

There are two possible large time evolutions, or fixed points:

A BH forms with arbitrarily small mass

Or the system bounces and it is radiated away to infinity leaving behind flat space

The critical solution has an unstable mode, or relevant direction.

The eigenvalue of the relevant direction leads to the BH critical exponent.

Basic results

Spherical collapse, no gravitational radiation

$$ds^2 = -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2 d\Omega^2_{\mathrm{d}-2}$$

The critical solution is independent of the initial conditions. On the supercritical side, the size of the small BH satisfies a universal scaling law. The critical solution exhibits DSS:

$$r_{BH} \sim (p - p^*)^\gamma \qquad Z_*(e^{n\Delta}\, t, e^{n\Delta}\, r) = Z_*(t,r)$$

Numerics

Do a stability, Liapounov analysis

$$Z_p(\tau,\zeta) \approx Z_*(\tau,\zeta) + \sum_{k=1}^{\infty} C_k(p) e^{\lambda_k \tau} \delta_k Z(\tau,\zeta)$$

D	Δ	γ
4	$3.37 \pm 2\%$	$0.372 \pm 1\%$
5	$3.19 \pm 2\%$	$0.408 \pm 2\%$
6	$3.01 \pm 2\%$	$0.422 \pm 2\%$
7	$2.83 \pm 2\%$	$0.429 \pm 2\%$
8	$2.70 \pm 2\%$	$0.436 \pm 2\%$
9	$2.61 \pm 2\%$	$0.442 \pm 2\%$
10	$2.55 \pm 3\%$	$0.447 \pm 3\%$
11	$2.51 \pm 3\%$	$0.44 \pm 3\%$

$$\gamma = \frac{1}{\lambda_1}$$

Perfect fluid collapse

In the relevant scaling limit in YM, there is no echo parameter.

We want a similar symmetry in gravity. This is achieved by studying the collapse of perfect fluids.

$$ds^2 = -\alpha(t,r)^2 dt^2 + a(t,r)^2 dr^2 + R(t,r)^2 d\Omega_{d-2}^2$$

The critical solution will have CSS rather than DSS. A region of the space time before the singularity forms has homothety, i.e. a conformal Killing vector of weight 2.

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p\, g_{\mu\nu}$$

We choose comoving coordinates to describe the spherical collapse of the fluid. The equations are simpler.

$$p = k\,\rho, \qquad 0 \leq k \leq 1$$

Cahill-Taub, Bicknell-Henriksen, Coleman-Evans, Hara-Koike-Adachi, Harada-Maeda. We follow and complete these authors in any d

Equations of motion

Introducing the Hawking-Misner mass:

$$a^{-1} = 1 - \frac{2\,m}{r^{d-3}}$$

[illegible] $_{,r}$ = $\frac{16}{-2}$ [illegible]

[illegible] $_{,t}$ = $-\frac{16}{-2}$ [illegible]

[illegible] = $d-3$ $\left(1 + \ldots\right)$ [illegible]

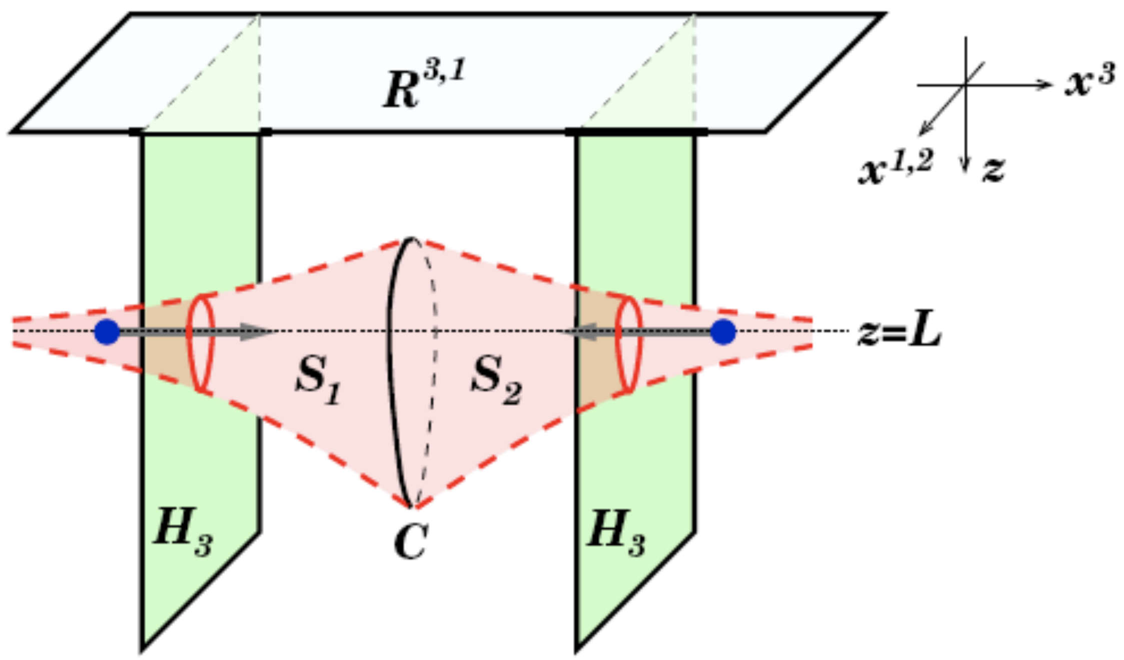
[illegible] + [illegible]

[illegible] $_{,t}$ = $-\frac{_{,t}}{+}$ [illegible]

All equations have a simple physical interpretation. The 4th one is related to the formation of a trapped surface R=const

Criticality in the formation of MTS

Recently, Gubser, Pufu and Yarom considered the formation of trapped surfaces in the head-on collision of gravitational shock waves in AdS to estimate the entropy of generated in the collision of HI's



We study the formation of MTS as a function of the spread of the energy density in transverse space. In D=4,5 we find critical behavior of Type I,II, but now in a control situation where we can be at weak gravity.

