The Wroblewski parameter from lattice QCD

Workshop on Field Theories Near Equilibrium

Rajiv V. Gavai

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Introdution

λ_s from Quark Number Susceptibility

Pressure for small baryon density

Summary

Introduction

- Quark-Gluon Plasma in Heavy Ion Collisions.
- Reliable signals needed to establish it.
- Enhancement of strangeness production as a promising signal of QGP (Rafelski-Müller, Phys. Rev. Lett '82, Phys. Rept '86..).
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Strangeness Enhancement

- Key Idea: $T_{QGP} > T_c \approx m_s \approx 150$ MeV
- Energy Threshold

$$q + \bar{q} \to s + \bar{s} \qquad g + g \to s + \bar{s} \qquad E_{thres} \approx 2m_s \approx 300 \text{ MeV}$$

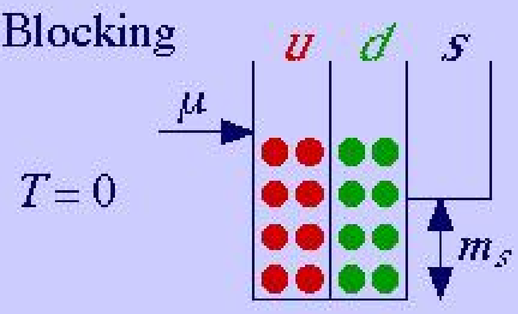
$$\pi + \text{N} \to \Lambda + \text{K} \qquad E_{thres} \approx 530 \text{ MeV}$$

$$\text{K} + \pi \to \bar{\Lambda} + \text{N} \qquad E_{thres} \approx 1420 \text{ MeV}$$

- Production Rate

$$\sigma_{QGP}(s\bar{s}) > \sigma_{HG}(s\bar{s})$$

- Pauli Blocking



Expect an enhancement especially for *multi*-strange *anti*-baryons.

Measure: $\Lambda = (uds) \to p\pi^-$ 64%

$\Xi^- = (dss) \to \Lambda\pi^-$ 100%

$\Omega^- = (sss) \to \Lambda K^-$ 68%

and their anti-particles.

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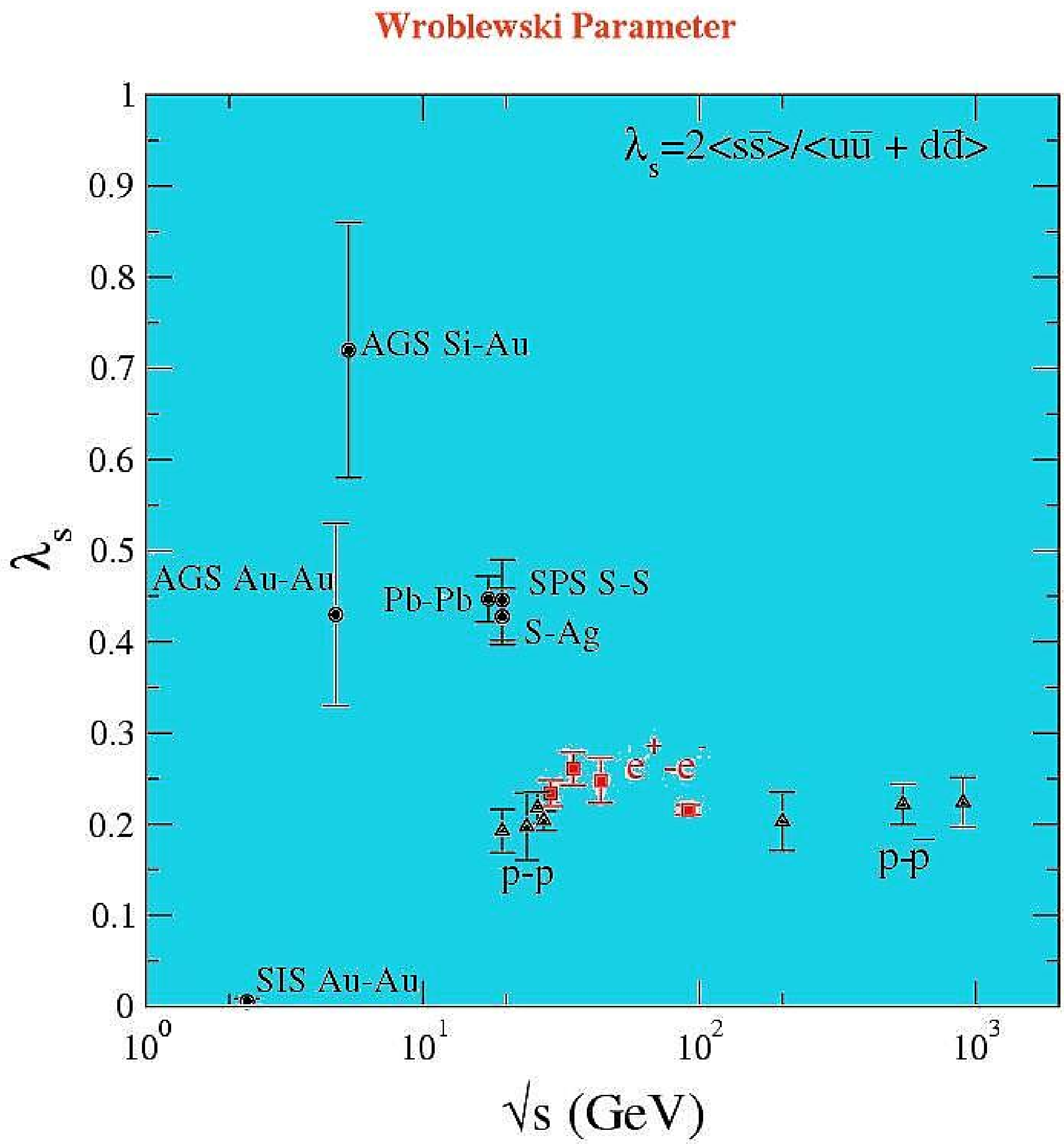
From STAR Webpage

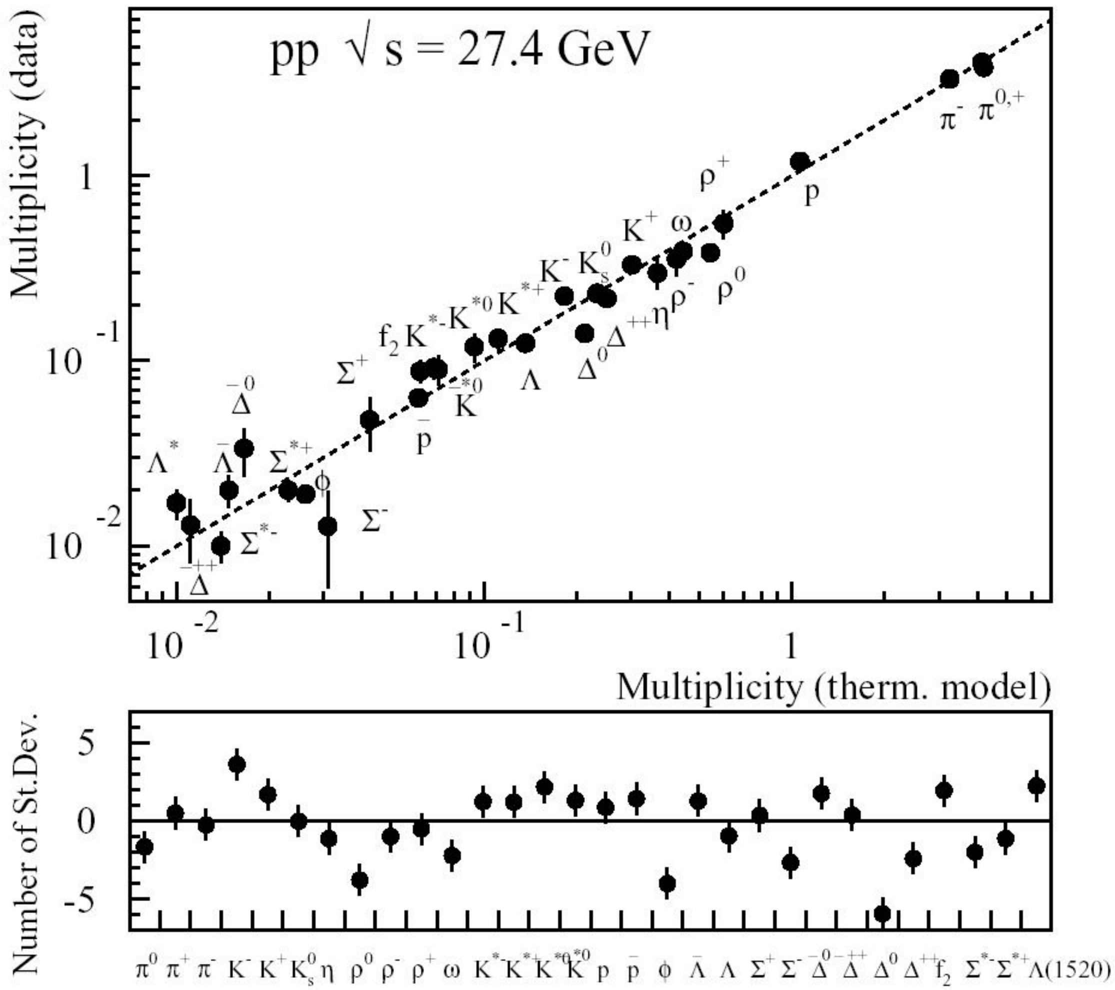


Ratio of newly created strange quarks to light quarks :

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} + d\bar{d}\rangle} \quad (1)$$

Wroblewski Parameter

- Hadron gas fireball model (Becattini-Heinz '97).
- 3 Free parameters : T, V, and $N_{s\bar{s}}$.
- Fit many hadron abundunces.
- Obtain λ_s from data.
- Find $\lambda_s \sim 0.4$ (0.2) for AA (pp).



