Hybrid Formalism Approach to OSV Conjecture

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based on Black Hole Partition Function using Hybrid Formalism of Superstrings
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In hep-th/0405146: Ooguri, Strominger and Vafa, conjectured a relationship of the form:

$$Z_{\rm BH} = |Z_{\rm top}|^2$$

$Z_{\rm BH}$ is the (indexed) entropy of four dimensional BPS black holes in Type II Calabi-Yau compactifications.

$Z_{\rm top}$ is the topological string partition function evaluated at the attractor point in the Calabi-Yau.

Note: Several refinements in literature

Large charge Q expansion:

$$\ln Z_{BH} = -4\pi Q^2 \mathrm{Im}\left[\sum_h F_h(X^\Lambda)\left(\frac{8}{Q}\right)^{2h}\right]. \quad (1)$$

where $X^\Lambda = \frac{p^\Lambda + i\frac{\phi^\Lambda}{\pi}}{2Q}$.

$F_h(X^\Lambda)$ appear in the low energy effective action of Type IIA superstrings on Calabi-Yau three folds as F-terms:

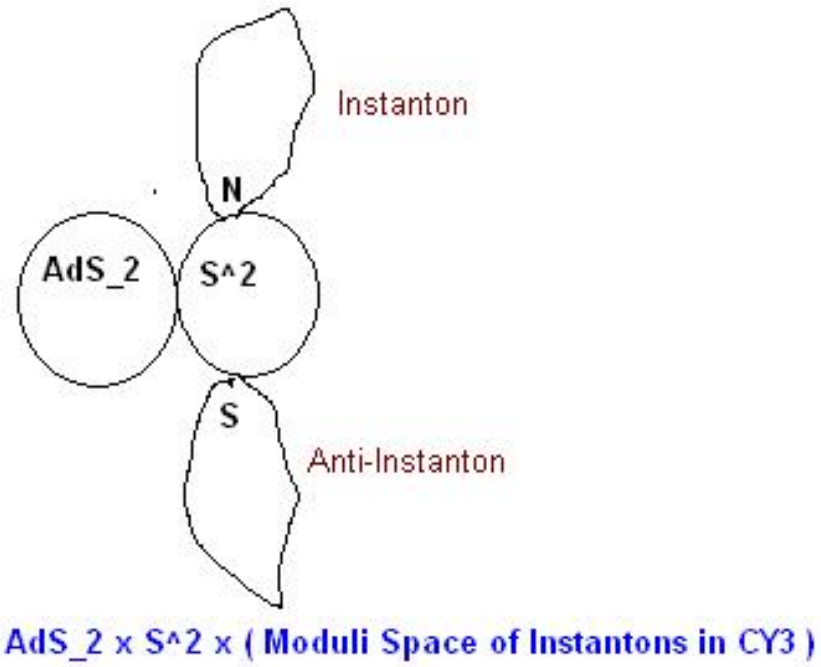
$$\int d^4x d^4\theta (W_{\alpha\beta} W^{\alpha\beta})^h F_h(X^\Lambda). \quad (2)$$

where W^{2h} involves the graviphoton field strength and the Weyl tensor.

These F-terms are computed by Topological string amplitudes.
Bershadsky, Cecotti, Ooguri, Vafa - hep-th/9309140
Antoniadis, Gava, Narain, Taylor - hep-th/9307158

A proposal to understand OSV formula (beasley et. al., hep-th/0608021)

The Green-Schwarz formalism is used to calculate the IIA partition function in a perturbative expansion with appropriate genus g instantons and anti-instantons insertions, giving $Z_{\rm top}$ and $\bar{Z}_{\rm top}$.



Disadvantage of the Green-Schwarz formalism:

The over all normalization coming from regularizing divergence over the AdS_2 part could not be fixed.

The instanton partition functions are assumed to produce topological string partition function. As there is no explicit calculation yet.

Topological string partition function can be nicely dealt with in Hybrid Formalism – (Berkovits and Vafa, hep-th/9407190)

World-sheet action for Hybrid Formalism in $AdS_2 \times S^2 \times CY_3$

$$S = S_{AdS_2 \times S^2} + S_\rho + S_{CY}$$

$S_{AdS_2 \times S^2}$: Sum of Coset space and Wess-Zumino sigma model actions for the supergroup $\frac{PSU(1,1|2)}{U(1) \times U(1)}$, giving a CFT with $N = 2, c = -3$.

S_ρ: Conformal field theory of the chiral boson ρ.