Some details

$$ds^2 = \frac{L^2}{z^2}\Big(- du dv + d\vec{x}_T^2 + dz^2 \Big) + \frac{L}{z} \Phi(z, \vec{x}_T)\, \delta(u)\, du^2.$$

$$T_{uu} = \rho(z, \vec{x}_T) \delta(u).$$

$$\left(\Box_{\mathbb{H}_{D-2}} - \frac{D-2}{L^2} \right) \Phi(z, \vec{x}_T) = -16 \pi G_N \frac{z}{L} \rho(z, \vec{x}_T)$$

$$q \equiv \frac{(z-L)^2 + \vec{x}_T^2}{4Lz}.$$

$$ds^2_{\mathbb{H}_{D-2}} = L^2 \left[\frac{dq^2}{q(q+1)} + 4q(q+1) d\Omega^2_{D-3} \right]$$

$$ds^2 = \frac{L^2}{z^2}\Big(- du dv + d\vec{x}_T^2 + dz^2 \Big) + \frac{L}{z} \Phi_1(z, \vec{x}_T) \theta(v) \delta(u) du^2 + \frac{L}{z} \Phi_2(z, \vec{x}_T) \theta(u) \delta(v) dv^2$$

Introducing the dilution parameter:

$$\overline{\rho}(q) = \frac{2(2L)^{2-D}}{\mathrm{Vol}(S^{D-3})} \frac{\mu}{[q(1+q)]^{\frac{D-4}{2}}} \delta(q-\epsilon).$$

$$\overline{\rho}(q) = \frac{2(2L)^{2-D}}{\mathrm{Vol}(S^{D-3})} \frac{\mu}{[q(1+q)]^{\frac{D-4}{2}}} F(\omega, q).$$

$$\int_0^\infty dq\, F(\omega, q) = 1.$$

$$\lim_{\omega \to 0+} F(\omega, q) = \delta(q-\epsilon)$$

The MTS condition becomes:

$$\frac{8\pi G_N \mu (2L)^{3-D}}{\mathrm{Vol}(S^{D-3})} \int_0^{qc} dq\, (1+2q) F(\omega, q) = (1+2qc)[qc(1+qc)]^{\frac{D-3}{2}}$$

AdS / Minkowski

D=4

AdS: There is a critical value in the dilution parameter, and the size of the trapped surface is finite. Like in Type-I phenomena. Furthermore the critical exponents is 1/2

Minkowski: There is a critical value in the dilution parameter, and the size of the trapped surface is finite. Like in Type-I phenomena. Furthermore the critical exponents is 1/2

D=5

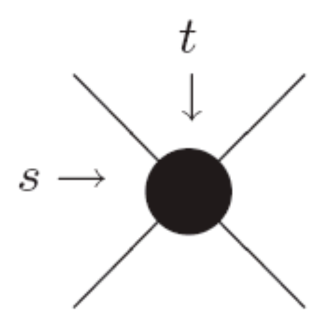
AdS: There is a critical value in the dilution parameter, and the size of the trapped surface is zero. Like in Type-II phenomena. Furthermore the critical exponents is 1

Minkowski: There is a critical value in the dilution parameter, and the size of the trapped surface is zero. Like in Type-II phenomena. Furthermore the critical exponents is 1/2

For higher dimensions there is always a solution for any dilution parameter

BFKL the Regge limit of YM

BFKL is an equation which describes the high-energy limit of weakly coupled YM



$$s >> |t|\,, Q^2$$
$$\alpha_s(Q^2) << 1$$

$$\text{Large } \ln s \text{ compensate small } \alpha_s\text{: } \alpha_s \ln s \sim 1 \left[\text{Balitsky–Fadin–Kuraev–Lipatov}\right]$$

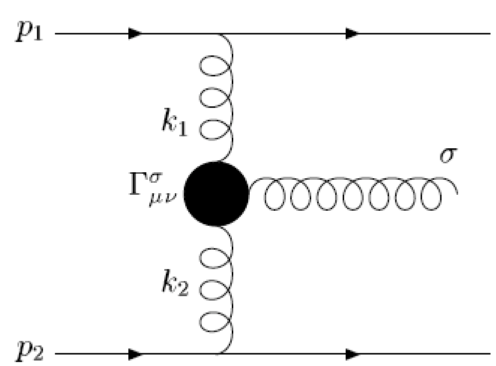
$$\mathcal{A}(s,t) \sim s^{\alpha(t)} \quad ; \quad s >> 1 \qquad \sigma^{\text{total}} \sim s^{\alpha(0)-1}$$

$$\alpha(0) = 1 + (4 \log 2)\, \alpha_s + \mathrm{O}[\alpha_s^2]$$

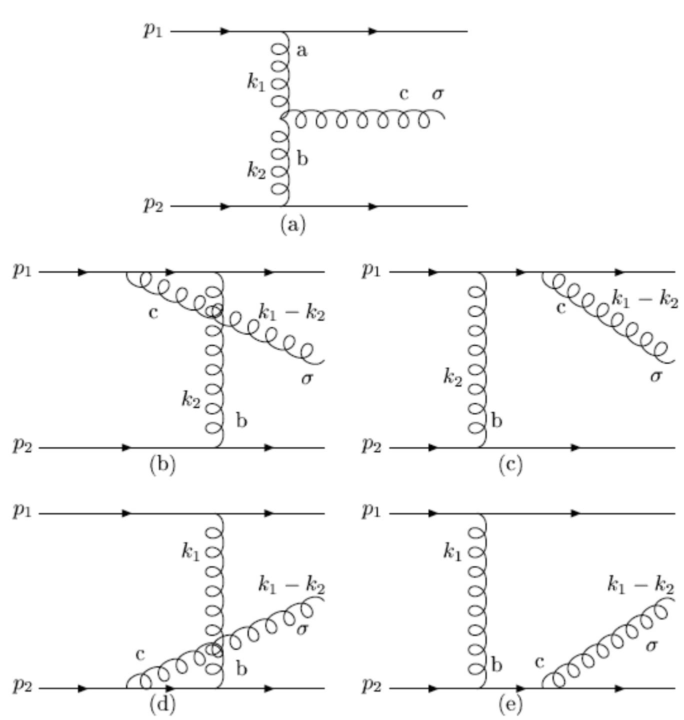
Unitarity violation, Froissart-Martin bound. We will be looking at weak coupling. Reggeization processes have also been studied at strong coupling by Brower, Polchinski, Strassler and Tan