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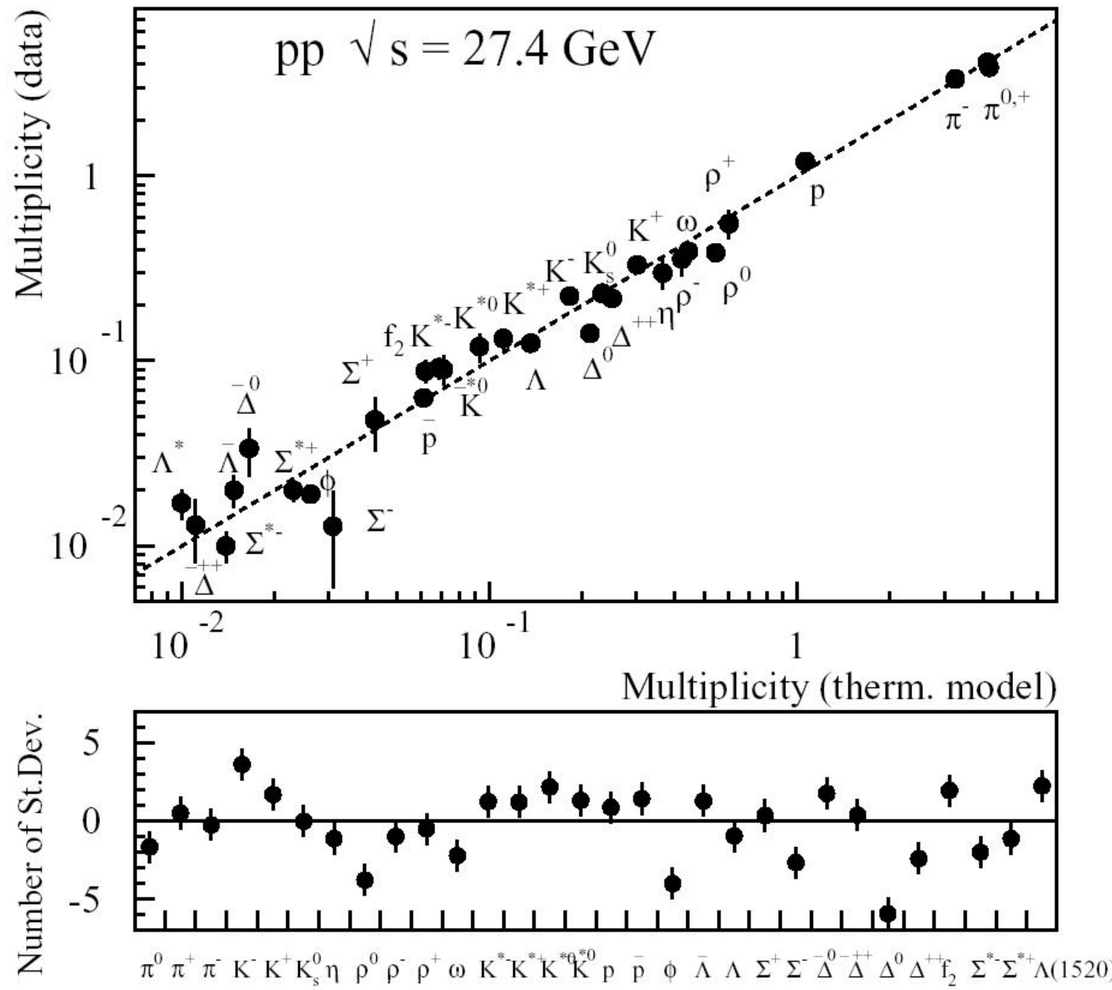

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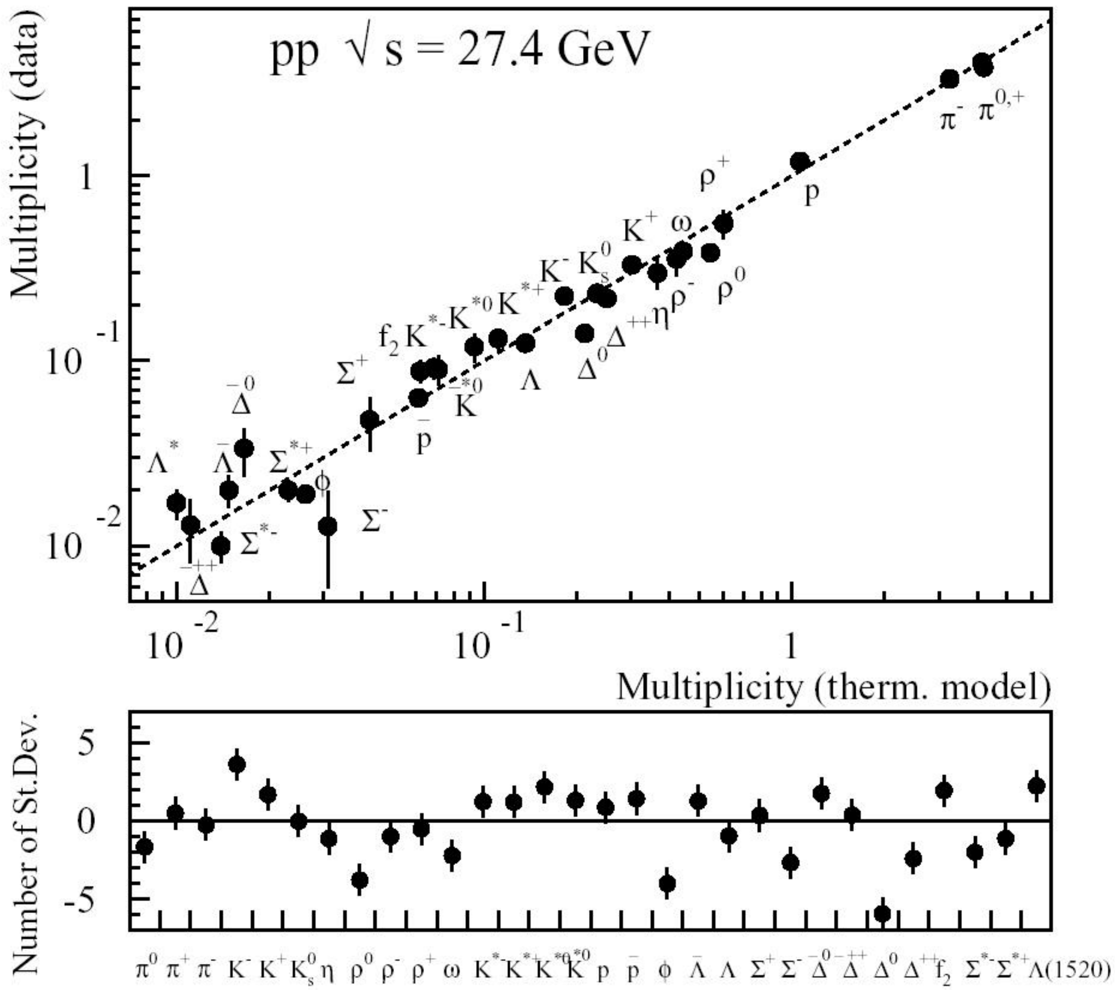

