Lattice QCD approach for finite density and finite temperature systems

- Transport Coefficients and Related Topics -

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Field Theories Near Equilibrium
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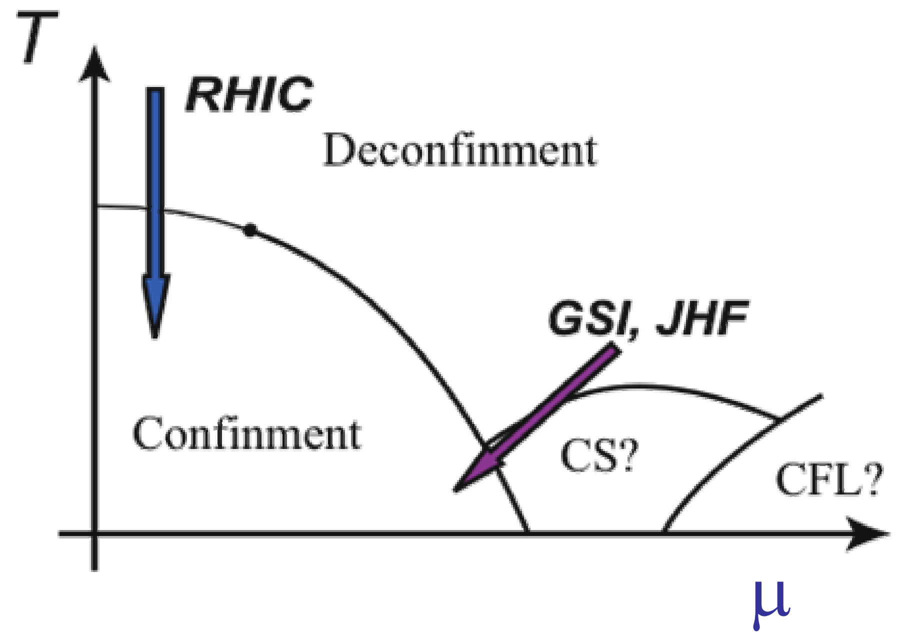
Plan of the Talk

- Introduction and Formulation
- Several Topics of Lattice QCD Approach for finite temperature and density systems
- Transport Coefficients by Lattice QCD
- Summary

Introduction

- QCD is Standard Theory of Hadrons/ Quark-Gluon system at T=0 and μ=0
- QCD is expected to be very rich at finite T and μ.
- The new form of matter can be explored in Relativistic Heavy Ion Collision Experiments.
- Lattice QCD can provide information based on the first principle calculation.



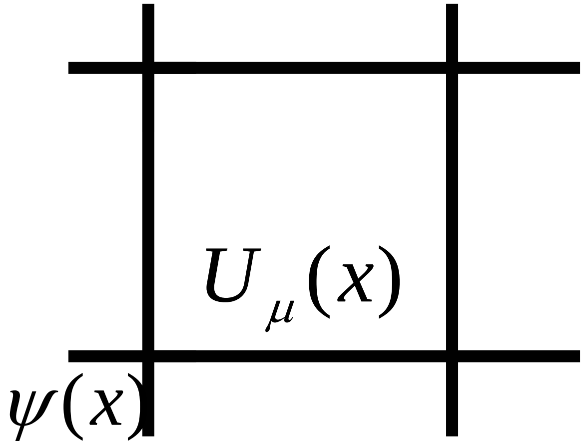
Lattice QCD

$$Z = \mathrm{Tr}\, e^{-\beta(H-\mu N)} = \int DUD\overline{\psi}D\psi\, e^{-(S_G+\overline{\psi}\Delta\psi)}$$

$$= \frac{1}{Z}\int DU \det\Delta\, e^{-S_G}$$

$$S_G = \beta \,\square$$

$$\Delta = D_\nu \gamma_\nu + m + \mu\gamma_0$$



$$U_\mu(x) = e^{iA_\mu(x)}$$

$$\det\Delta \to const$$

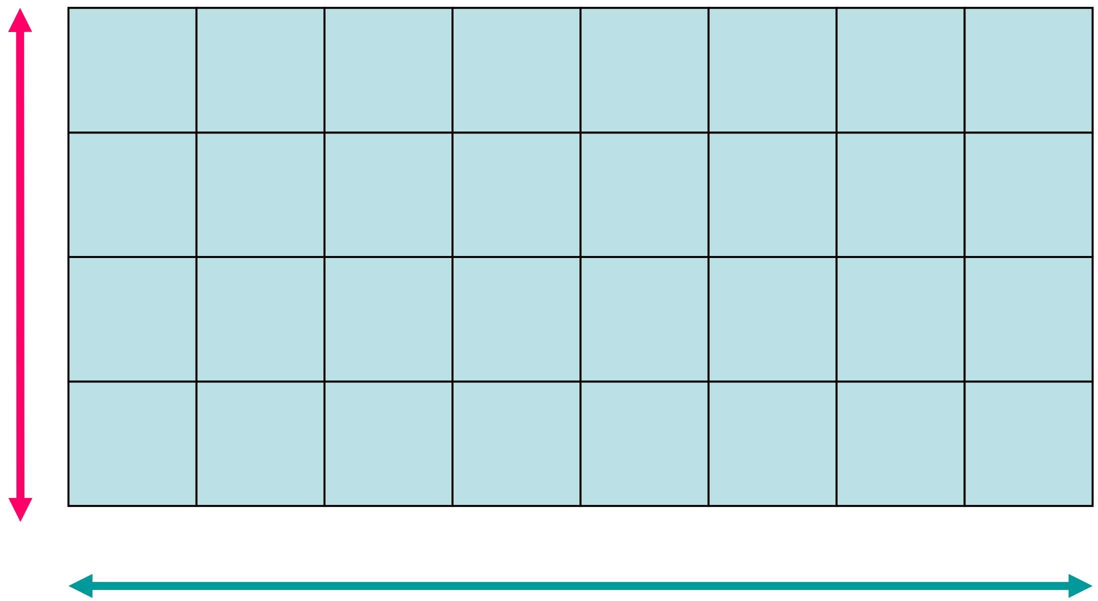
Quench Approximation

$A_\mu(x)$: Gluon Fields

$\psi(x), \overline{\psi}(x)$: Quark Fields

Some Special Features of Lattice QCD at Finite Temperature and Density

$$N_t a_t = \frac{1}{kT}$$



$$N_s a_s$$

High Temperature ⇒ $N_t a_t$: small

Finite Density

$$Z = \mathrm{Tr}e^{-\beta(H-\mu N)} = \int DUD\overline{\psi}D\psi\, e^{-(S_G+\overline{\psi}\Delta\psi)}$$

$$= \frac{1}{Z}\int DU \det\Delta\, e^{-S_G}$$

$$U_\mu(x) = e^{iA_\mu(x)}$$

$$U_t(x) \to e^{\mu}U_t(x),$$

$$U_t^\dagger(x) \to e^{-\mu}U_t^\dagger(x)$$

$$\Delta = D_\nu\gamma_\nu + m + \mu\gamma_0$$

At $\mu = 0$

$$(\det\Delta)^* = \det\Delta^\dagger = \det\gamma_5\Delta\gamma_5 = \det\Delta$$ ➡ $\det\Delta$: real

