Lifshitz-like fixed point

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Assumptions:

- **There exists gauge/gravity duality to systems perserving (only) scaling symmetry.**

- **There exists an effective description to either side of the duality.**

- **A critical point in the field theory is described as a fixed point (Solution) from the gravitational side.**

- **The symmetry of such systems do not necessarily preserve Lorentz symmetry.**

Outline:

- **Schrödinger group**
- **Its geometric realization**
- **Dynamical exponent**
- **Lifshitz point**
- **Its geometric realization**

The generators of the group are:

- **Translations** (H, P_i)
- **Rotations,** M_{ij}
- **Galilean boosts,** K_i
- **Scaling,** D
- **Special conformal transformations,** C
- **Mass Operator,** N

$$\begin{aligned}
&[M_{ij}, N] = [M_{ij}, D] = 0, \quad [M_{ij}, P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i), \\
&[M_{ij}, K_k] = i(\delta_{ik}K_j - \delta_{jk}K_i), \\
&[M_{ij}, M_{kl}] = i(\delta_{ik}M_{jk} - \delta jkM_{il} + \delta_{il}M_{kj} - \delta_{jl}M_{ki}), \\
&[P_i, P_j] = 0 = [K_i, K_j], \quad [K_i, P_j] = i\delta_{ij}N, \quad [D, P_i] = iP_i, \\
&[D, K_i] = (1-z)iK_i, \quad [H, N] = [H, P_i] = [H, M_{ij}] = 0, \\
&[H, K_i] = -iP_i, \quad [D, H] = izH, \quad [D, N] = i(2-z)N. \quad (1)
\end{aligned}$$

For $z = 2$

$$[M_{ij}, C] = 0, \quad [K_i, C] = 0, \quad [D, C] = -i2C, \quad [H, C] = -iD. \quad (2)$$

- **The algebra with this choice of z is also said as the centrally extended Galilei group.**

In $4+1$**-dim bulk spacetime, this contains 13 generators.**

- **The Galilei group can also be seen containing $O(3) \times SL(2,R)$. The SL(2,R) comes from a suitable redefinitation of H,C,D generators and the O(3) from the commutative algebra of angular momentum and S_i**

- $S_1 = \frac{H+C}{2}, \; S_2 = \frac{H-C}{2}, \quad S_3 = \frac{D}{2}$.

- $[S_i, S_j] = -iS_k, \quad (i,j,k = 1,2,3)$

•The metric

$$ds^2 = L^2[-2\frac{(dx^+)^2}{r^4} + \frac{dx_i^2 - 2dx^+dx^-}{r^2} + \frac{dr^2}{r^2}] \tag{3}$$

• The action

$$S = \int \sqrt{-g}(R - \Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu) \tag{4}$$

$$< \mathcal{O}(x,t)\mathcal{O}(0,0) >\sim \theta(t)\frac{\Gamma(1-\nu)}{\Gamma(\nu)}\frac{1}{|\epsilon^2 t|^\Delta}e^{-i\frac{lx^2}{2|t|}}, \tag{5}$$

$\nu = \sqrt{(5/2)^2 + l^2 + m^2}$
$\Delta_\pm = 5/2 \pm \nu$

• **Lifshitz point is a critical point on the curve describing the λ line of the 2nd order PT.**

• **The anisotropy exponent (z) is also called as the the Dynamical exponent.**

$$x_i \to \lambda\ x_i, \quad t \to \lambda^z\ t \tag{6}$$

$$C(\lambda x_i; \lambda^z t) = \lambda^{-2\delta} C(x_i; t) \tag{7}$$

[δ is the conformal dimension.]