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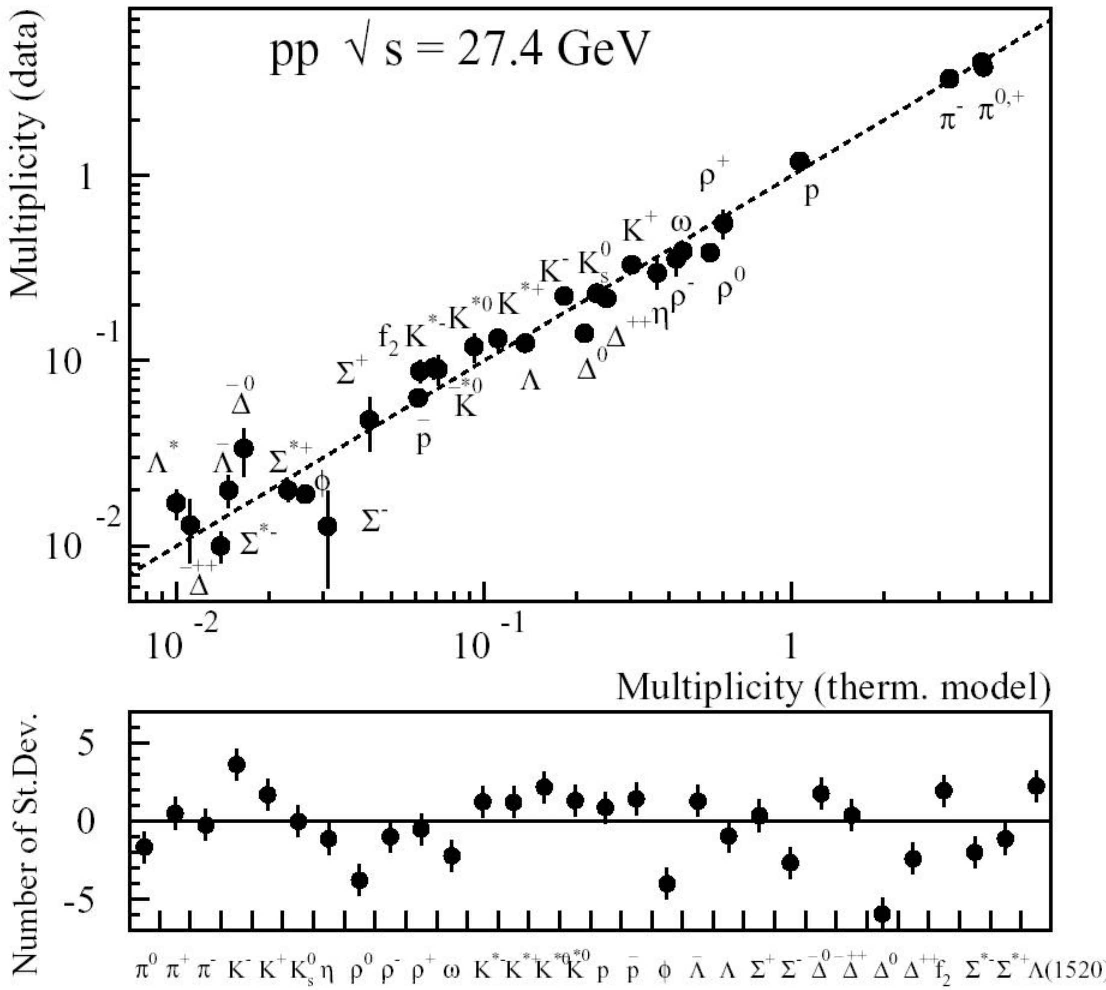

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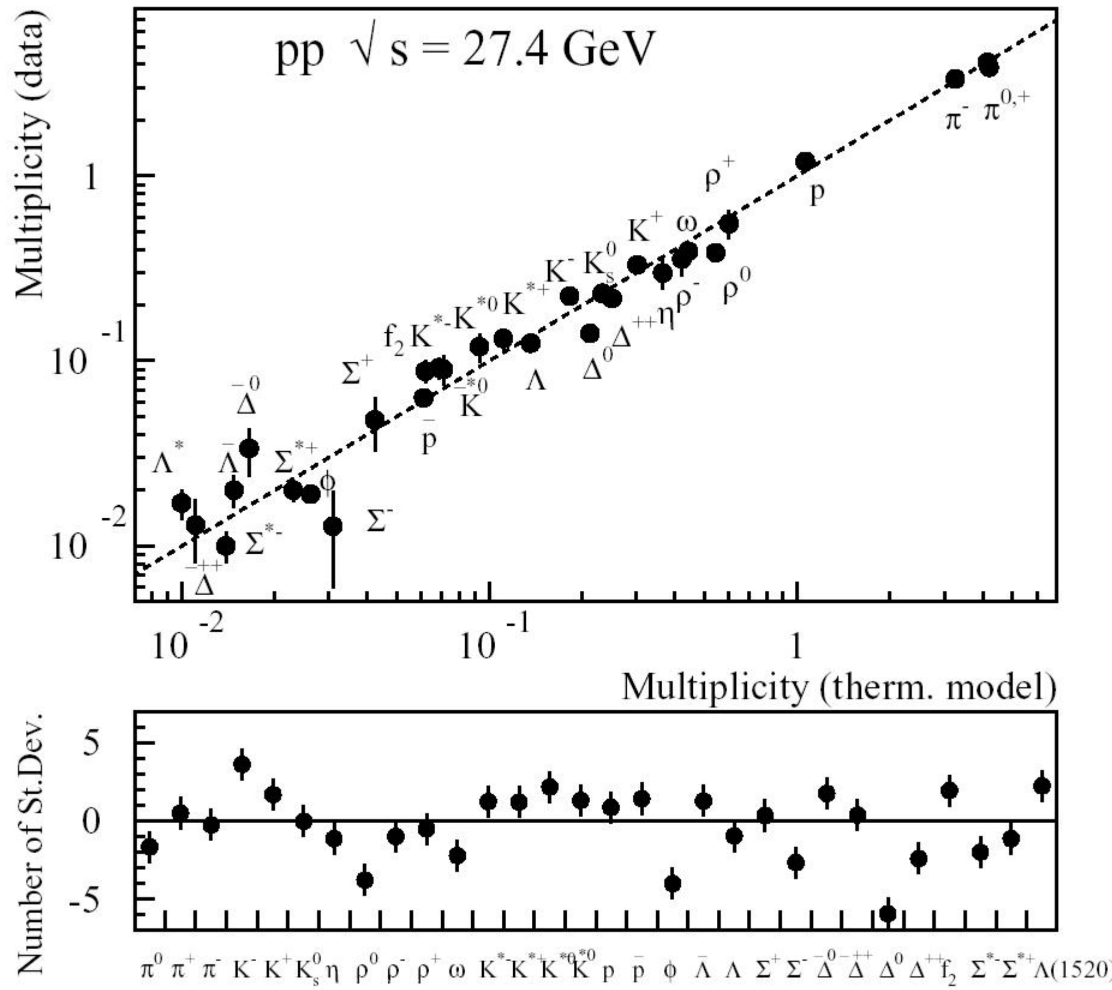

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$$\lambda_s = \frac{2\chi_s}{\chi_u + \chi_d} \, . \qquad (2)$$

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♠ Theoretical Checks : Resummed Perturbation expansions, Dimensional Reduction..

♠ Our improvement: Fixed m_q/T_c, Continuum limit...

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These are Taylor coefficients of the pressure P in its expansion in μ.

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Here $\mathcal{O}_2 = \mathrm{Tr}\ M_u^{-1}M_u'' - \mathrm{Tr}\ M_u^{-1}M_u'M_u^{-1}M_u'$, and $\mathcal{O}_{11}(m_u) = (\mathrm{Tr}\ M_u^{-1}M_u')^2$, and the traces are estimated by a stochastic method: $\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L} (\mathrm{Tr}\ A)_i (\mathrm{Tr}\ A)_j / L(L-1)$, where R_i is a complex vector from a set of N_v subdivided in L independent sets.

Gavai & Gupta PR D '01; Gavai, Gupta & Majumdar, PR D 2002

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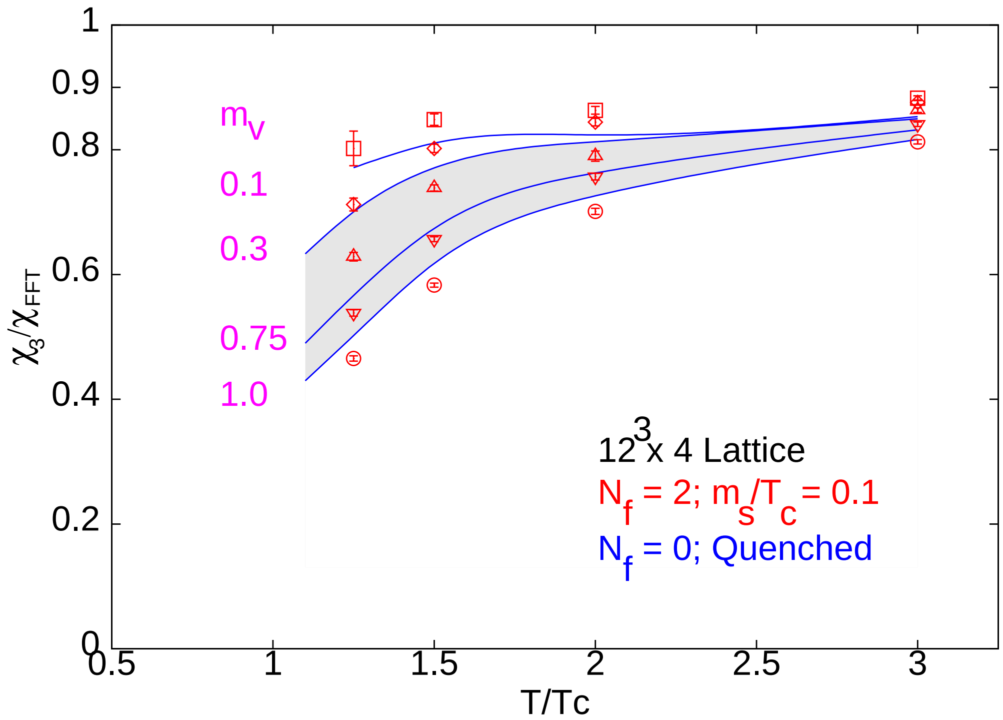