At $\mu \neq 0$

$$\Delta^+ = -D_\nu\gamma_\nu + m + \mu\gamma_0 \neq \gamma_5\Delta\gamma_5$$ ➡ $\det\Delta$: complex

Real Time Green function vs. *Temperature* Green function

Hashimoto, A.N. and Stamatescu, Nucl.Phys.B400(1993)267

$$<<\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]>>\equiv\frac{1}{Z}\mathrm{Tr}(\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]e^{-\beta H})$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Lambda(\omega,\vec{p})$$

$$\phi(t,\vec{x})=e^{itH}\phi(0,\vec{x})e^{-itH}$$

$$G_{\beta}^{ret/adv}(t,\vec{x};t',\vec{x}')=\pm\theta(t-t'/t'-t)<<...>>$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}K_{\beta}^{ret/adv}(\omega,\vec{p})$$

$$K_{\beta}^{ret/adv}(\omega,\vec{p})=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Lambda(\omega')}{\omega-\omega'\pm\varepsilon}$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') = << T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}') >>$$

$$\phi(t,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

$$G_\beta(\tau,\vec{x};0,0) = G_\beta(\tau+\beta,\vec{x};0,0)$$

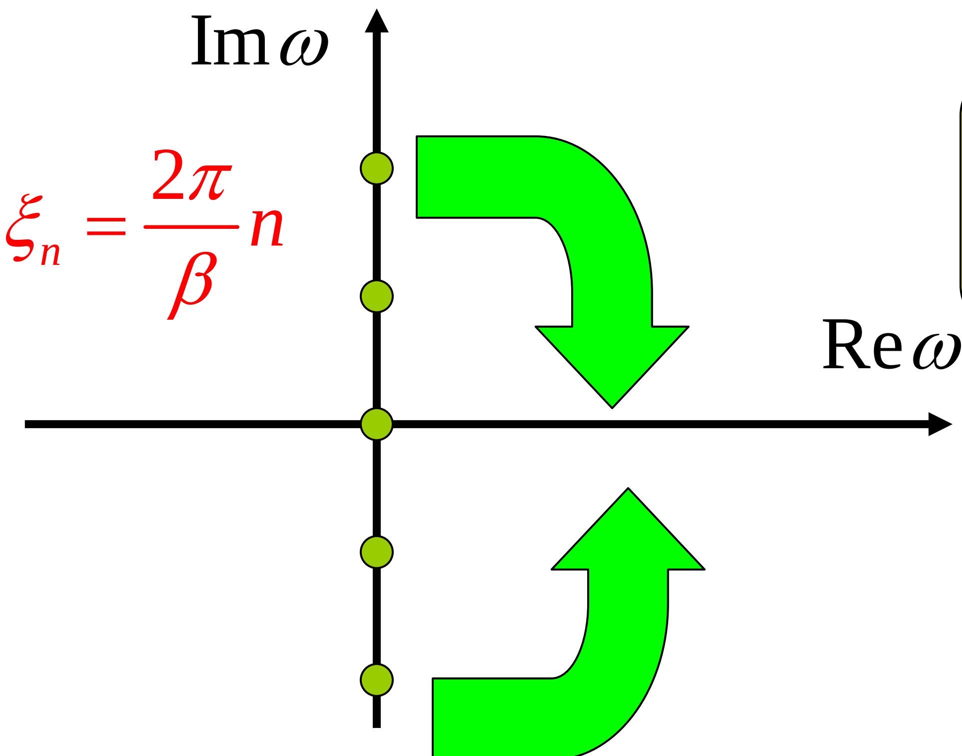
$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau e^{-i\xi_n(\tau-\tau')}G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

$$\xi_n = \frac{2\pi}{\beta}n, n = 0, \pm 1, \pm 2,,,$$

Matsubara-frequencies

Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

$$\hat{K}_\beta(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_\beta(i\xi_n)$$



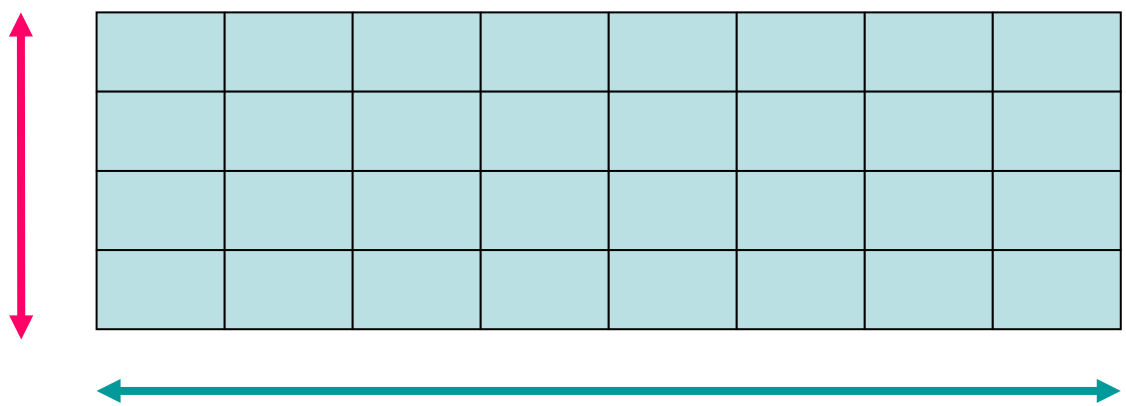
On the lattice, we measure Temperature Green function at
$$\omega = \xi_n$$

We must reconstruct Advance or Retarded Green function.

Development of Tools

- Anisotropic Lattice
  - Burgers, Karsch, Nakamura and Stamatescu, Nucl.Phys. B204 (1988) 587

