THE FOUR DERIVATIVE BLG ACTION

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- Based on ongoing work with Sunil Mukhi and Costis Papageorgakis

Recently Bagger, Lambert and independently Gustavsson constructed a $\mathcal{N} = 8$ Superconformal action in 3D

$$\mathcal{L}_{BLG} = -\frac{1}{2}D_\mu X^{a(I)} D^\mu X_a^{(I)} + \frac{i}{2}\bar{\Psi}^a \Gamma^\mu D_\mu \Psi_a + \frac{i}{4} f_{abcd} \bar{\Psi}^b \Gamma^{IJ} X^{c(I)} X^{d(J)} \Psi^a$$
$$\frac{-1}{12}(f_{abcd} X^{a(I)} X^{b(J)} X^{c(K)})(f_{efg}{}^{d} X^{e(I)} X^{f(J)} X^{g(K)})$$
$$+\frac{1}{2}\epsilon^{\mu\nu\rho}(f_{abcd} A_\mu^{ab} \partial_\nu A_\lambda^{cd} + \frac{2}{3} f_{aef}{}^{g} f_{bcdg} A_\mu^{ab} A_\nu^{cd} A_\rho^{ef}) \quad (1)$$

The f^{abcd} which appears in the interaction terms of the lagrangian, both bosonic and fermionic is completely antisymmetric in its indices and satisfies an identity, which is named the fundamental identity, which comes out of demanding closure of the supersymmetry algebra.

$$f^{[abc}{}_{g} f^{e]fg}{}_{d} = 0 \quad (2)$$

The Lagrangian and interactions are completely specified by the form of f^{abcd}. The only solution of this identity turned out to be $f^{abcd} = f\epsilon^{abcd}$ with $f = \frac{2\pi}{k}$, where k is the quantized Chern-Simons level.

Later this Lagrangian was reformulated as a $SU(2) \times SU(2)$ Chern-Simons Matter theory, by Van Raamsdonk :

$$\begin{aligned}
\mathcal{L}_{BLG} = \quad & \frac{k}{2\pi}\mathrm{Tr}[- (\tilde{D}_\mu X^I)^\dagger \tilde{D}_\mu X^I + i\bar{\Psi}^\dagger \Gamma^\mu \tilde{D}_\mu \Psi - \frac{8}{3} X^{IJK\dagger} X^{IKJ} \\
& - i\bar{\Psi}^\dagger \Gamma_{IJ}[X^I, X^{J\dagger}, \Psi] + i\bar{\Psi}\Gamma_{IJ}[X^{I\dagger}, X^J, \Psi^\dagger] \\
& + \frac{1}{2}\epsilon^{\mu\nu\lambda}(A^{(1)}{}_\mu \partial_\nu A^{(1)}{}_\lambda + \frac{2}{3} A^{(1)}{}_\mu A^{(1)}{}_\nu A^{(1)}{}_\lambda) \\
& - \frac{1}{2}\epsilon^{\mu\nu\lambda}(A^{(2)}{}_\mu \partial_\nu A^{(2)}{}_\lambda + \frac{2}{3} A^{(2)}{}_\mu A^{(2)}{}_\nu A^{(2)}{}_\lambda)]
\end{aligned} \tag{3}$$

where the matter fields are complex valued, transforming in the bi-fundamental representation $(2, \bar{2})$ of the gauge group, and obey the following reality condition:

$$\begin{aligned}
X_{a\dot{b}} &= \epsilon_{ab}\epsilon_{\dot{b}\dot{a}} X^{\dagger a\dot{b}} \\
\text{while } X^{IJK} &= X^{[I} X^J X^{K]} \\
\text{and } [X^I, X^{J\dagger}, \Psi] &= \frac{1}{3}(X^{[I} X^{J]\dagger}\Psi - X^{[I}\Psi^\dagger X^{J]} + \Psi X^{[I\dagger} X^{J]} \\
\text{also } \tilde{D}_\mu X^I &= \partial_\mu X^I + A^{(1)}_\mu X^I - X^I A^{(2)}_\mu
\end{aligned} \tag{4}$$

In this way of writing the BLG action, the ϵ^{abcd} structure is encoded in the way the interactions occur, namely through X^{IJK} and $[X^I, X^{J\dagger}, \Psi]$