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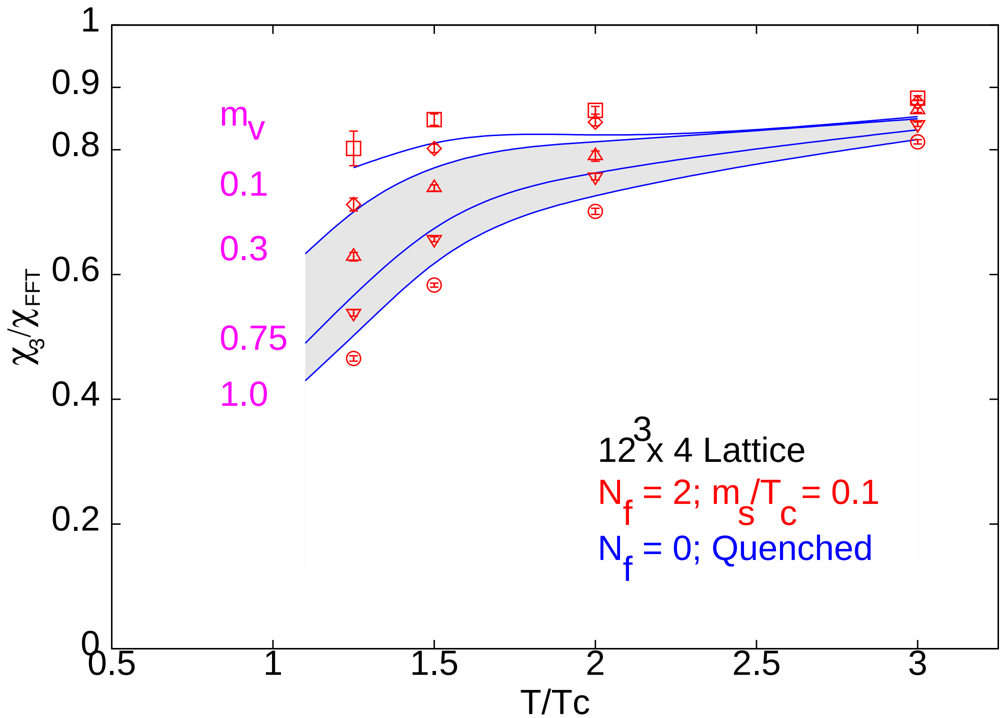

Note that PDG values for strange quark mass $\Longrightarrow$ $m_v^{strange}/T_c$ $\simeq$ 0.3-0.7 (N_f=0); 0.45-1.0(N_f=2).

Perturbation Theory

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Weak coupling expansion gives:

$$\frac{\chi}{\chi_{FFT}} = 1 - 2(\frac{\alpha_s}{\pi}) + 8\sqrt{(1 + 0.167 N_f)}(\frac{\alpha_s}{\pi})^{\frac{3}{2}}$$

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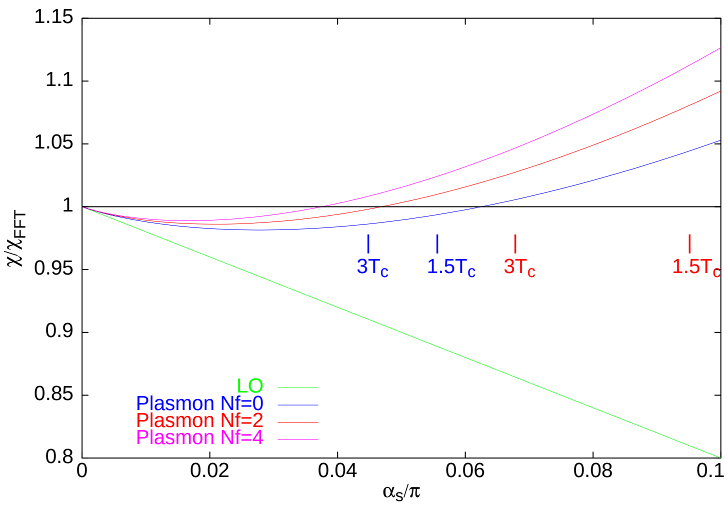

♣ Minm 0.981 (0.986) at 0.03 (0.02) for $N_f = 0$ (2).

♣ For $1.5 \leq T/T_c \leq 3$ pert. theory $\longrightarrow$ 0.99-0.98 (1.08=1.03) for $N_f = 0$ (2).

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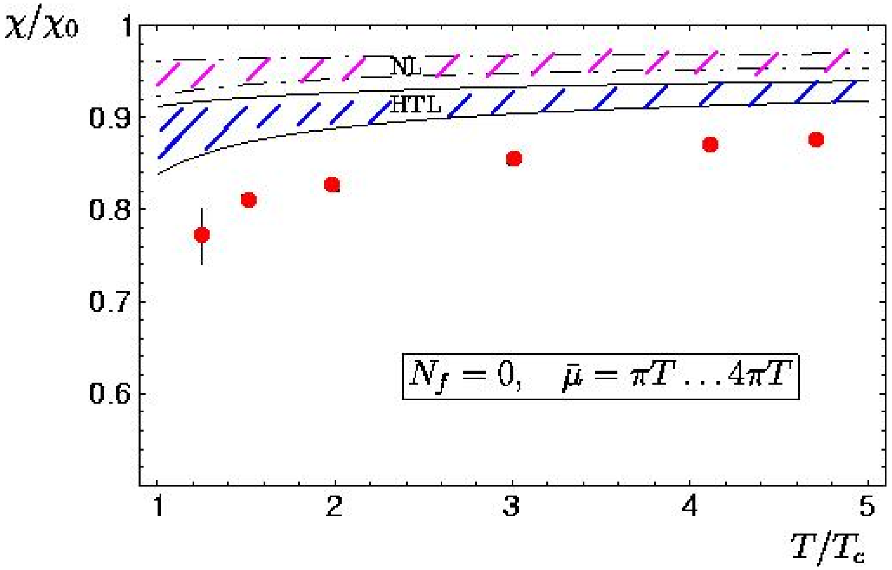

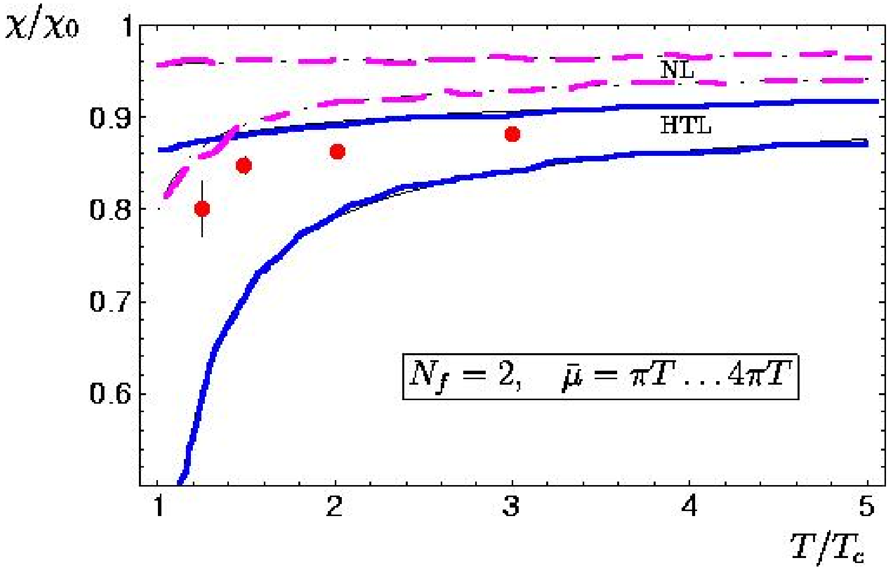

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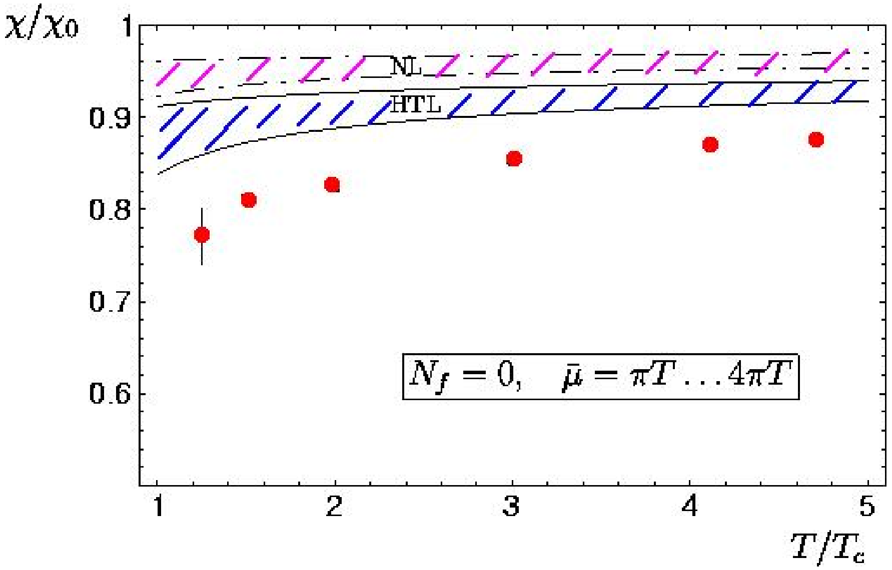

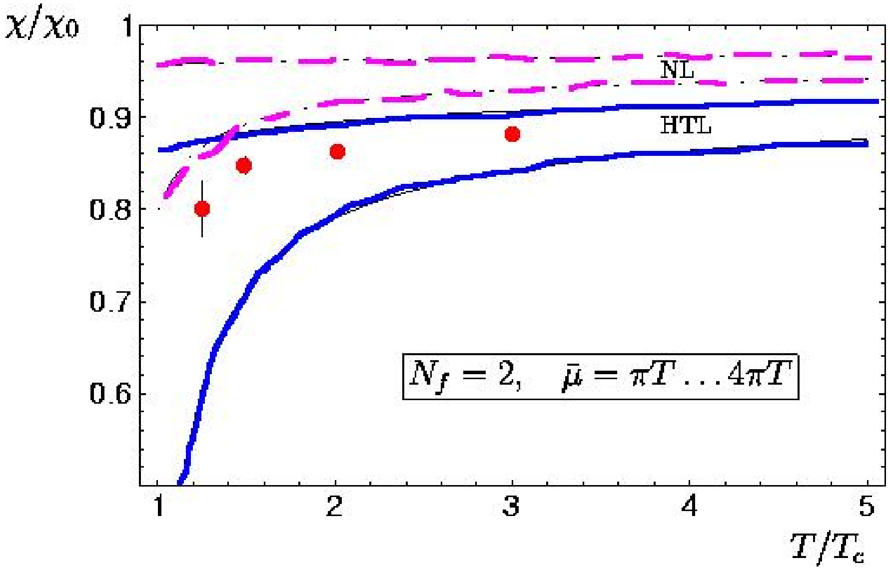

Our results for $N_t = 4 \rightsquigarrow$ Lattice artifacts ?
Check for larger N_t and improved actions.