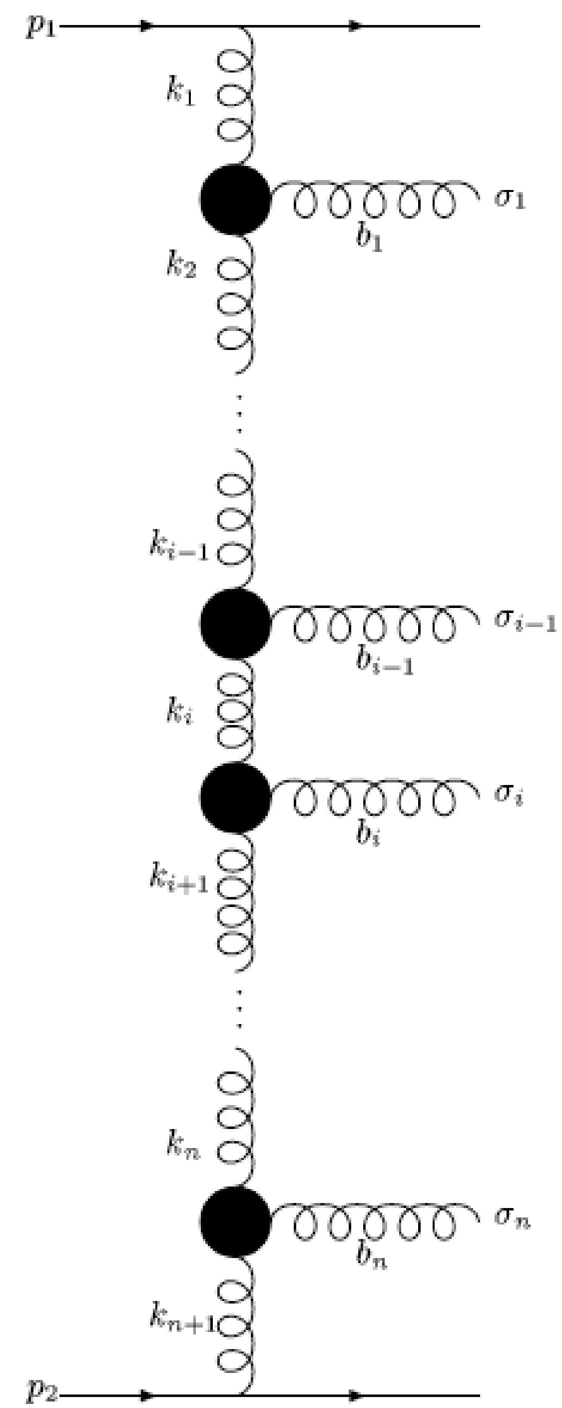
Dominants graphs in the Regge region

$k_1^+ \gg k_2^+,\ k_2^- \gg k_1^-$

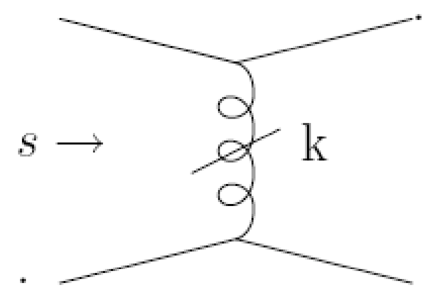


$$\Gamma^{\sigma}_{+-}(k_1,k_2) = 2gf^{abc}\left(k_1^+ + \frac{2\mathbf{k_1}^2}{k_2^-}, k_2^- + \frac{2\mathbf{k_2}^2}{k_1^+}, -(\mathbf{k_1}+\mathbf{k_2})\right)$$





Reggeized gluon

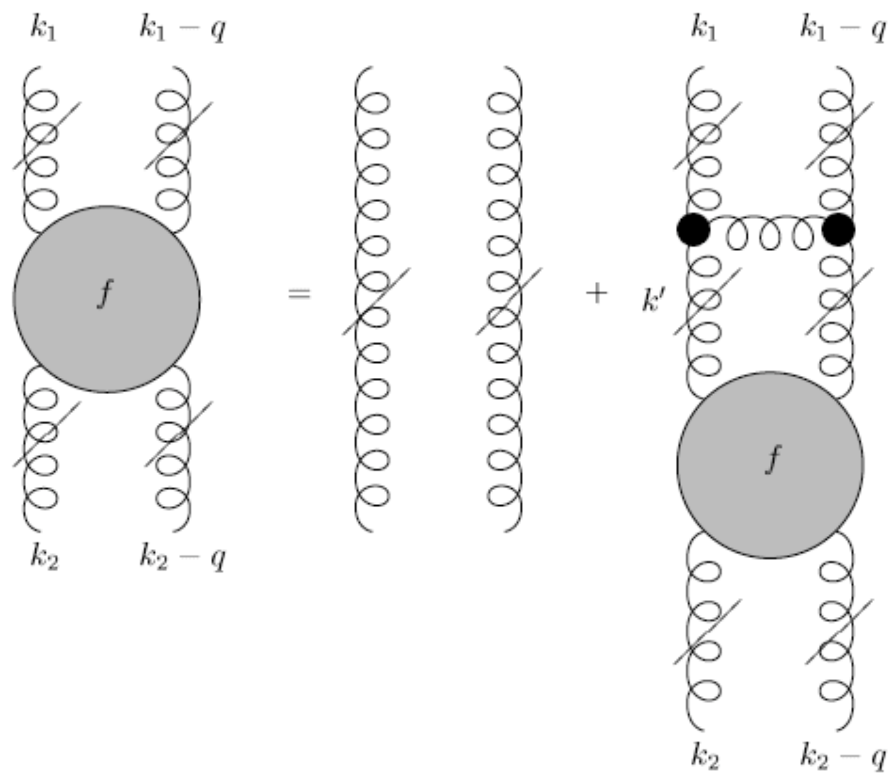


$$\frac{1}{\mathbf{k}^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\varepsilon_G(\mathbf{k}^2)}$$

$$\epsilon_G(\mathbf{q}^2) \;=\; -\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}\,\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

Some details

Like a Bethe-Salpeter equation at leading log



This is similar to integrating out the fast, longitudinal degrees of freedom and working with the effective transverse hamiltonian.

This hamiltonian exhibits scale (SL(2,C)) invariance

$$f(\sqrt{s},\mathbf{k_1},\mathbf{k_2}) = \delta^2(\mathbf{k_1}-\mathbf{k_2}) - \frac{\alpha C_A}{2\pi^2}\int d^2\mathbf{k}' \int^{\sqrt{s}} dk^+ f(k^+,\mathbf{k}',\mathbf{k_2})$$

$$\times \left(\frac{k_1^+}{k^{+\prime}}\right)^{2\varepsilon_G(\mathbf{k}'^2)} \frac{\Gamma^{\sigma}_{+-}(k_1,k')\Gamma^{\sigma}_{+-}(k_1,k')}{\mathbf{k}'^4}$$

Eigenfunctions and eigenvalues

Eigenfunctions:, $\phi_{n,\nu}(\mathbf{k}) = (k^2)^{-1/2+i\nu} e^{in\theta}$

Eigenvalues: $\frac{\alpha_s C_A}{\pi}\chi_n(\nu)$

$$\chi(\nu) = 2\Psi(1) - \Psi\left(\frac{(n+1)}{2} + i\nu\right) - \Psi\left(\frac{(n+1)}{2} - i\nu\right)$$

General solution $(\mathbf{k} = (k, \theta))$

$$\tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2 \mathbf{k_1}\mathbf{k_2}} \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{i\nu} \frac{e^{in(\theta_1-\theta_2)}}{\omega - \overline{\alpha_s}\chi_n(\nu)}$$

where

$$\overline{\alpha_s} \equiv \frac{\alpha_s C_A}{\pi}$$

Unitarization, energy conservation

BFKL is valid up to a “saturation scale’’ beyond which nonlinear effects from overlapping wave function of gluons and partons cannot be neglected.

BFKL can be modified to introduce nonlinear effects to restore unitarity. These lead to the saturation phenomena, easier to explain in terms of pictures. This is the BK behavior of the gluon distribution function. We also need to worry about energy conservation.