Soon after the BLG action appeared, Mukhi-Papageorgakis, showed a connection between this BLG action and the action of (2+1 D) $\mathcal{N} = 8$ SYM. Giving a vev (v) to one of the scalar fields and expanding around it,

$$
\begin{aligned}
X^8 &= \frac{1}{2}(v + \tilde{x}^8) + \mathbf{x}^8 \\
X^i &= \frac{1}{2}\tilde{x}^i + \mathbf{x}^i
\end{aligned}
$$

where $\tilde{x}$ and $\mathbf{x}$ are the trace and traceless fluctuations of the field. Upon this substitution, one linear combination of the two gauge fields becomes massive. Upon solving and substituting for the eom of this linear combination of the gauge fields, the other linear combination becomes dynamical and one ends up the SYM action with coupling $g^2_{YM} = \frac{2\pi v^2}{k}$, in the limit where v, k are large such that g^2_{YM} is finite.

$$
\mathcal{L}_{SYM} = \frac{k}{2\pi v^2}[\,\mathrm{Tr}(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + D^\mu X^i D_\mu X^i + \frac{1}{2}[X^i, X^j][X^i, X^j])
$$

$$+\mathrm{Tr}(-i\bar{\Psi}'\Gamma^{\mu}D_{\mu}\Psi^{'} - i\bar{\Psi}'\Gamma^{i}[X^{i},\Psi])\,] + \mathrm{O}(1/v)\ . \qquad (5)$$

with the identification:

$$\Psi^{'} = \frac{1}{\sqrt{2}}(1+\Gamma^{8})\Psi \qquad (6)$$

Later, a geometric interpretation of this result was given in terms of a compactification of M2 branes on an orbifold and in this interpretation the vev was simply the distance from the origin of the orbifold. In this picture, as one moves the M2 branes far away from the orbifold fixed point one is effecively compactifying over a cylinder and so should get back a D2 brane, whose low energy limit is the SYM action. However a moduli space analysis of the BLG theory showed that it cannot be describing M2 branes in a conventional orbifold. So its not clear how relevant such an interpretation is for the BLG theory.

Nevertheless the connection between the BLG and SYM is interesting, and a natural question is wheter one can make this connection at the level of the full DBI action of which SYM is a low energy truncation, independent of the relation of BLG theory to M2 branes.

In particular the first $O(\alpha'^2)$ correction to the SYM has been worked out by Tsetyln, Bergshoeff-Bilal-de Roo-Sevrin, Cederwall-Nielsson-Tsimpis using symmetrised trace(STr) and its form is given below.

$$\begin{aligned}\mathcal{L}_b \;=\; & \frac{\alpha'^2}{g_{YM}^2}\mathrm{STr}[\frac{1}{4}F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} - \frac{1}{16}F^{\mu\nu}F_{\mu\nu}F^{\rho\sigma}F_{\rho\sigma} - \frac{1}{4}D_\mu X^i D^\mu X^i D_\nu X^j D^\nu X^j \\ & +\frac{1}{2}D_\mu X^i D^\nu X^i D_\nu X^j D^\mu X^j + \frac{1}{4}X^{ij}X^{jk}X^{kl}X^{li} - \frac{1}{16}X^{ij}X^{ij}X^{kl}X^{kl} \\ & -F_{\mu\nu}F^{\nu\rho}D_\rho X^i D^\mu X^i - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}D_\rho X^i D^\rho X^i - \frac{1}{8}F_{\mu\nu}F^{\mu\nu}X^{kl}X^{kl} \\ & -\frac{1}{4}D_\mu X^i D^\mu X^i X^{kl}X^{kl} - X^{ij}X^{jk}D^\mu X^k D_\mu X^i - F_{\mu\nu}D^\nu X^i D^\mu X^j X^{ij}]\,, \qquad (7)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_f = \; & \frac{\alpha'^2}{g_{YM}^2}\mathrm{STr}(-\frac{1}{8}\bar\Psi\Gamma^\mu D^\nu\Psi\;\bar\Psi\Gamma_\nu D_\mu\Psi - \frac{1}{4}\bar\Psi\Gamma^i D^\nu\Psi\;\bar\Psi\Gamma_\nu[X^i,\Psi] \\ & -\frac{1}{8}\bar\Psi\Gamma^i[X^j,\Psi]\;\bar\Psi\Gamma^j[X^i,\Psi] + \frac{i}{2}\bar\Psi\Gamma_\mu D^\nu\Psi F^{\mu\rho}F_{\rho\nu} - \frac{i}{2}\bar\Psi\Gamma_\mu D^\nu\Psi D^\mu X^l D_\nu X^l\end{aligned}$$