χ_{ud}

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Off-diagonal Susceptibility : $\chi_{ud} = \langle \frac{T}{V} \text{ Tr } M_u^{-1} M_u' \text{Tr } M_d^{-1} M_d' \rangle$

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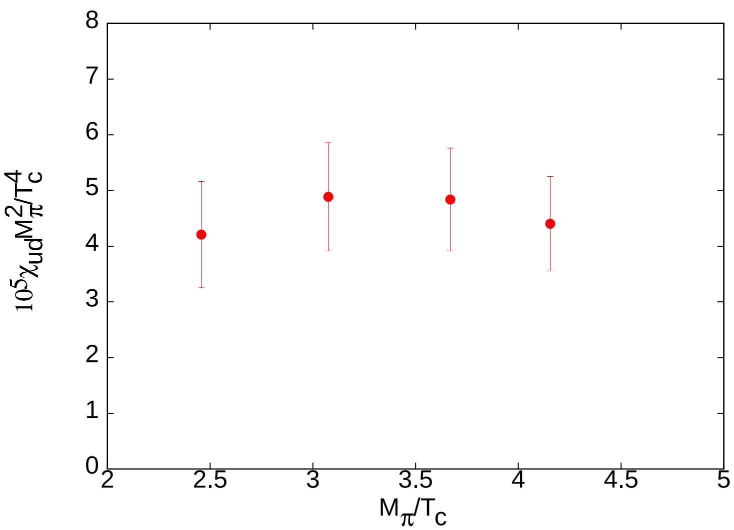

♣ $12^3 \times 4$ Lattice; Quenched.
♣ $T = 0.75T_c$

♣ Gavai, Gupta & Majumdar, PR D 2002

Taking Continuum Limit

(Gavai & Gupta, PR D '02 and PR D '03)

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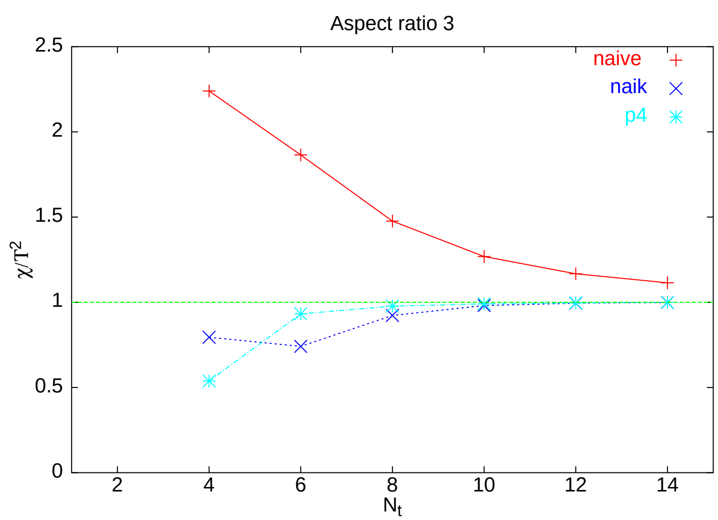
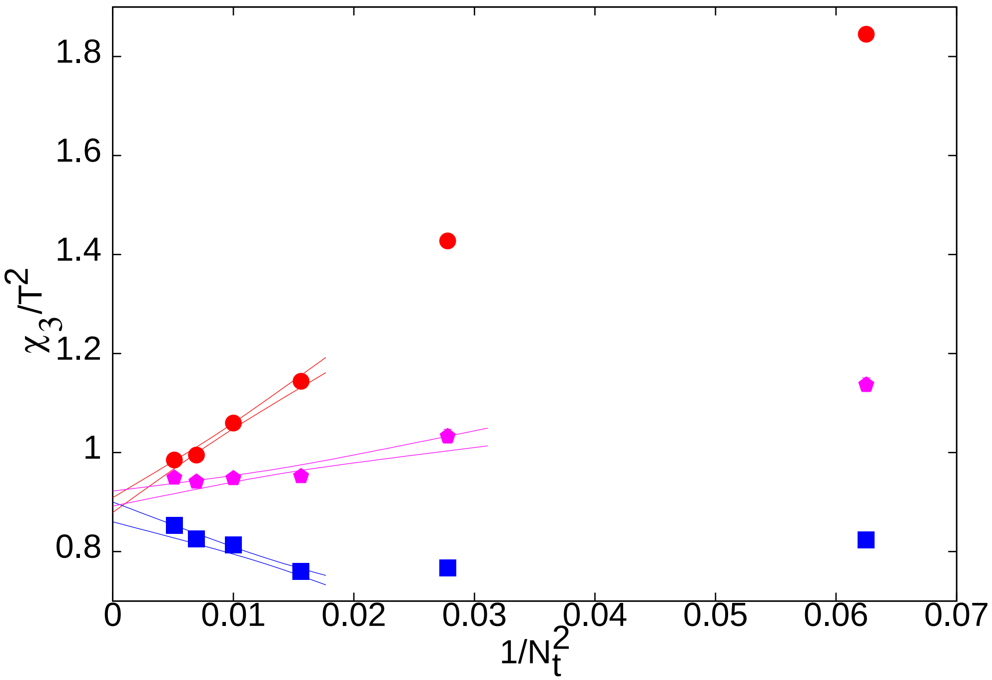


♠ Does improve the N_t-dependence of the free fermions.

Results at $2T_c$:



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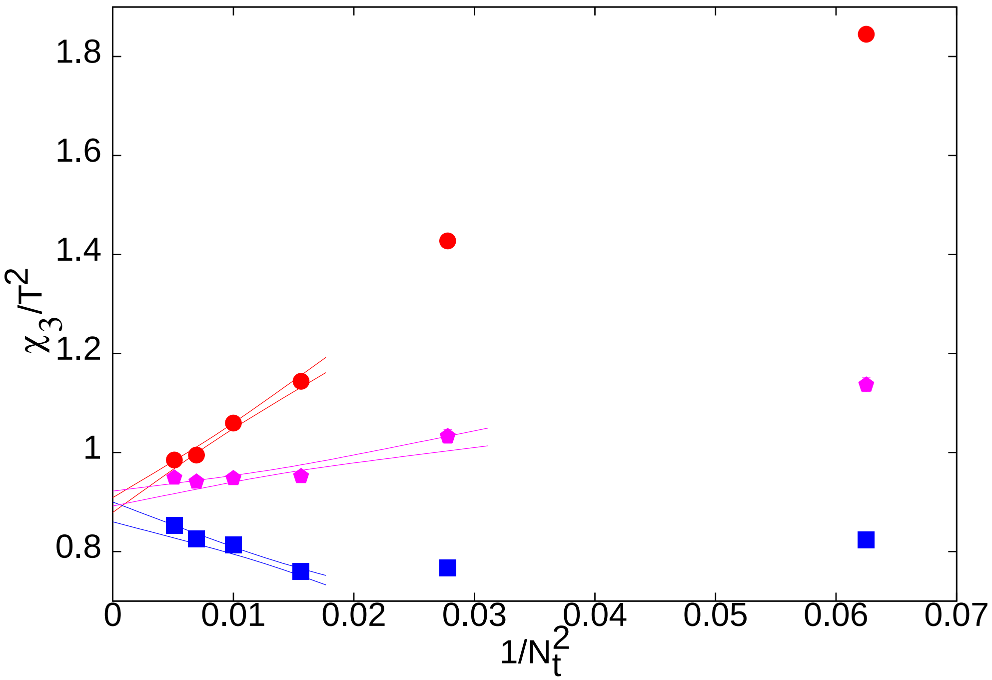