One form of BK

$$\frac{\partial\Phi\left(k_A,k_B,\mathrm{Y}\right)}{\partial(\bar{\alpha}_s\mathrm{Y})} = -\,\Phi\left(k_A,k_B,\mathrm{Y}\right)^2$$

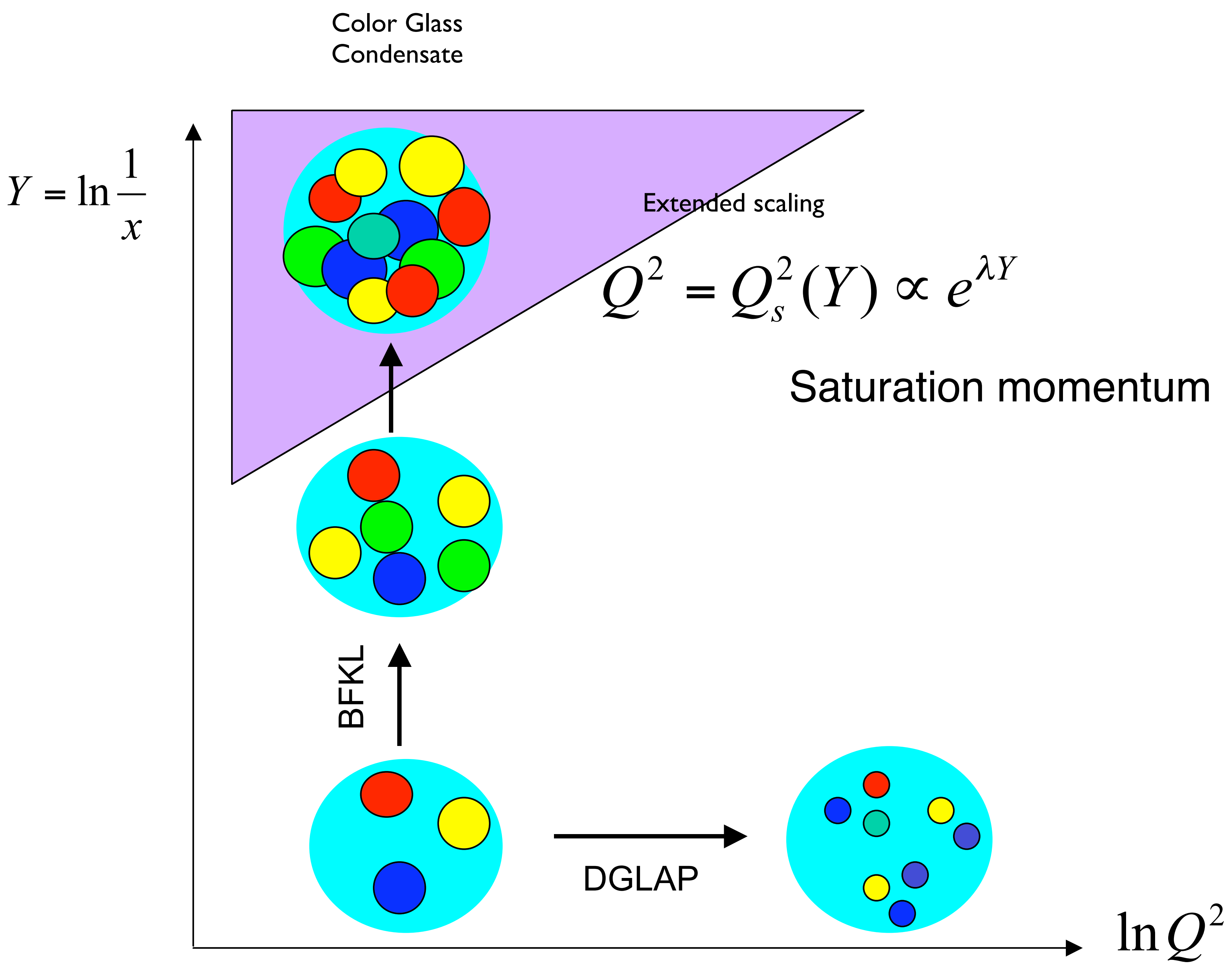
$$+\int_0^1\frac{dx}{1-x}\left[\Phi\left(\sqrt{x}k_A,k_B,\mathrm{Y}\right)+\frac{1}{x}\Phi\left(\frac{k_A}{\sqrt{x}},k_B,\mathrm{Y}\right)-2\Phi\left(k_A,k_B,\mathrm{Y}\right)\right],$$

With the initial condition:

$$\Phi\left(k_A,k_B,\mathrm{Y}=0\right) = \frac{1}{\pi}\int\frac{d\gamma}{2\pi i}\left(\frac{k_A^2}{k_B^2}\right)^{\gamma-\frac{1}{2}}.$$

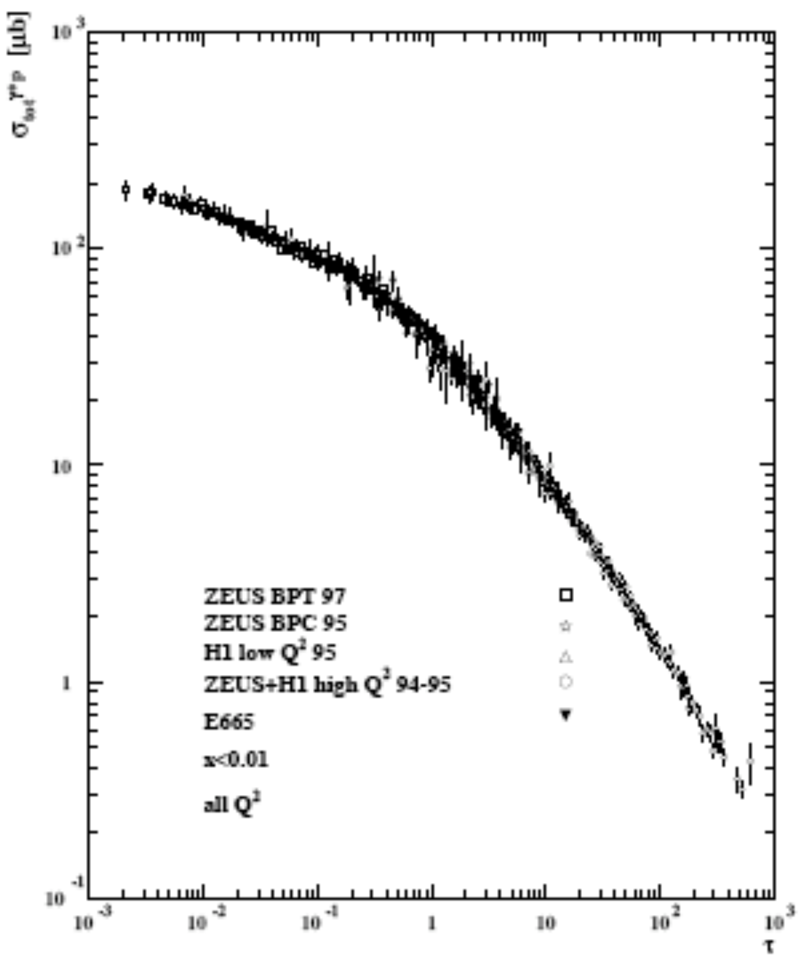
The two kernels are related by simple scaling factors that we do not include here, and we have introduced the rapidity and a different way of writing the integral transform. To be precise, this is the large N limit, or the Hartree-Fock approximation of BK. Otherwise we get an infinite hierarchy like the BBGKY hierarchy in kinetic theory