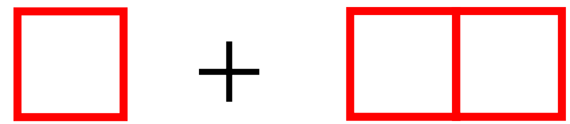
$$N_t a_t = \frac{1}{kT}$$



$N_s a_s$

$a_t < a_s$ anisotropic lattice

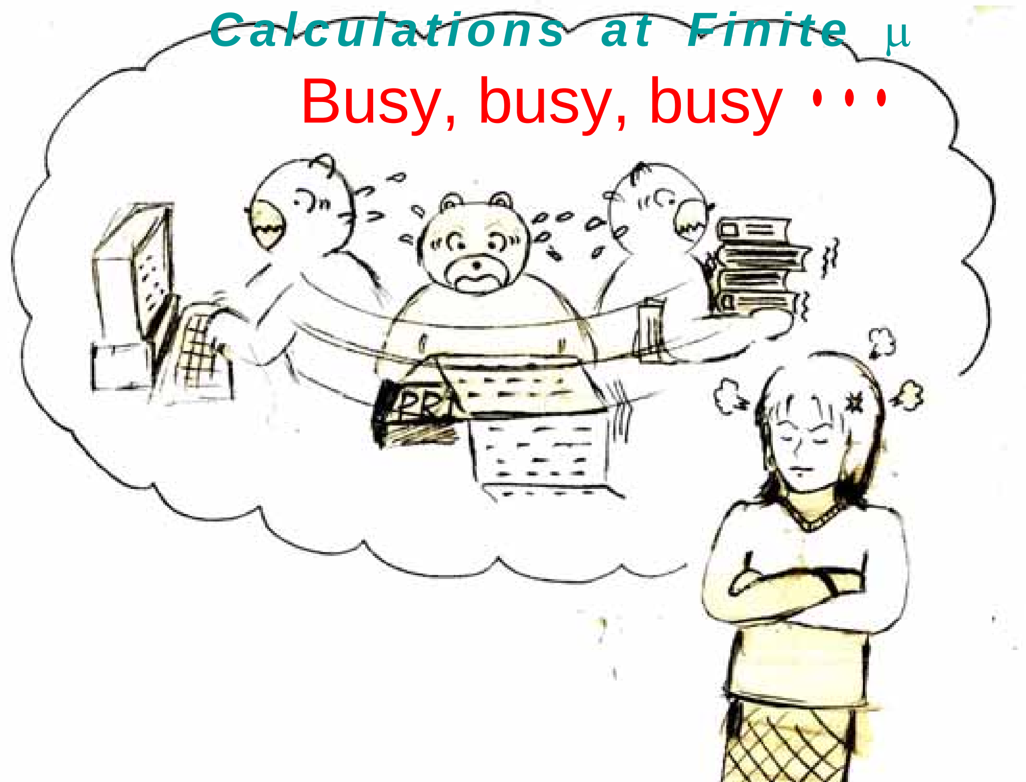
fine resolution along temperature direction



- Improved Actions
  - QCDTARO Collaboration,Nucl.Phys. B577, (2000) 263
- Anisotropic improved actions
  - Sakai, Saito and A. N, Nucl.Phys. B584, (2000) 528, Sakai and A.N.
- Maximum Entropy Method for Spectral Functions, QCD-TARO Collaboration, Nucl.Phys. B(Proc.Suppl.)63, 1998, 460
  - (Later Asakawa, Hatsuda and Nakahara developped MEM in more sophistic way.)
- Gauge Fixing
  - Gluon probagators at finite tempearture
  - Grinov Copy Problem

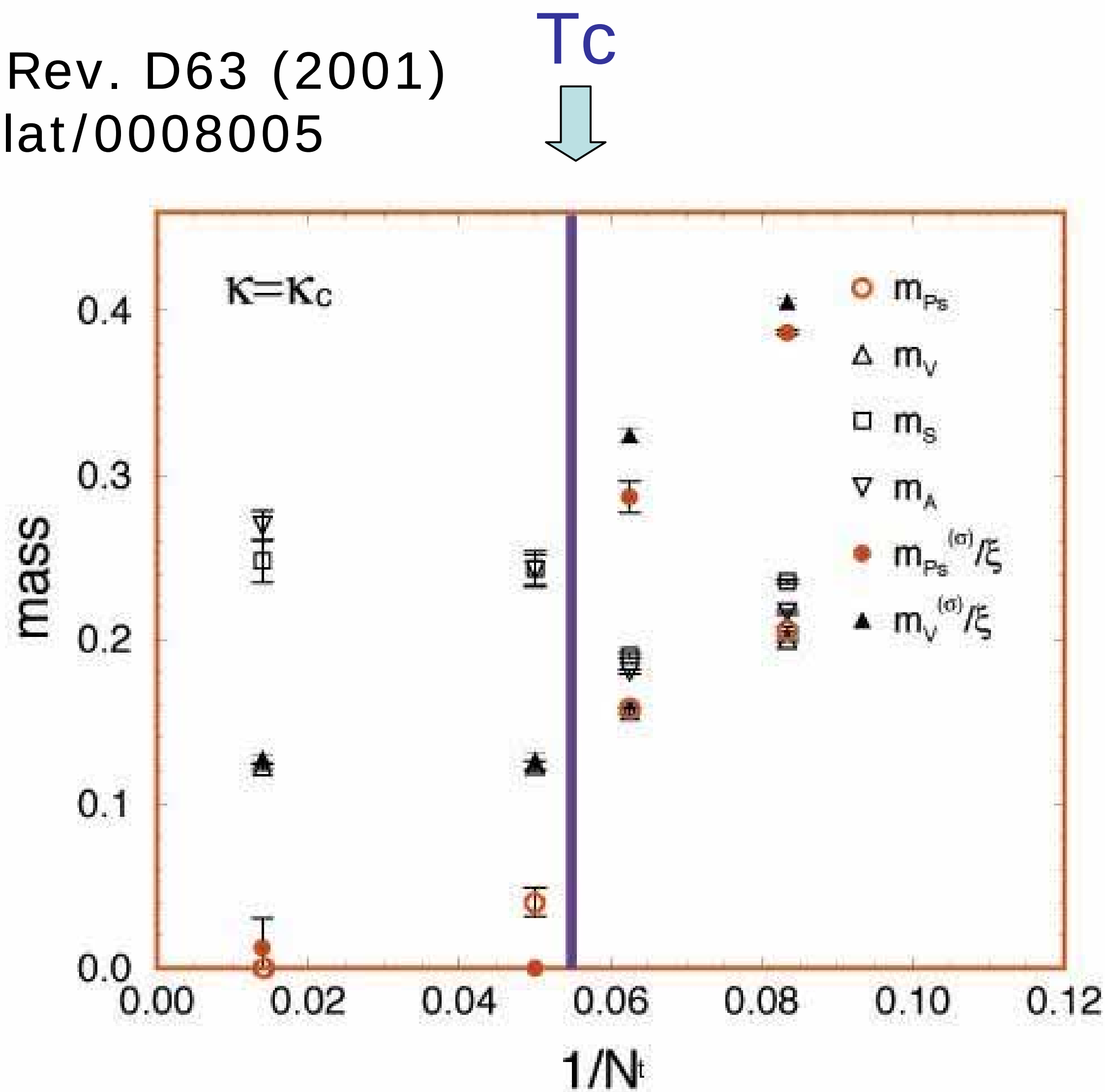
Hadronic Properties at finite Temperature and Density

- Pole and Screening Masses at finite temperature
- Response of Screening Masses at small finite chemical potential
- Vector meson mass at finite chemical potential
- Gluon Screening Masses