• **Scaling suggests**

$$< O(\mathbf{x}, t) O(\mathbf{0}, 0) > \sim \frac{1}{|\mathbf{x}|^{2\delta}} F(\frac{\mathbf{x}^z}{t}) \tag{8}$$

• **A Lifshitz field theory with a line of fixed point parametrized by κ obeying such a scaling symmetry**

$$S = \int d^D x dt [(\partial_t \chi)^2 - \kappa (\nabla^2 \chi)^2] \tag{9}$$

• **The scaling dimension of $[\chi] = \frac{D-z}{2}$**

• The exact two point correlator in the Lifshitz theory

$$< \mathcal{O}_n(\mathbf{x}, t)^\dagger \mathcal{O}_n(\mathbf{x}', t') >= e^{-n^2 G(\mathbf{x}-\mathbf{x}', t-t')} \tag{10}$$

where

$$G = \frac{1}{8\pi\kappa}[Log(\frac{|\mathbf{x}|^2}{\epsilon^2}) + \Gamma(0, \frac{|\mathbf{x}|^2}{4\kappa|t})] \tag{11}$$

$$\mathcal{O}_n(x) = e^{-in\chi(x)}, \quad \Gamma(0, z) = \int_z^\infty \frac{ds}{s} e^{-s} \tag{12}$$

• **The scaling symmetry**

$$r \to \frac{r}{\lambda},\ x_i \to \lambda x_i,\ t \to \lambda^z t \qquad (13)$$

The metric

$$ds^2 = L^2[-r^{2z}dt^2 + r^2 dx_i^2 + \frac{dr^2}{r^2}] \qquad (14)$$

• **It has got lesser symmetries than the Galilei group, note the number of spacetime dimensions.**

• **Geodesically incomplete, at $r = 0$.**

$$S \sim \int \sqrt{-g}[R - 2\Lambda - \frac{F_2^2}{4} - \frac{F_3^2}{12}] - c \int A_2 \wedge F_2 \qquad (15)$$

• **The operators dual to massless scalar field has the conformal dimension $\Delta(\Delta - 4) = 0$**

$$< \mathcal{O}(\mathbf{x}, t), \mathcal{O}(\mathbf{0}, 0) > \sim \frac{1}{\mathbf{x}^8}, \quad |\mathbf{x}| \to \infty \qquad (16)$$

• **There exists a flow from $z = 2$ to $z = 1$.**

$$
\begin{aligned}
ds^2 &= L^2[-r^4 f^2(r)dt^2 + r^2 ds_2^2 + \frac{g^2(r)}{r^2}dr^2], \\
F_2 &= \frac{2}{L}h(r)e^r \wedge e^t, \quad H_3 = \frac{2}{L}j(r)e^r \wedge e^i \wedge e^j \\
rf' &= -\frac{5f}{2} + \frac{fg^2}{2}(5 + j^2 - h^2 + \frac{1}{r^2}), \\
rg' &= \frac{3g}{2} - \frac{g^2}{2}(5 - j^2 - h^2 + \frac{1}{r^2}), \qquad (17) \\
rj' &= 2gh + \frac{j}{2} - \frac{jg^2}{2}(5 + j^2 - h^2 + \frac{1}{r^2}), \qquad (18) \\
rh' &= 2gj - 2h \qquad (19)
\end{aligned}
$$

• **Lifshitz point:** $f = g = h = j = 1$.

• $AdS_4 : \ h = j = 0$.

• **From the linearlised analysis, there are two decaying modes and one zero modes.**

• **Generalize the number of exponents in 3+1 dim, with the scaling**

$$r \to \frac{r}{\lambda},\; y \to \lambda y,\; x \to \lambda^{z_2} x,\; t \to \lambda^{z_1} t \quad (20)$$

The metric

$$ds^2 = L^2[-r^{2z_1}dt^2 + r^{2z_2}dx^2 + r^2 dy^2 + \frac{dr^2}{r^2}] \quad (21)$$

• **It has got lesser symmetries than the previous one.**

$$S = \int \sqrt{-g}[R - 2\Lambda - \frac{F_2^2}{4} - \frac{F_3^2}{12} - \frac{H_3^2}{12} - \frac{m_0^2}{2}B_2^2] - c\int A_2 \wedge F_2 \quad (22)$$