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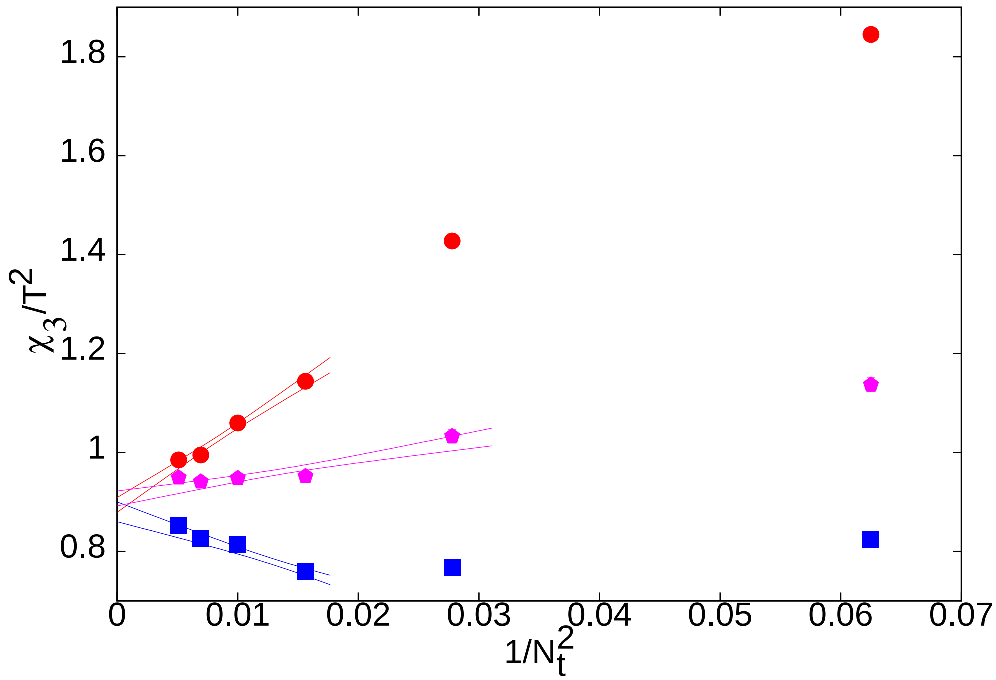

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$\diamondsuit$ Milder $N_t^{-2} \sim a^2$-dependence for Naik fermions.

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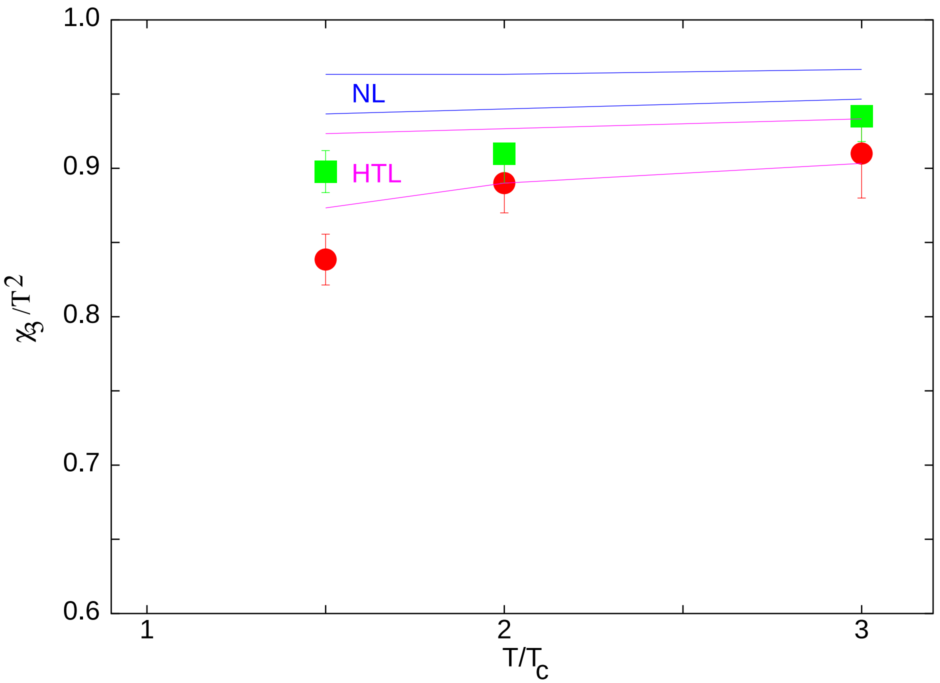

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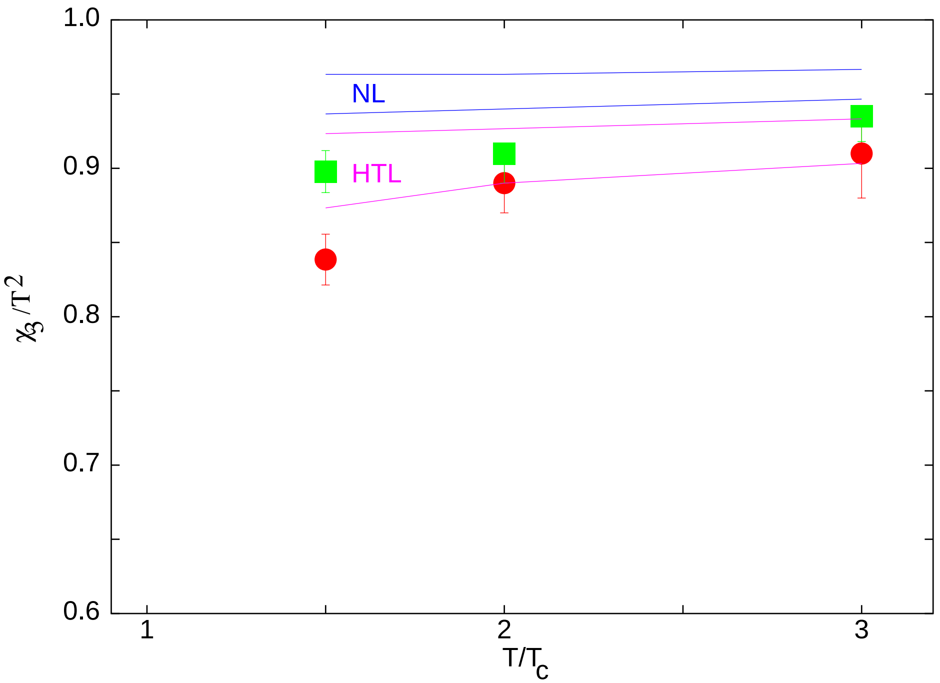

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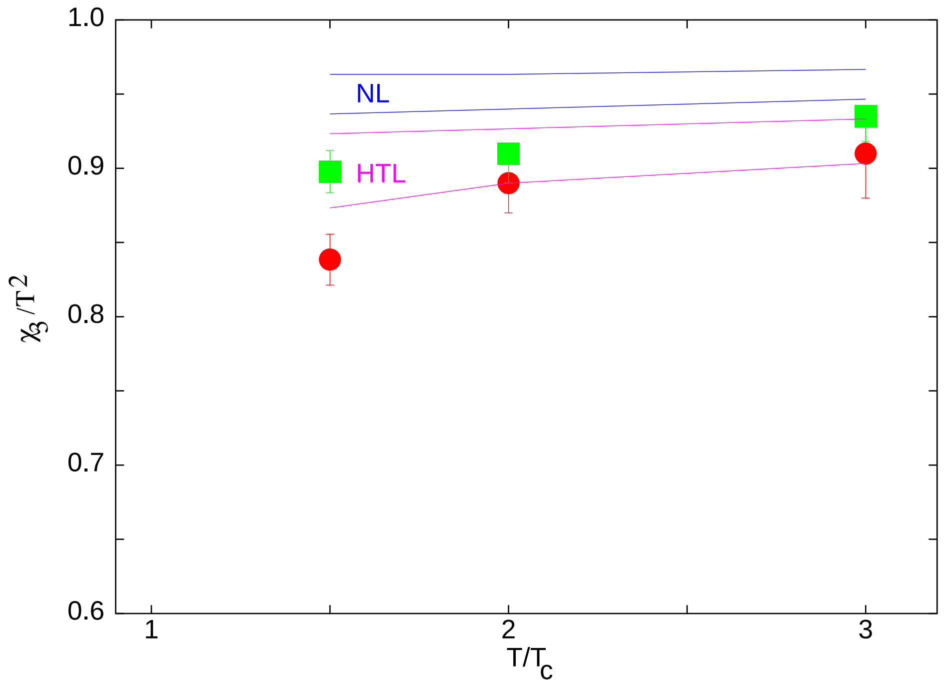

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♡ Note that χ_{ud} behaves the same way for ALL N_t and both fermions, leading to the same $O(10^{-6})$ values in continuum too.