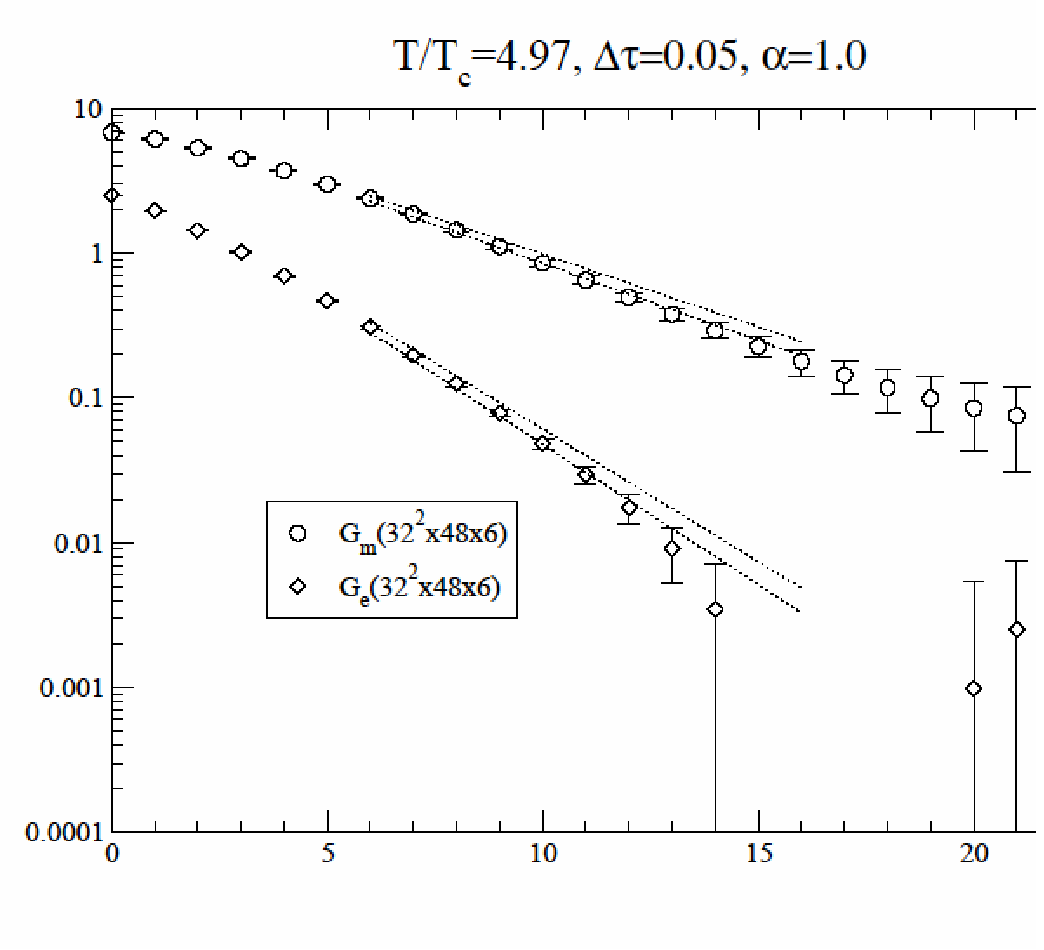


Pole mass and Screening mass

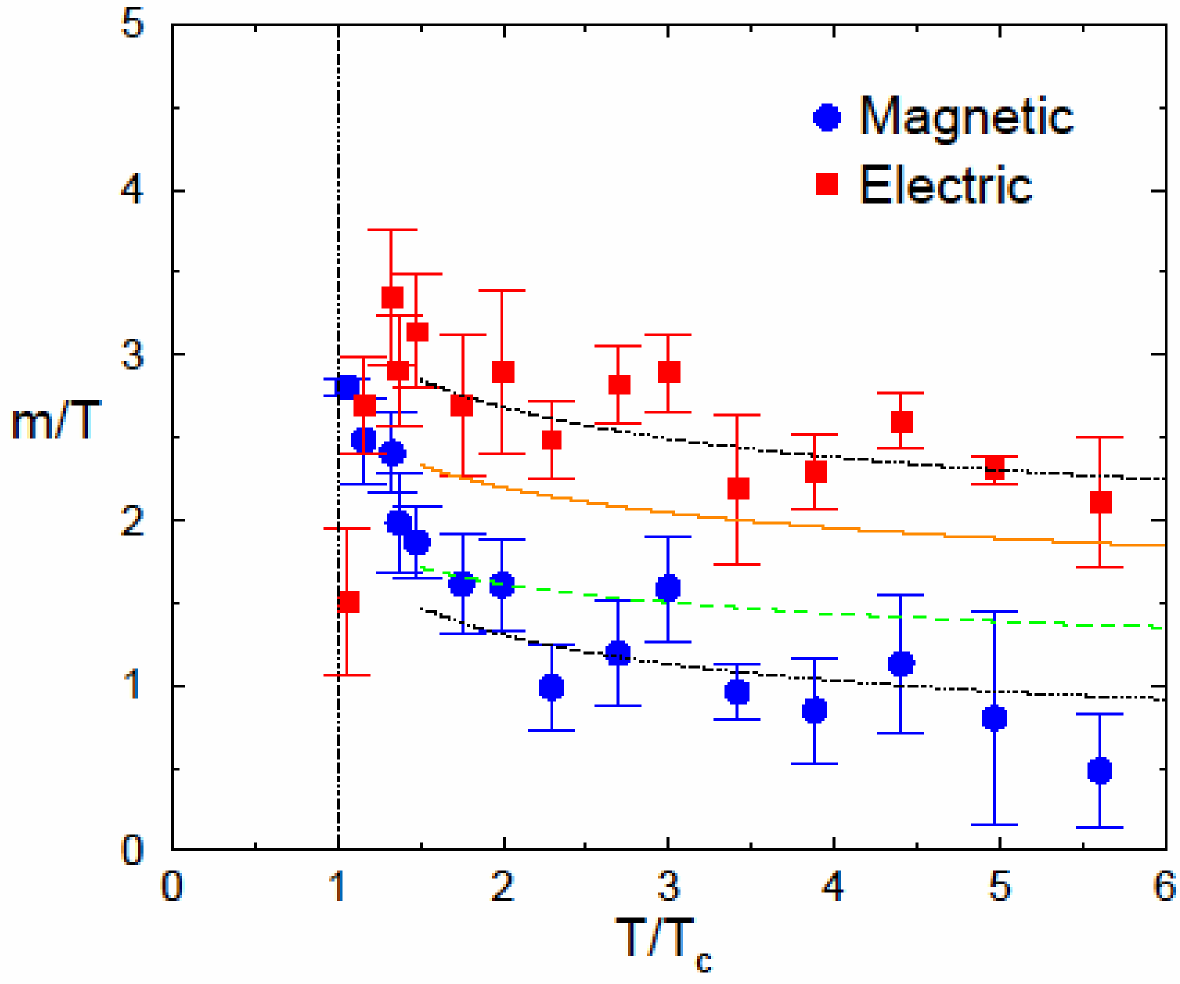
QCD-Taro, Phys.Rev. D63 (2001) 054501,hep-lat/0008005