Saturation in QCD



Geometric scaling

$\sigma^{tot}_{|\gamma^* p}(Y,Q) = \sigma^{tot}_{|\gamma^* p}(\tau)$; $\tau = \dfrac{Q^2}{Q_s^2(Y)} = Q^2 x^{\lambda_s}$ HERA data for $\sigma_{\gamma^* p}$ with $x < 0.01$ versus τ



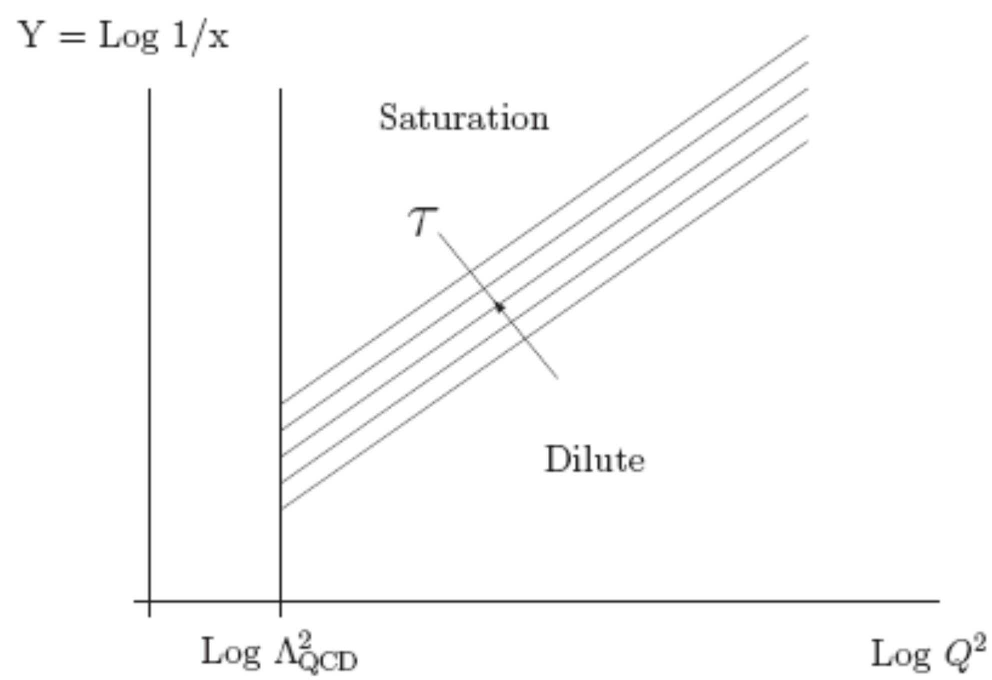
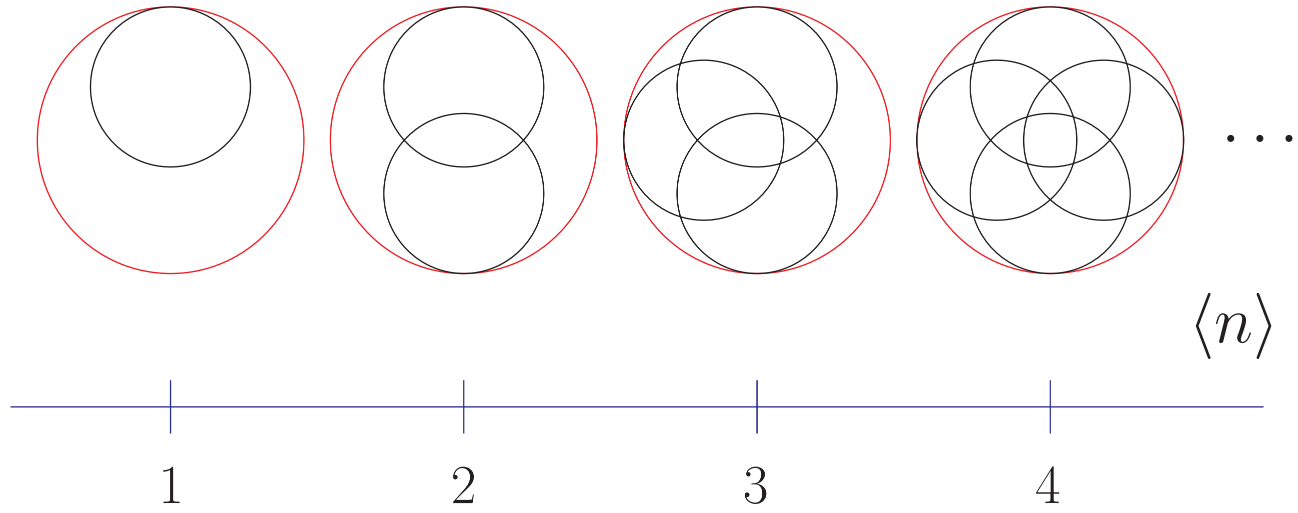
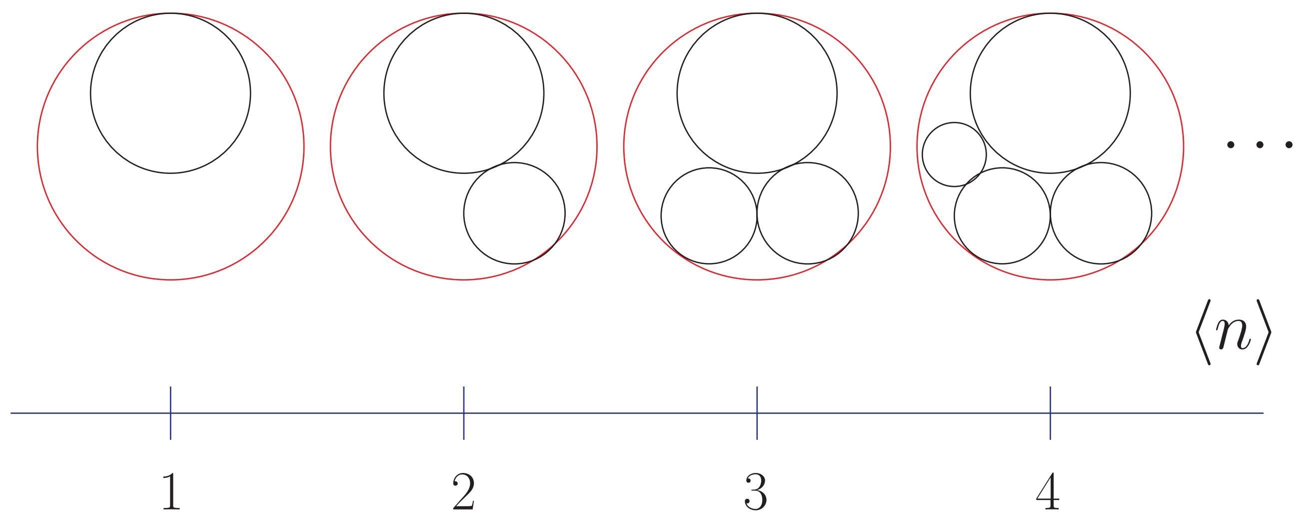


Fig. 5: Continuous self–similarity on the $(Y, \log Q^2)$ plane.