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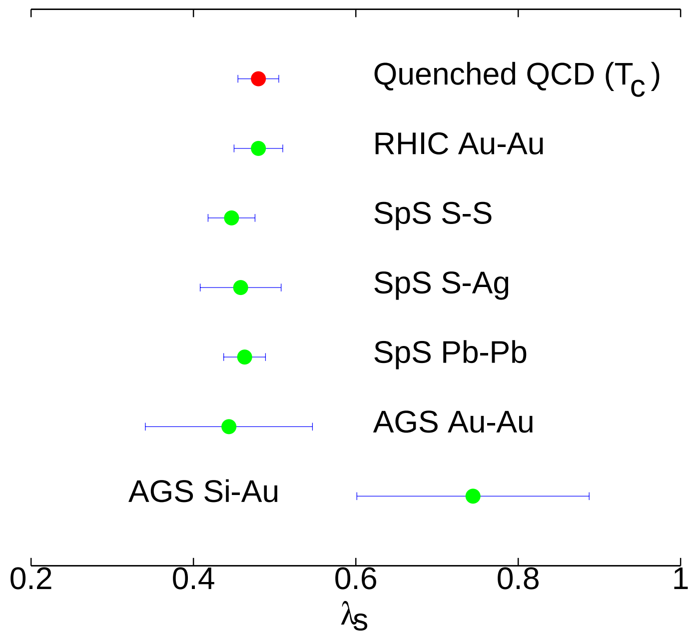

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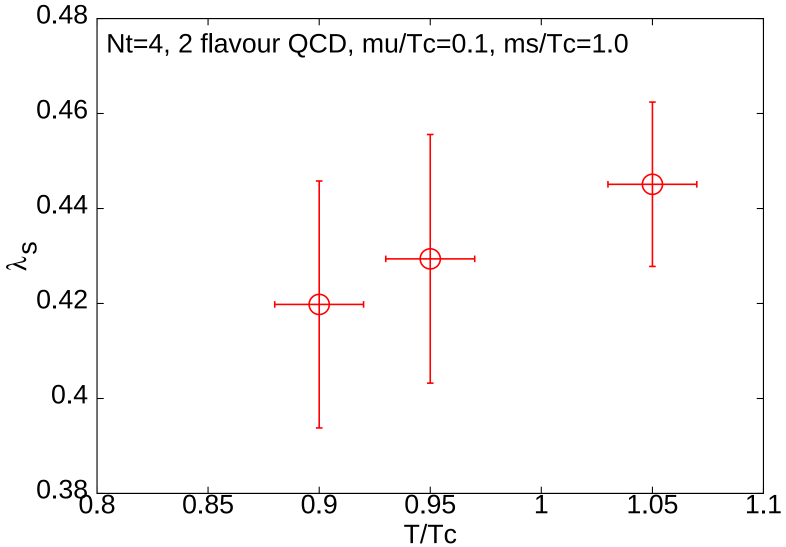

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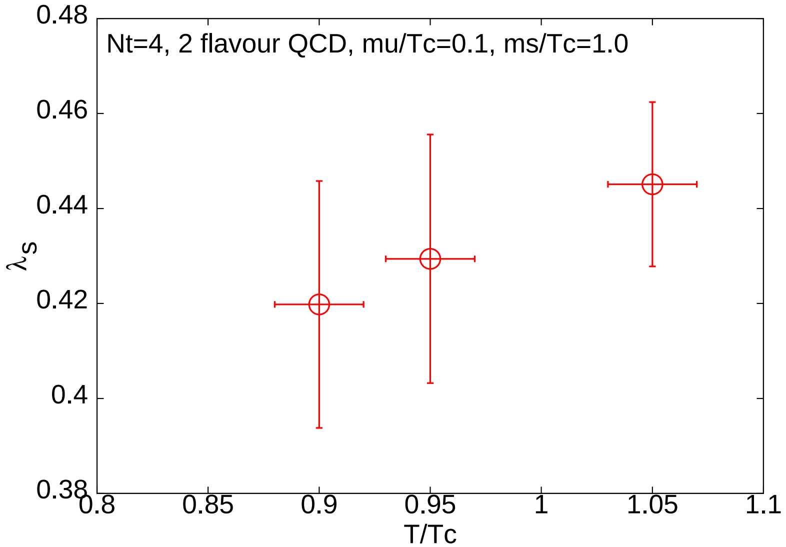

♣ Large finite volume effects below T_c
♣ Up to 12^3 Lattices used.
♣ Strong dependence on m_s expected.
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- Assumed : Chemical equilibration in the plasma.

EoS for nonzero baryon density

Recall,

$$\chi_{fg\cdots} = \frac{T}{V}\frac{\partial^n \log Z}{\partial\mu_f\partial\mu_g\cdots} = \frac{\partial^n P}{\partial\mu_f\partial\mu_g\cdots} . \tag{8}$$

Thus χ_{uuuu} involves terms having fourth derivative w. r. to μ while χ_{uudd} only second derivatives.

In continuum, $f(a\mu) = 1 + a\mu \to f''(0) = 0$.
On lattice, in general, all derivatives exist and depend on the nature of function : prescription dependence !

Fodor-Katz used f_{HK} and got $\mu_E = 725$ MeV for $N_t = 4$. If they were to use f_{BG}, then $\mu_E = 692$ MeV.

Easy to show that $f''(0) = 1$ always but all higher derivatives depend on choice of

f. Thus, one can write

$$\chi_{uuuu} = \chi_{uuuu}^{HK} + \Delta f^{(3)} \left(\frac{\chi_{uu}}{T^2}\right) \left(\frac{4}{N_t^2}\right) \quad , \tag{9}$$

where $\Delta f^{(3)} = f^{(3)} - 1$ is 2 for f_{BG}.

Prescription dependence must go away for small a or large enough N_t.
How large an N_t needed ? $N_t \geq 10$, see below.

Defining

$$\frac{\mu_*}{T} = \sqrt{\frac{12\chi_{uu}/T^2}{|\chi_{uuuu}|}} \ , \tag{10}$$

and $\Delta P = P(\mu) - P(\mu = 0)$, the Taylor series expansion for Pressure P for 2 flavours can be re-organized as,

$$\frac{\Delta P}{T^4} = \left(\frac{\chi_{uu}}{T^2}\right) \left(\frac{\mu}{T}\right)^2 \left[1 + \left(\frac{\mu/T}{\mu_*/T}\right)^2 + \mathcal{O}\left(\frac{\mu^4}{\mu_*^4}\right)\right] . \tag{11}$$

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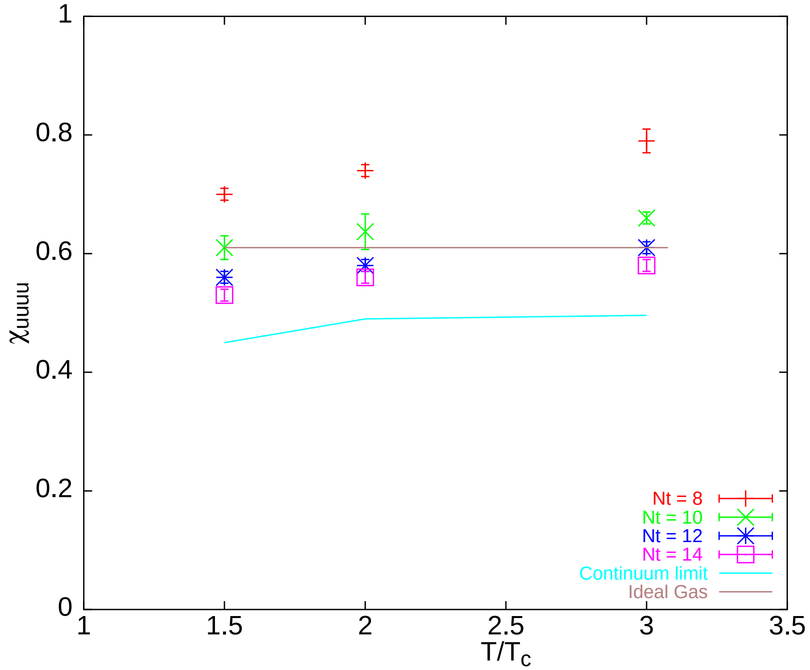
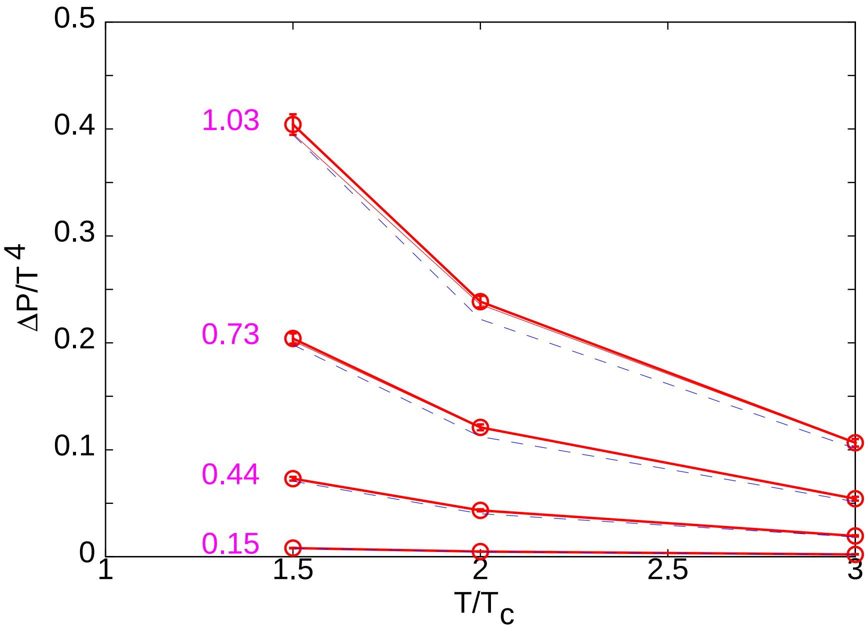
- (T_E, μ_E) may be identified from the radius of convergence using many higher susceptibilities obtained in continuum limit term by term. What about series on finite lattice and estimate of (T_E, μ_E) as done presently ?