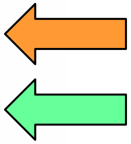
Gluon Propagators



Gluon's screening mass



A.N., Pushkina, Saito and Sakai Phys. Lett. B549 (2002), 133 (hep-lat/0208075): A. N., Saito and Sakai hep-lat/0311024, to appear in Phys. Rev. D

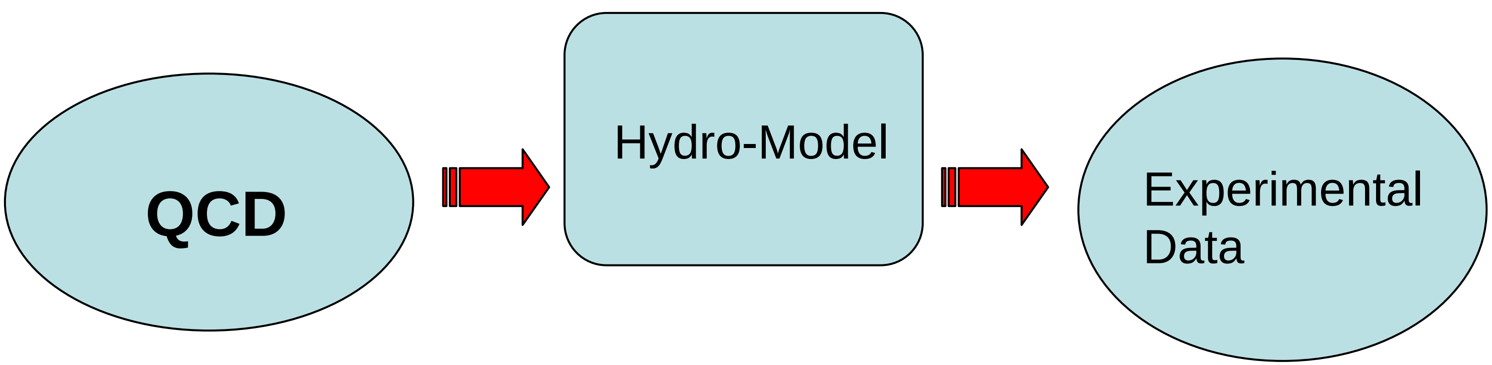


Hard Thermal Loop

Leading Order Perturbation

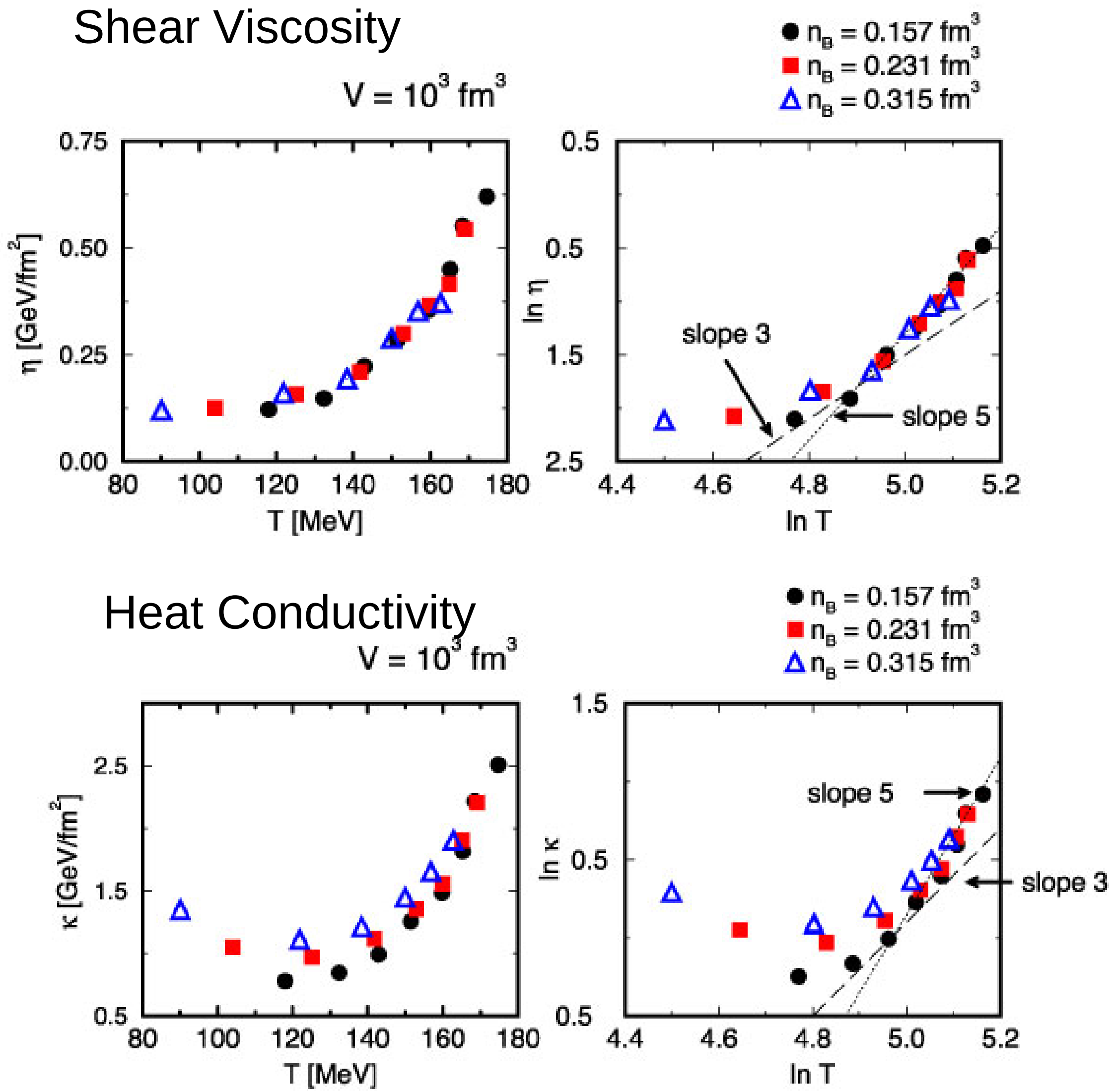
Transport Coefficients

- A Step towards Gluon Dynamical Behavior.
- They can be (in principle) calculated by a well established formula (Kubo Formula).
- They are important to understand QGP which is realized in RHIC (and CERN-SPS)