S_{CY}: $N = 2, c = 9$ SCFT representing the Calabi-Yau.

$$
\begin{aligned}
S_{AdS_2\times S^2} &= \frac{1}{\alpha'}\int d^2z\left[\frac{1}{2}\eta_{cd}J^c\bar{J}^d - \frac{1}{4Ng_s}\delta_{\alpha_L\beta_R}\left(J^{\alpha_L}\bar{J}^{\beta_R} - \bar{J}^{\alpha_L}J^{\beta_R}\right)\right. \\
&- \frac{1}{4Ng_s}\delta_{\dot{\alpha}_L\dot{\beta}_R}\left(J^{\dot{\alpha}_L}\bar{J}^{\dot{\beta}_R} - \bar{J}^{\dot{\alpha}_L}J^{\dot{\beta}_R}\right) + d_{\alpha_L}\bar{J}^{\alpha_L} + \bar{d}_{\dot{\alpha}_L}\bar{J}^{\dot{\alpha}_L} \\
&+ d_{\alpha_R}J^{\alpha_R} + \bar{d}_{\dot{\alpha}_R}J^{\dot{\alpha}_R} \\
&+ \left. Ng_s d_{\alpha_L}d_{\beta_R}\delta^{\alpha_L\beta_R} + Ng_s\bar{d}_{\dot{\alpha}_L}\bar{d}_{\dot{\beta}_R}\delta^{\dot{\alpha}_L\dot{\beta}_R}\,.\right]
\end{aligned}
$$

- $J^A = (g^{-1}\partial g)^A$, $A = c, \alpha_{L,R}, \dot{\alpha}_{L,R}$ are left-invariant currents.
- N: Related to the value of the graviphoton field strength
- η_{cd} and $\delta_{\alpha_L\beta_R}$ are metrics for bosonic and fermionic directions.
- The presence of d_α's breaks Kappa-Symmetry , but, allows for world-sheet Superconformal Invariance (Berkovits et. al., hep-th/9907200).

The $AdS_2 \times S^2 \times CY_3$ $N = 2, c = 6$ Superconformal generators:

$$\begin{aligned} T &= \frac{1}{2}\partial\Pi^c\partial\Pi_c + d_\alpha\partial\theta^\alpha + \bar{d}_{\dot\alpha}\partial\bar\theta^{\dot\alpha} + \frac{\alpha'}{2}\partial\rho\partial\rho \\ &\quad + \partial Y^j\partial\bar{Y}_j + \frac{1}{2}(\psi^j\partial\bar\psi_j + \bar\psi_j\partial\psi^j)\ , \\ G^- &= \frac{1}{\sqrt{\alpha'}}e^{\rho}(d\,)^2 + \psi^j\partial\bar{Y}_j\,, \\ G^+ &= \frac{1}{\sqrt{\alpha'}}e^{-\rho}(\bar{d}\,)^2 \ + \bar\psi_j\partial Y^j, \\ J &= \alpha'\partial\rho \ + \psi^j\bar\psi_j\ , \end{aligned}$$

where, $(d)^2$ means $\frac{1}{2}\epsilon^{\alpha\beta}d_\alpha d_\beta$.

Procedure:

- Embed the $c = 6, N = 2$ CFT of $AdS_2 \times S^2 \times CY_3$ in a small $N = 4$ topological algebra and calculate the partition function of instantons and anti-instantons.

- Unlike the Green-Schwarz formalism: There are four bosonic and eight fermionic zero modes to be integrated over. There is no action associated with them, giving a divergence.

A BRST exact term can be added to deform the action and cure the divergence.

- World-sheet instantons which are holomorphic in CY satisfy, $\bar{\partial} Y = 0$.
- Holomorphy condition implies that: $G_L^- = \frac{1}{\sqrt{\alpha'}} e^{\rho_L} (d_L)^2 + \psi_L^j \partial \bar{Y}_j$ and also G_R^- annihilate the instanton configuration.
- Thus, we perform an A-twist, so that G_L^- and G_R^- acquire spin 1. Then, we can choose the BRST operator as $Q = G_L^- + G_R^-$.
- It can be shown that, the instanton action for $AdS_2 \times S^2$ is closed under Q. Thus, a BRST exact operator is constructed and added to the action as:

$$S_{AdS_2 \times S^2} = S_{AdS_2 \times S^2} + t\ V \tag{3}$$

where V is Q-exact and t is a free parameter which can be taken to infinity.

The final result for the Instanton partition function:

$$ln\ Z_{IIA}^{\rm Instanton} = \sum_h\ g_{\rm top}^{2h-2}\ F_h \tag{4}$$

where the A-model topological string amplitude is:

$$F_h = \prod_{j=1}^{3h-3} \int d^2 m_j < \prod_{j=1}^{3h-3} (\int \mu_j^L G_{CY}^{-L})(\int \mu_j^R G_{\rm CY}^{+R}) > \ . \tag{5}$$

Adding the anti-instanton partition function as well, one reproduces:

$$Z_{BH} \ = \ Z^{IIA} \ = \ |Z_{top}|^2 \tag{6}$$

Comments

- In Hybrid formalism the topological string partition function emerges naturally.
- The divergence over the AdS_2 variables was fixed as $g_{\rm top}^{-2}$. This needs a derivation.
- Partition function for $N = 4$ and $N = 8$ string theories in four dimensions should be interesting.
- How to take holomorphic anomaly in to account.