A pictorial description





Dilute to dense transition, with a fractal dim equal to the BFKL exponent

The Commutant of M_{+-}

High energy collisions in the 3rd direction. The BFKL conformal symmetry is the part of the conformal group commuting with the boost generator

$$
\begin{aligned}
[P_1, P_2] &= [K_1, K_2] = [D, M_{12}] = 0 \\
[D, P_{1,2}] &= -i\,P_{1,2} \qquad [D, K_{1,2}] = +i\,K_{1,2} \\
[M_{12}, P_1] &= -iP_2 \qquad [M_{12}, P_2] = +iP_1 \\
[M_{12}, K_1] &= -iK_2 \qquad [M_{12}, K_2] = +iK_1 \\
[P_1, K_1] &= [P_2, K_2] = -2iD \\
[P_1, K_2] &= 2iM_{12}, \qquad [P_2, K_1] = -2iM_{12}
\end{aligned}
$$

$$
\begin{aligned}
&(\,P_1 + iP_2,\ K_1 - iK_2,\ iD - M_{12}\,) \\
&(\,P_1 - iP_2,\ K_1 + iK_2,\ iD + M_{12}\,)
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
[iD - M_{12}, P_1 + iP_2] &= +2\,(P_1 + iP_2) \\
[iD - M_{12}, K_1 - iK_2] &= -2\,(K_1 - iK_2) \\
[P_1 + iP_2, K_1 - iK_2] &= -4\,(iD - M_{12})
\end{aligned}
}
$$

Exotic form of BFKL

Integrate out the longitudinal momenta

$$\begin{aligned} H_{jk} &= P_j^{-1} \log z_{jk} P_j + P_k^{-1} \log z_{jk} P_k + \log P_j P_k + 2\gamma_E \\ &= 2 \log z_{jk} + z_{jk} \log P_j P_k \frac{1}{z_{jk}} + 2\gamma_E, \\ &= \sum_{l \geq 0} \left(\frac{2l+1}{l(l+1) - M_{jk}^2} - \frac{2}{l+1} \right) \end{aligned}$$

Scaling limit at any coupling

$$x^\alpha \to \lambda\, x^\alpha \qquad s' = \lambda\, s \qquad \lambda \sim \frac{1}{\sqrt{s}} \to 0$$

The principal series of SL(2,C) gives the correct eigenvalues in the weak coupling limit, but it should also provide the correct eigenfunctions in the general case

Farewell

Shock wave collisions on the boundary should be related by HRG to phenomena similar to the collision of gravitational waves in the bulk. The understanding of what unitarises one, should allow us to understand what unitarizes the other in detail

RHIC, or LHC may explore the BFKL region where interesting BH phenomena should be directly related to measurable quantities (dream)

THANK YOU

CSS conditions

Scaling variables

$$\tau = -\log(-t), \qquad z = -\frac{r}{t}$$

$$\begin{aligned} \eta(\tau, z) &= 8\pi r^2 \rho(t, r), \\ S(\tau, z) &= \frac{R(t, r)}{r}, \\ m(t, r) &= r^{d-3} M(t, r), \end{aligned}$$

Equations of motion

$$\begin{aligned} \frac{d\log M}{d\log z} &= \frac{(d-3)k}{k+1}\left(\frac{1}{y} - 1\right) \\ \frac{d\log S}{d\log z} &= \frac{1}{k+1}(y-1) \\ \frac{d\log \eta}{d\log z} &= \frac{1}{V_z^2 - k}\left[\frac{(1+k)^2}{d-2}\eta^{\frac{k-1}{k+1}} S^{4-2d} - (d-2)(y-1)V_z^2 - 2k\right] \end{aligned}$$