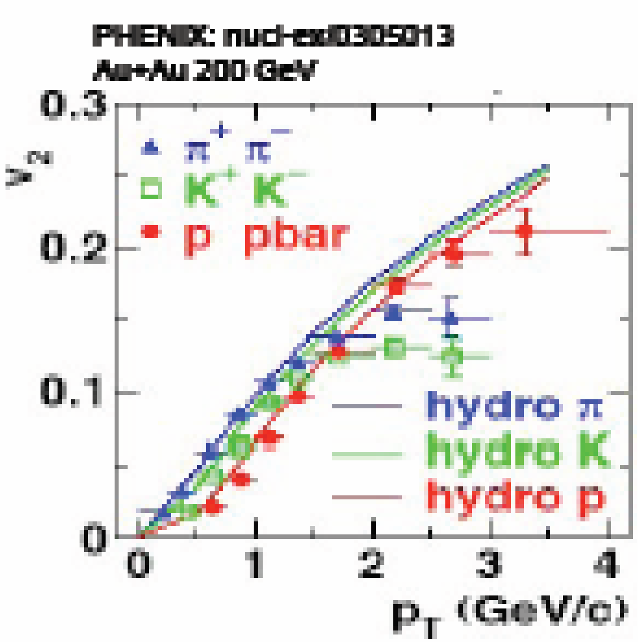


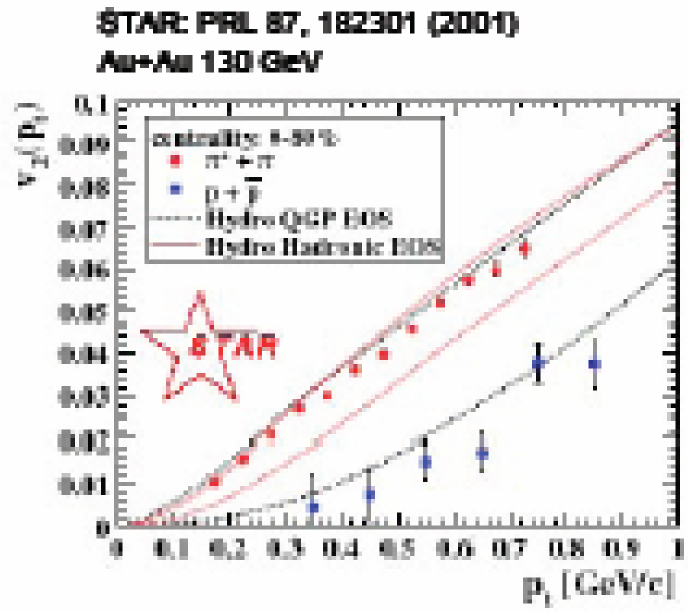
Transport Coefficients of Hadronic System

- Results by Event generator
- Sasaki, Muroya and Nonaka
  - Private com-munication



Hydro describes well v2





Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

Literature

- Iso, Mori and Namiki, Prog. Theor. Phys. 22 (1959) pp.403-429
  - Applicability Conditions of the Hydrodynamical Model of Multiple Production of Particles from the Point of View of Quantum Field Theory,
- Hosoya, Sakagami and Takao, Ann. Phys. 154 (1984) 228.
- Hosoya and Kayantie, Nucl. Phys. B250 (1985) 666.
- Horsley and Shoenmaker, Phys. Rev. Lett. 57 (1986) 2894; Nucl. Phys. B280 (1987) 716.
- Karsch and Wyld, Phys. Rev. D35 (1987) 2518.
- Aarts and Martinez-Resco, JHEP0204 (2002)053, hep-lat/0209033(Lattice02), hep-ph/0203177.

Literature (2)

- A.N, Saito and Sakai
  Nucl.Phys. B(Proc.Suppl.)63, (1998), 424
- Sakai, A.N. and Saito
  Nucl.Phys. A638, (1998), 535c
- A.N., Sakai and Amemiya
  Nucl.Phys. B(Proc.Suppl.)53, (1997), 432
- Masuda, A.N.,Sakai and Shoji
  Nucl.Phys. B(Proc.Suppl.)42, (1995),526

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 gF_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = (U_{\mu\nu} - 1) / ia^2 g$$

Kubo’s Linear Response Theory

- Zubarev
  “Non-Equilibrium Statistical Thermo-dynamics”
- Kubo, Toda and Saito
  “Statistical Mechanics”

Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors $< T_{\mu\nu}(0)T_{\mu\nu}(\tau) >$

Convert them (Matsubara Green Functions) to Retarded ones (real time).

Transport Coefficients (Shear Viscosity, Bulk Visicosity and Heat Conductivity)

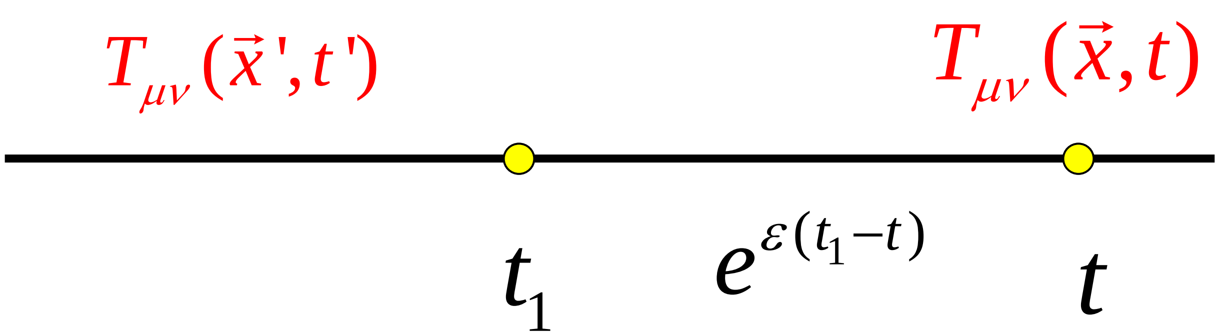
$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x}, t) T_{12}(\vec{x}', t') >_{ret}$$

$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x}, t) T_{11}(\vec{x}', t') >$$

$$\chi = -\frac{1}{T} \int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x}, t) T_{01}(\vec{x}', t') >_{ret}$$

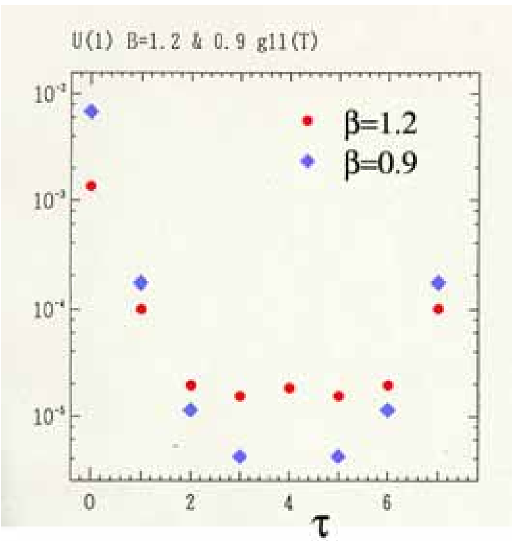
η **: Shear Viscosity** ς **: Bulk Viscosity**