$$
\begin{aligned}
&-\frac{i}{2}\bar{\Psi}\Gamma^i D^\nu\Psi\ D^\rho X^i F_{\rho\nu} - \frac{i}{2}\bar{\Psi}\Gamma^i D^\nu\Psi X^{il} D_\nu X^l + \frac{i}{2}\bar{\Psi}\Gamma_\mu [X^j,\Psi] F^{\mu\rho} D_\rho X^j \\
&-\frac{i}{2}\bar{\Psi}\Gamma^i [X^j,\Psi] D^\rho X^i D_\rho X^j + \frac{i}{2}\bar{\Psi}\Gamma_\mu [X^j,\Psi] D^\mu X^l X^{lj} + \frac{i}{2}\bar{\Psi}\Gamma^i [X^j,\Psi] X^{il} X^{lj} \\
&-\frac{i}{4}\bar{\Psi}\Gamma_{\mu\nu\rho} D_\sigma\Psi F^{\mu\nu} F^{\rho\sigma} - \frac{i}{4}\bar{\Psi}\Gamma_{\mu\nu\rho}[X^k,\Psi] F^{\mu\nu} D^\rho X^k \\
&+\frac{i}{4}\bar{\Psi}\Gamma_{\mu\nu l} D_\sigma\Psi F^{\mu\nu} D^\sigma X^l - \frac{i}{4}\bar{\Psi}\Gamma_{\mu\nu l}[X^k,\Psi] F^{\mu\nu} X^{lk} \\
&-\frac{i}{2}\bar{\Psi}\Gamma_{\mu j\rho} D_\sigma\Psi D^\mu X^j F^{\rho\sigma} - \frac{i}{2}\bar{\Psi}\Gamma_{\mu j\rho}[X^k,\Psi]\ D^\mu X^j D^\rho X^k \\
&+\frac{i}{2}\bar{\Psi}\Gamma_{\mu jl} D_\sigma\Psi\ D^\mu X^j D^\sigma X^l - \frac{i}{2}\bar{\Psi}\Gamma_{\mu jl}[X^k,\Psi] D^\mu X^j X^{lk} \\
&-\frac{i}{4}\bar{\Psi}\Gamma_{ij\rho} D_\sigma\Psi X^{ij} F^{\rho\sigma} - \frac{i}{4}\bar{\Psi}\Gamma_{ij\rho}[X^k,\Psi] X^{ij} D^\rho X^k \\
&+\frac{i}{4}\bar{\Psi}\Gamma_{ijl} D_\sigma\Psi X^{ij} D^\sigma X^l - \frac{i}{4}\bar{\Psi}\Gamma_{ijl}[X^k,\Psi] X^{ij} X^{lk})
\end{aligned}
\tag{8}
$$

So the question one is asking is wheter there exists a BLG 4-Derivative action, by which one means an action with four derivative terms and all dimension six bosonic and fermionic interaction terms constructed out of the BLG structures X^{IJK} and $[X^I, X^{J\dagger}, \Psi]$ and which on higgsing gives rise to the first derivative corrections to SYM written above.

The main result is that such an action can indeed be constructed and in fact will turn out to be **unique**, once one constrains it to reduce to SYM + $O(\alpha'^2)$ correction on higgsing.

In proving the uniqueness, one has to use the properties of the 3-algebra structure to relate various potential terms that could have appeared in the final action. In the case of the BLG theory where the $f^{abcd} = \epsilon^{abcd}$, the useful property is simply the identity:

$$\epsilon^{abcd}\epsilon^{efgh} = \epsilon^{abch}\epsilon^{efgd} + \epsilon^{abhd}\epsilon^{efgc} + \epsilon^{ahcd}\epsilon^{efgb} + \epsilon^{hbcd}\epsilon^{efga}$$

The form of this 4 derivative action is given below:

$$
\begin{aligned}
\mathcal{L}^{b}_{4D} = \quad & \frac{k^2 l_{pl}^3}{4\pi^2}\mathrm{STr}\Big[\frac{1}{2}(\tilde{D}_\mu X^I \tilde{D}^\mu X^{J\dagger} \tilde{D}^\nu X^J \tilde{D}_\nu X^{I\dagger}) - \frac{1}{4}(\tilde{D}_\mu X^I \tilde{D}^\mu X^{I\dagger} \tilde{D}^\nu X^J \tilde{D}_\nu X^{J\dagger}) \\
& + \frac{16}{9} X^{IJK} X^{IKJ\dagger} X^{LNP} X^{PNL\dagger} \\
& + \frac{4}{3} X^{IJK} X^{IKJ\dagger} \tilde{D}^\mu X^L \tilde{D}_\mu X^{L\dagger} - 8 X^{IJK} X^{LKJ\dagger} \tilde{D}^\mu X^L \tilde{D}_\mu X^{I\dagger} \\
& - \frac{2}{3}\epsilon^{\mu\nu\lambda}(\tilde{D}_\mu X^{I\dagger} \tilde{D}_\nu X^J \tilde{D}_\lambda X^{K\dagger} X^{IJK} + c\tilde{D}_\mu X^I \tilde{D}_\nu X^{J\dagger} \tilde{D}_\lambda X^K X^{IJK\dagger})\Big] \quad (9)
\end{aligned}
$$

and the fermionic terms:

$$
\begin{aligned}
\mathcal{L}^{f}_{4D} = \quad & \frac{k^2 l_p^3}{4\pi^2}\mathrm{STr}[\frac{-1}{4}\bar{\Psi}^\dagger\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{KL}[X^I, X^{J\dagger}, \Psi] \\
& \frac{-1}{16}\bar{\Psi}^\dagger\Gamma^\mu\tilde{D}^\nu\Psi\Psi^\dagger\Gamma_\nu\tilde{D}_\mu\Psi + \frac{4i}{3}\bar{\Psi}^\dagger\Gamma^{IJKL}[X^M, X^{N\dagger}, \Psi]X^{IJL\dagger}X^{KMN} \\
& +\frac{1}{4}\bar{\Psi}^\dagger\Gamma^\mu[X^I, X^{J\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\mu\Psi + \frac{i}{4}\bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}\tilde{D}_\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^J \\
& \frac{-i}{4}\bar{\Psi}^\dagger\Gamma_\mu\tilde{D}^\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}_\nu X^I + \frac{i}{6}\bar{\Psi}^\dagger\Gamma^{IJKL}\tilde{D}_\nu\Psi X^{IJK\dagger}\tilde{D}^\nu X^L \\
& \frac{-i}{2}\bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\nu\Psi X^{IJK\dagger}\tilde{D}^\nu X^K - i\bar{\Psi}^\dagger\Gamma^{IJ}[X^J, X^{K\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}_\mu X^K \\
& i\bar{\Psi}^\dagger\Gamma^{\mu\nu}[X^I, X^{J\dagger}, \Psi]\tilde{D}_\mu X^{I\dagger}\tilde{D}_\nu X^J + i\bar{\Psi}^\dagger\Gamma_{\mu\nu}\Gamma^{IJ}[X^J, X^{K\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^K \\
& -2i\bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}X^{JKL} \\
& +2i\bar{\Psi}^\dagger\Gamma_\mu[X^I, X^{J\dagger}, \Psi]\tilde{D}^\mu X^{K\dagger}X^{IJK} \\
& \frac{-2i}{3}\bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJKL}[X^L, X^{M\dagger}, \Psi]X^{IJK\dagger}\tilde{D}^\mu X^M \\
& +2i\bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]X^{IJK\dagger}\tilde{D}^\mu X^L \\
& +4i\bar{\Psi}^\dagger\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]X^{IJM\dagger}X^{KLM} + \text{ h.c. with same coefficients}]
\end{aligned}
$$

SUMMARY AND CONCLUSIONS

Our main result is that one can construct a **UNIQUE** 4 derivative BLG action by assuming the existence of a three algebra structure as well as demanding that the connection between BLG and SYM go over to the next order in α'^2. It is natural to conjecture that such a connection holds to all orders of the full DBI action. ie, one can write down uniquely a full "3BI" action, with structure of the interactions being of the BLG type.

Bagger and Lambert have shown that ABJM theory also fits into a more general 3-algebra structure,with corresonding f^{abcd} complex and not completely antisymmetric in its indices, and so one might ask the same question in the context of the ABJM as well. This uniqueness result will go through atleast for the $U(2)$x$U(2)$ ABJM case, because the manupulations would be quite similar. Though its not obvious that this is true for the more general $U(N)$x$U(N)$ ABJMcase.

It would be more interesting to check this example, since the ABJM theory is conjectured to be the theory of M2 branes on orbifolds. In particular this might suggest some role for general 3-algebras in M2 brane dynamics. It would be nice to understand wheter these higher derivative 3-algebra theories have any role/relevance in the study of M2 brane dynamics.