Our Results

Our results for χ_{uuuu} and ΔP: Gavai and Gupta, PR D68, '03





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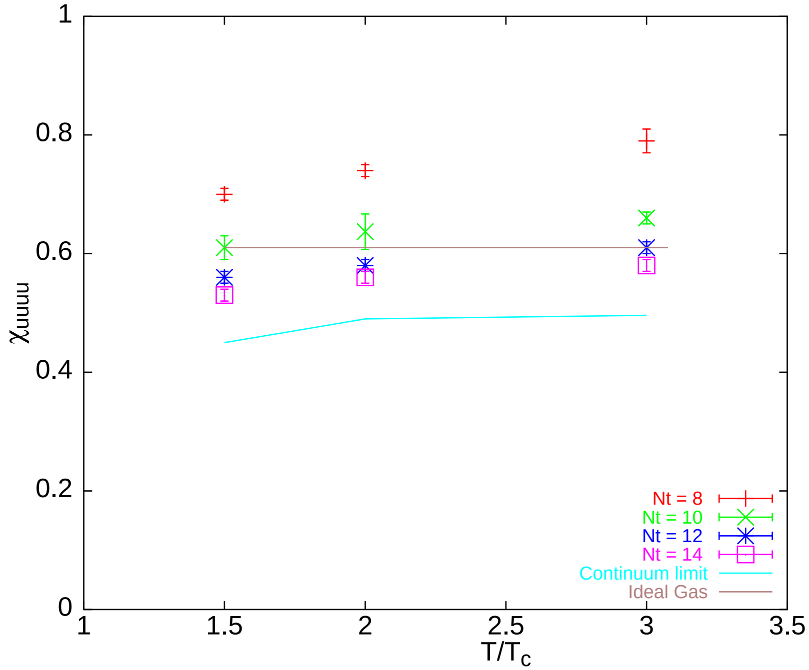

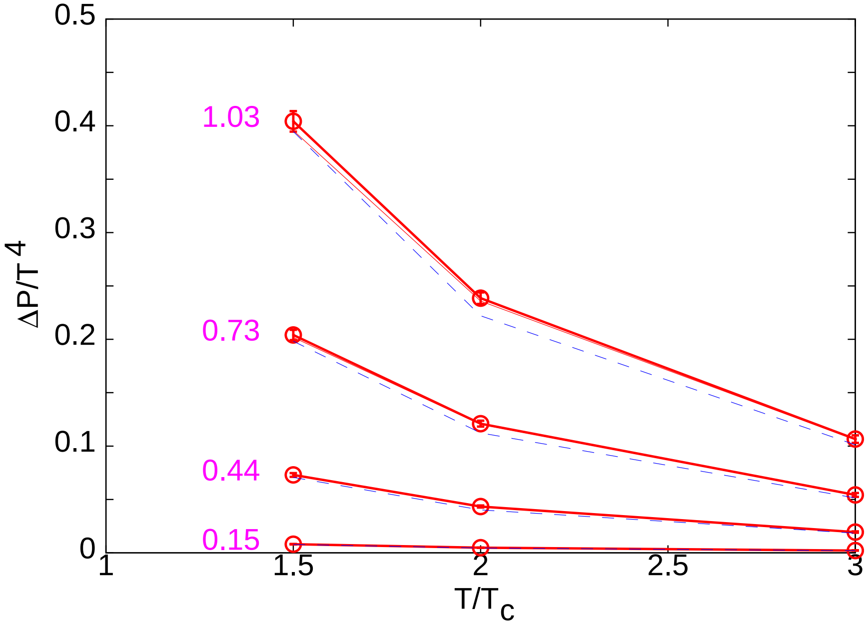

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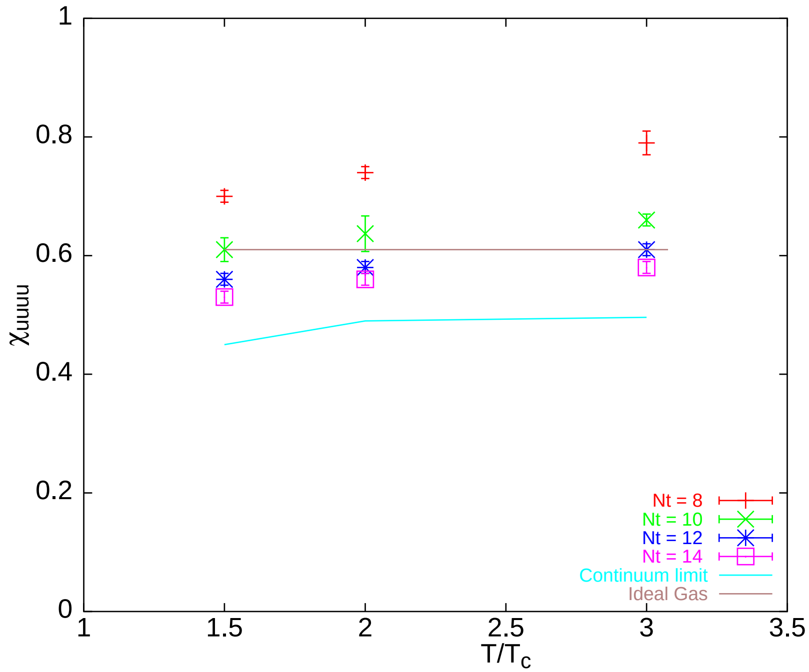

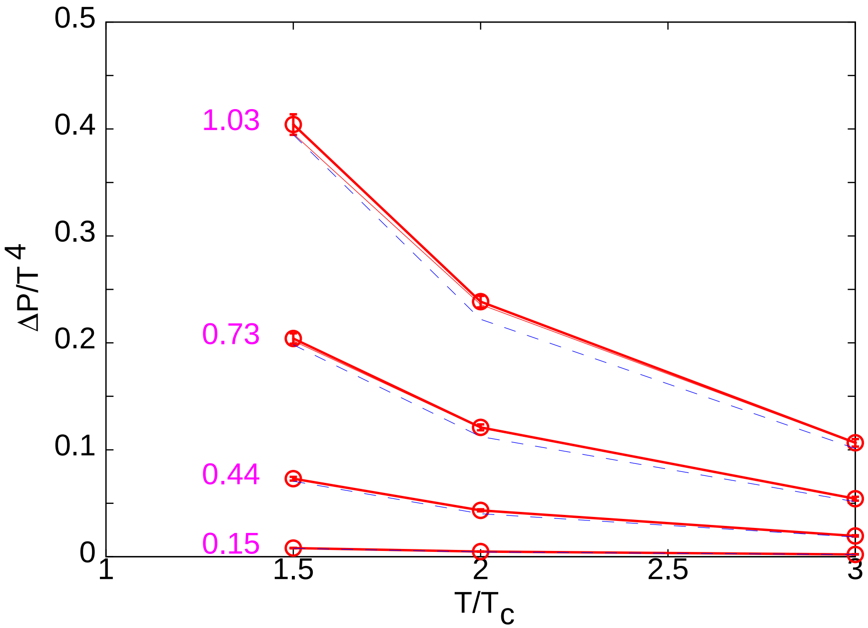

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♡ Our results for P agree with Fodor-Katz (PL B568, '03) and the recent Bielefeld results (PR D68, '03).

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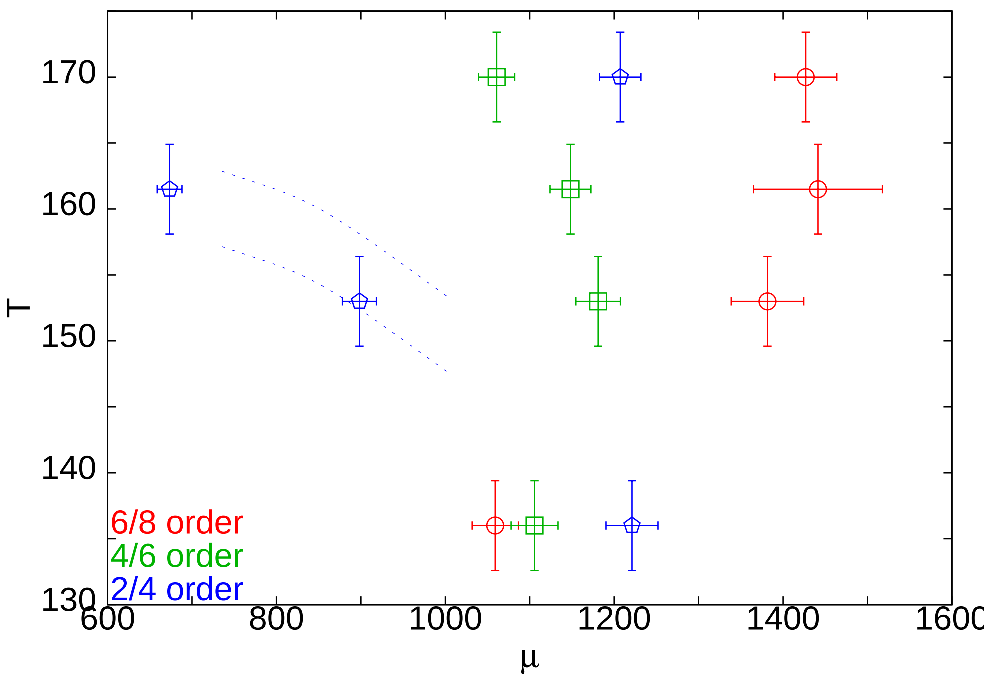

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- Pressure for nonzero μ obtained. At both SPS and RHIC, χ_{uu} is the major contribution.
- Phase diagram in $T - \mu$ on small $N_t = 4$ has begun to emerge: Different methods, $\rightsquigarrow$ same (T_E, μ_E). Beware of prescription dependence and look forward to larger N_t.