χ **: Heat Conductivity**



$$-\infty < t' < t_1 < t$$

Correlators



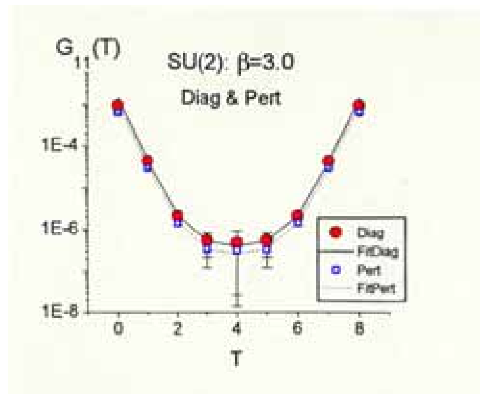
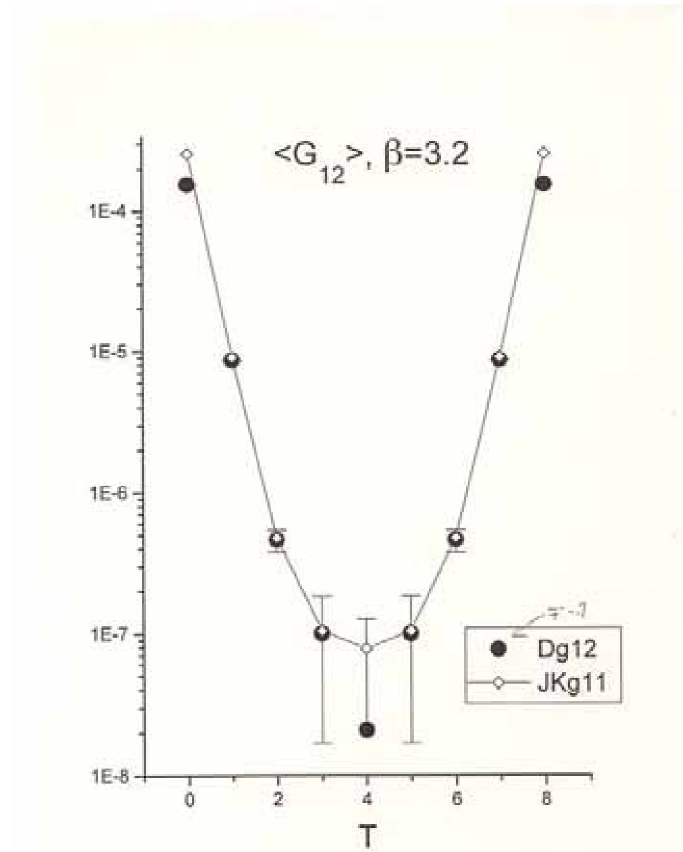


Figure 3. Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta = 3.0$



U(1)
Coulomb and Confinement Phases

SU(2)

Two Definitions:

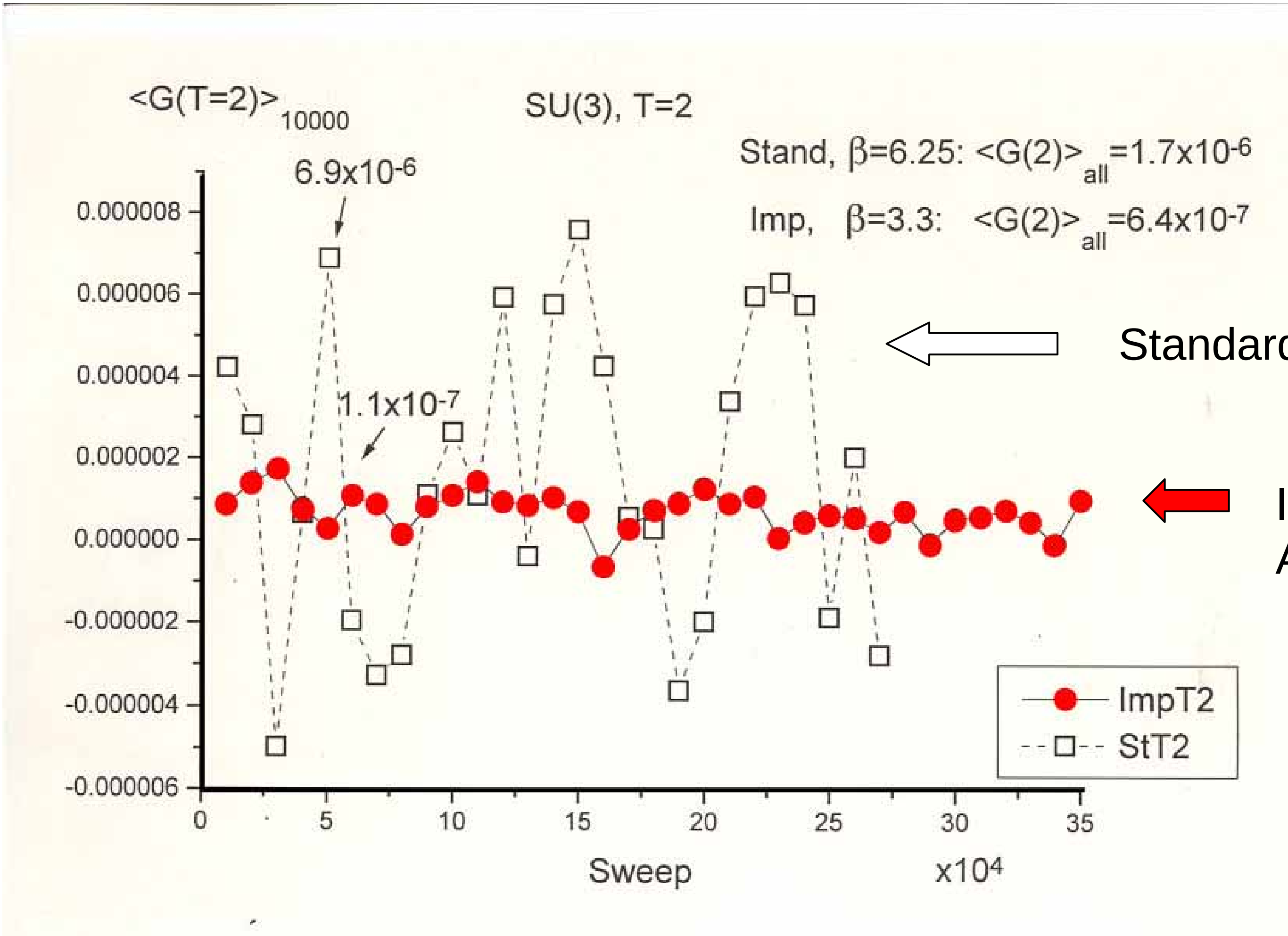
F=log U

F=U-1

SU(3)