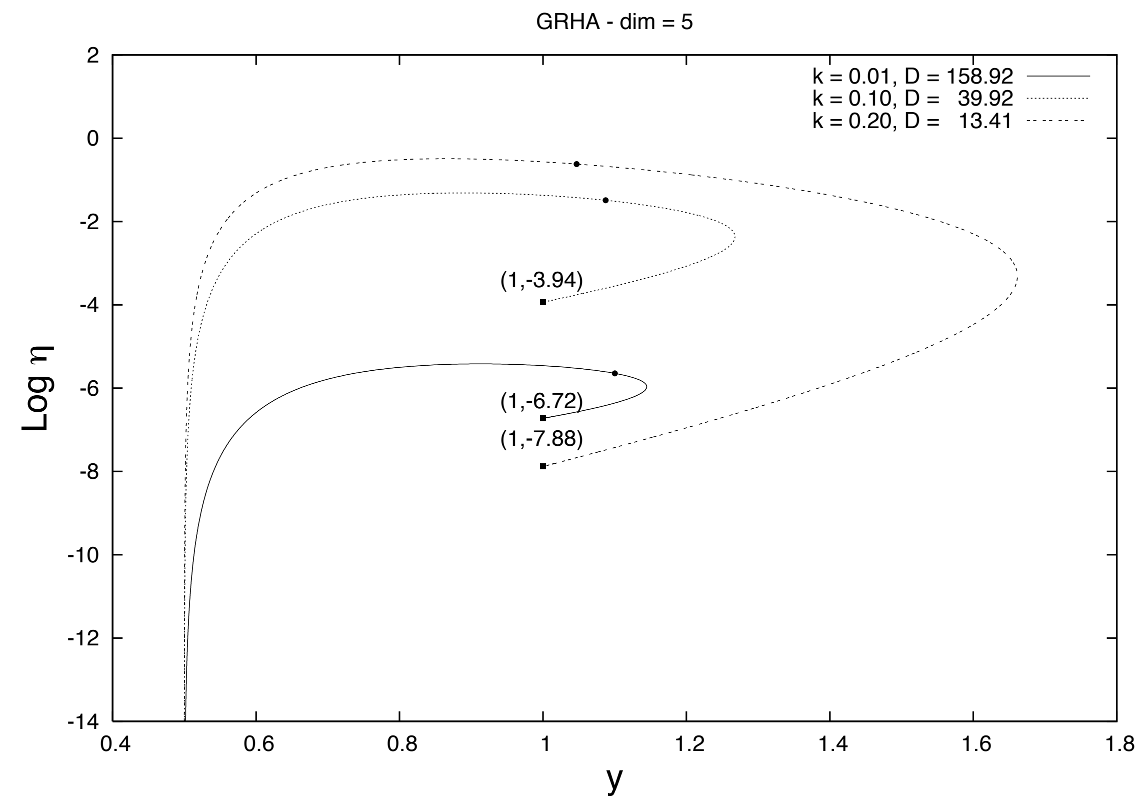
Conditions at the origin

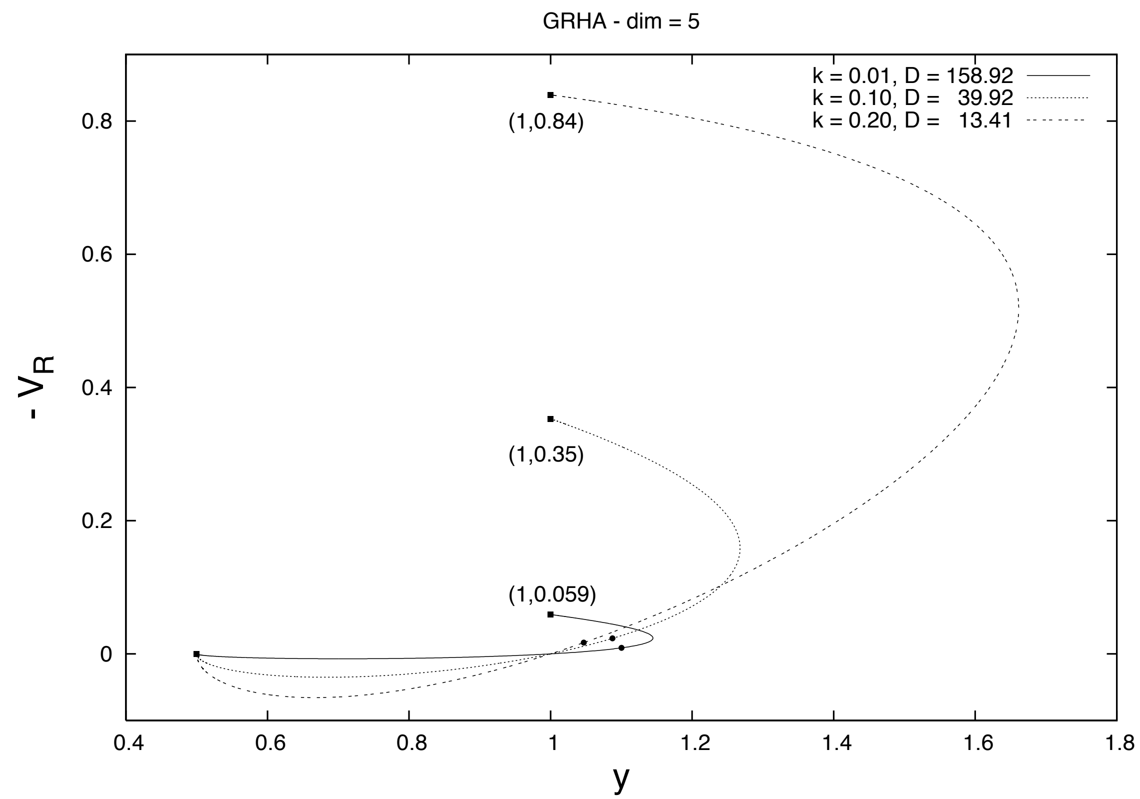
$$y(0^+) = \frac{d-3}{d-1}$$

$$\begin{aligned} M(z) &\simeq \frac{(2D)^{\frac{k}{k+1}}}{(d-2)}\left[\frac{k+1}{(d-1)k + d - 3}\right] z^{\frac{2k}{k+1}} \\ S(z) &\simeq \left[\frac{(2D)^{\frac{1}{k+1}}}{k+1}\left(k + \frac{d-3}{d-1}\right)\right]^{\frac{1}{1-d}} z^{-\frac{2}{(d-1)(k+1)}} \end{aligned}$$

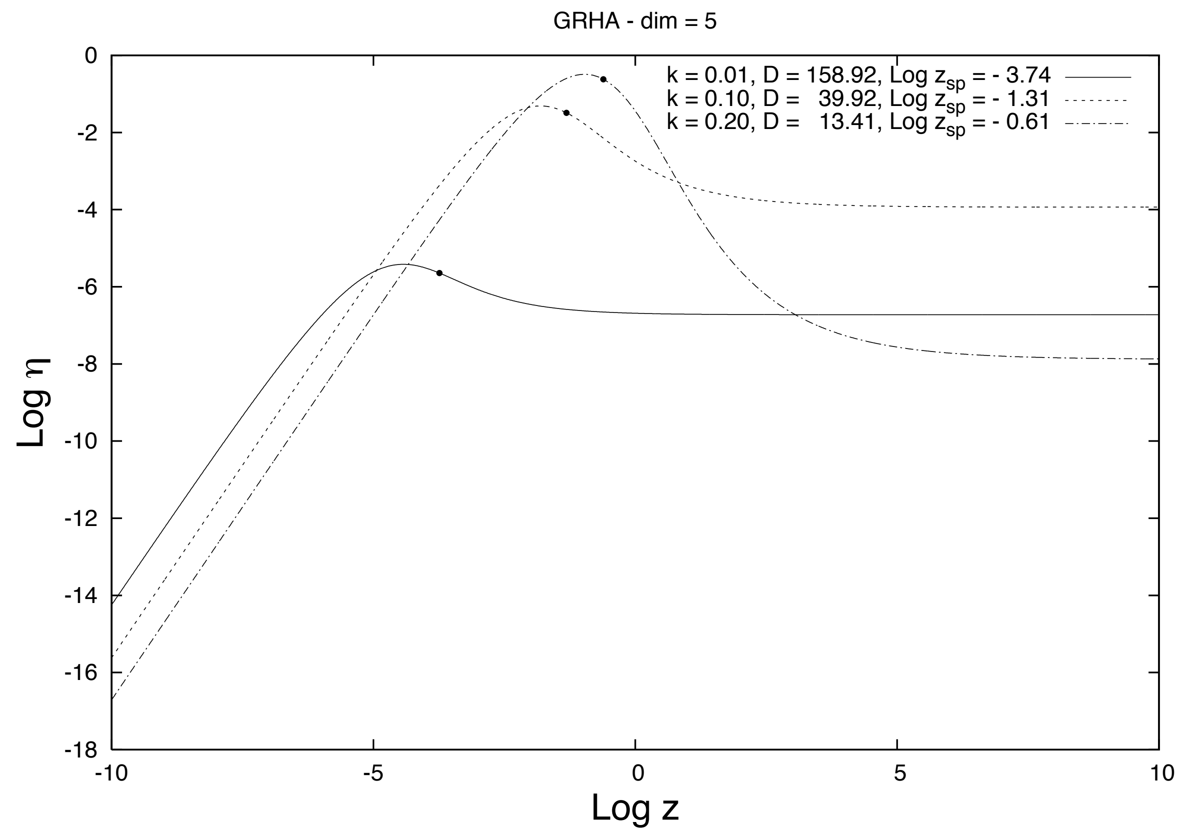
Only a discrete set of values for D are smooth at the sonic point (surface), related to the number of growing perturbations

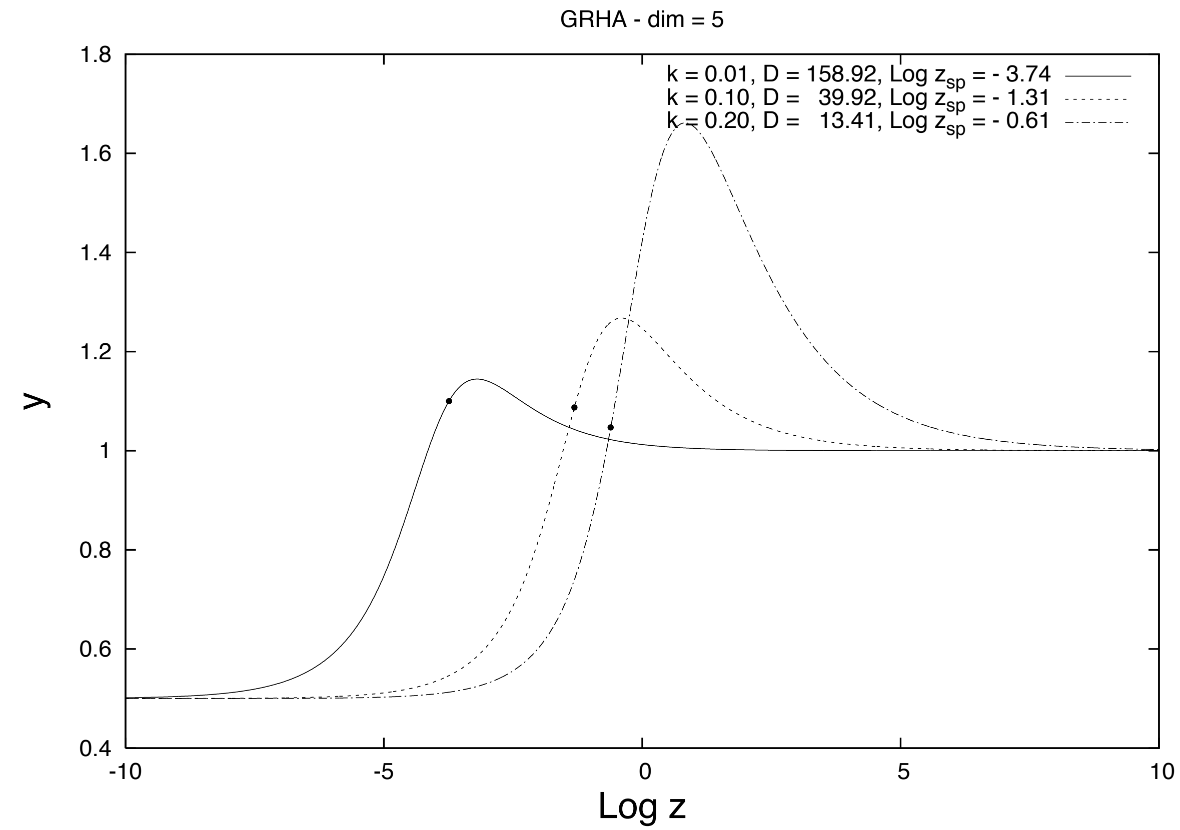
Like critical damping





Sonic point location





Choptuik exponents

k	$\lambda_{d=4}$	$\lambda_{d=5}$	$\lambda_{d=6}$	$\lambda_{d=7}$
0.01	8.747	4.435	3.453	3.026
0.02	8.140	4.288	3.376	2.974
0.03	7.617	4.152	3.302	2.924
0.04	7.163	4.027	3.233	2.876
0.05	6.764	3.911	3.169	2.831
0.06	6.412	3.804	3.107	2.788
0.07	6.099	3.703	3.049	2.746
0.08	5.818	3.609	2.993	2.706
0.09	5.565	3.521	2.940	2.668
0.10	5.334	3.438	2.890	2.631
0.11	5.124	3.360	2.841	2.595
0.12	4.932	3.286	2.795	2.561
0.13	4.756	3.216	2.751	2.527
0.14	4.593	3.149	2.708	2.494
0.15	4.442	3.086	2.667	2.464
0.16	4.301	3.026	2.627	2.433
0.17	4.170	2.968	2.589	2.414
0.18	4.048	2.913	2.552	2.377
0.19	3.933	2.860	2.517	2.348
0.20	3.825	2.809	2.482	2.321
0.21	3.723	2.760	2.449	2.297
0.22	3.627	2.713	2.417	2.272
0.23	3.536	2.668	2.386	2.246
0.24	3.449	2.625	2.355	2.224
0.25	3.367	2.583	2.325	2.202

k	$\gamma_{d=4}$	$\gamma_{d=5}$	$\gamma_{d=6}$	$\gamma_{d=7}$
0.01	0.114	0.225	0.290	0.330
0.02	0.123	0.233	0.296	0.336
0.03	0.131	0.241	0.303	0.342
0.04	0.140	0.248	0.309	0.348
0.05	0.148	0.256	0.316	0.353
0.06	0.156	0.263	0.322	0.359
0.07	0.164	0.270	0.328	0.364
0.08	0.172	0.277	0.334	0.369
0.09	0.180	0.284	0.340	0.375
0.10	0.187	0.291	0.346	0.380
0.11	0.195	0.298	0.352	0.385
0.12	0.203	0.304	0.358	0.390
0.13	0.210	0.311	0.364	0.396
0.14	0.218	0.318	0.369	0.401
0.15	0.225	0.324	0.375	0.406
0.16	0.232	0.330	0.381	0.411
0.17	0.240	0.337	0.386	0.416
0.18	0.247	0.343	0.392	0.421
0.19	0.254	0.347	0.397	0.426
0.20	0.261	0.356	0.403	0.431
0.21	0.259	0.362	0.408	0.435
0.22	0.276	0.368	0.414	0.440
0.23	0.283	0.375	0.419	0.445
0.24	0.290	0.381	0.425	0.450
0.25	0.297	0.387	0.430	0.454

Table 1: Values of the Lyapunov exponent (left) and the Choptuik exponent (right) as a function of k for d =4, 5, 6 and 7. All numbers are calculated with a precision ± 0.001.