Improved Action

Fluctuations in MC sweeps



Standard Action

Improved Action

Errors in U(1), SU(2), SU(3) standard and SU(3) improved

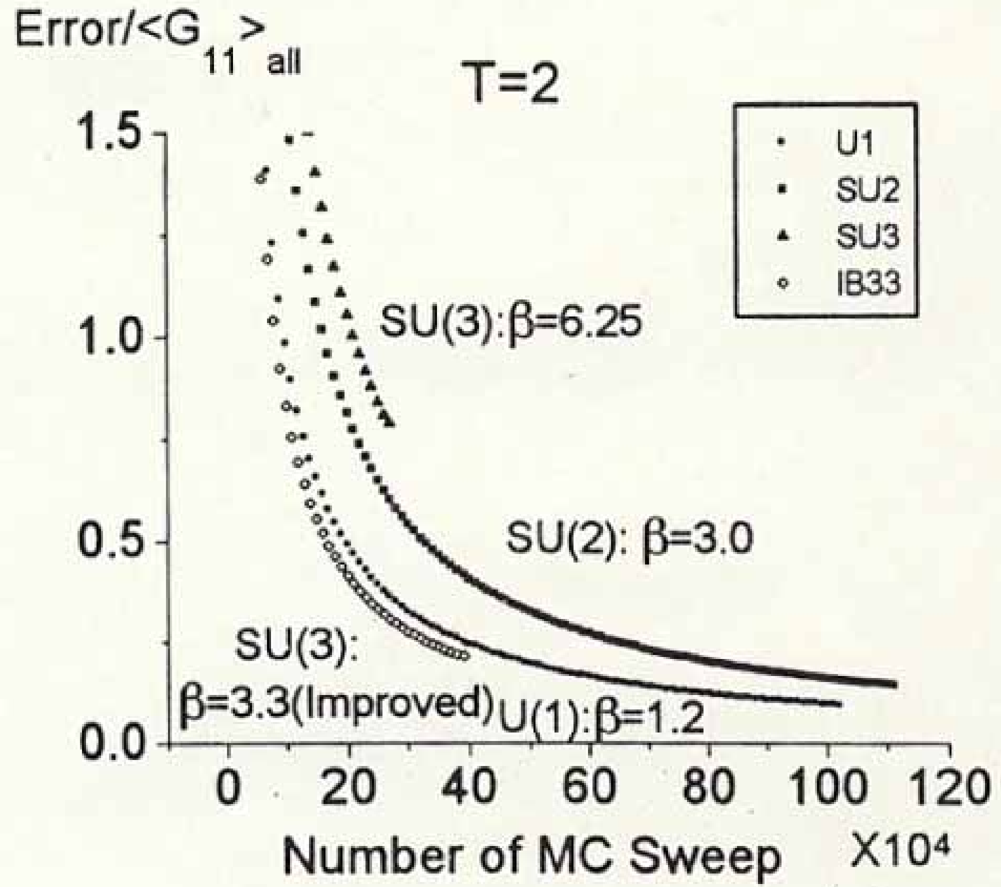


Figure 2. Error as a function of number of MC sweeps at $T = 2$ for U(1) $\beta = 1.2$, SU(2) $\beta = 3.0$, SU(3) $\beta = 6.25$ and improved action for SU(3)

Assumption for the spectral functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$< T_{\mu\upsilon}(t,\vec{x}) T_{\mu\upsilon}(0) >= G_\beta(t,\vec{x}) = F.T.G_\beta(\omega_n, \vec{p})$$

$$G_\beta(\vec{p}, i\omega_n) = \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

We assume

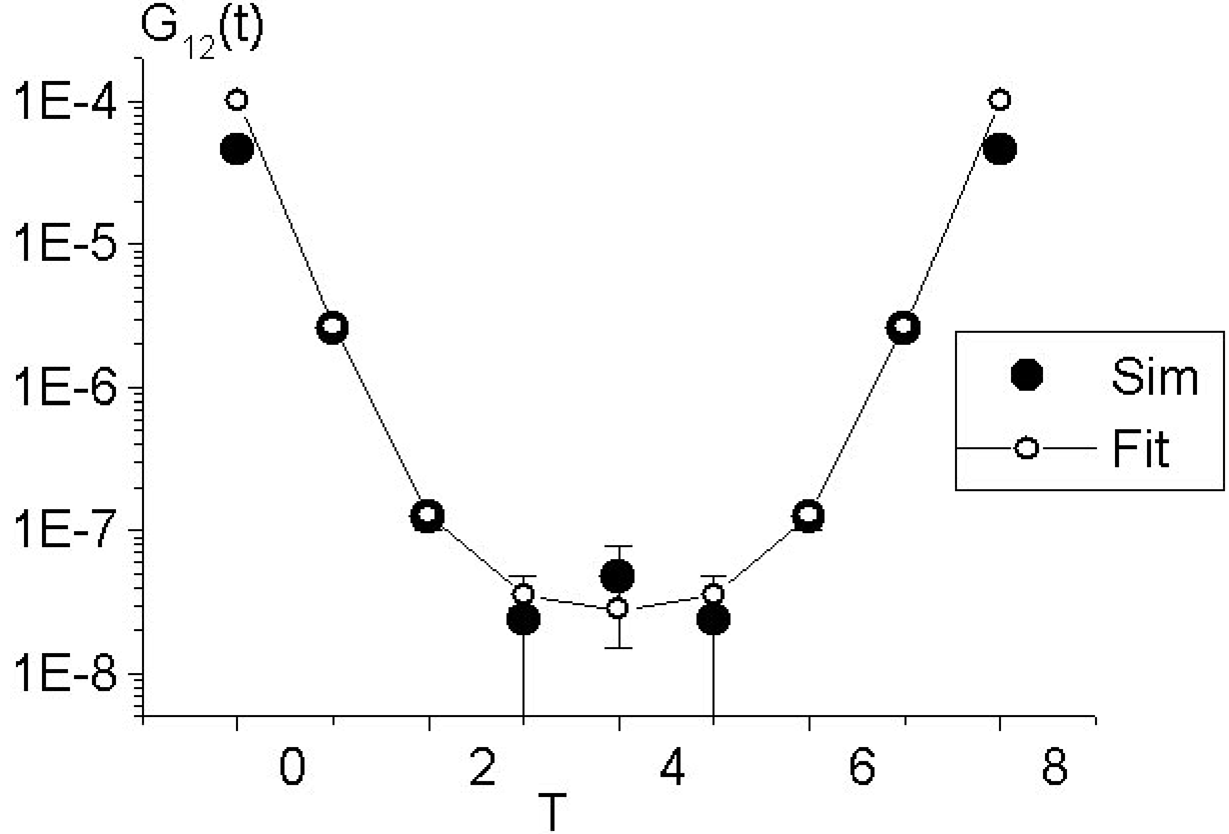
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2 + \gamma^2} + \frac{\gamma}{(m+\omega)^2 + \gamma^2} \right)$$

and determine three parameters,

A, m, γ.

We need large Nt !

Nt=8



Lattice and Statistics

Iwasaki Improved Action

$$16^3 \times 8$$

$\beta=3.05$: 1333900 sweeps

$\beta=3.20$: 1212400 sweeps

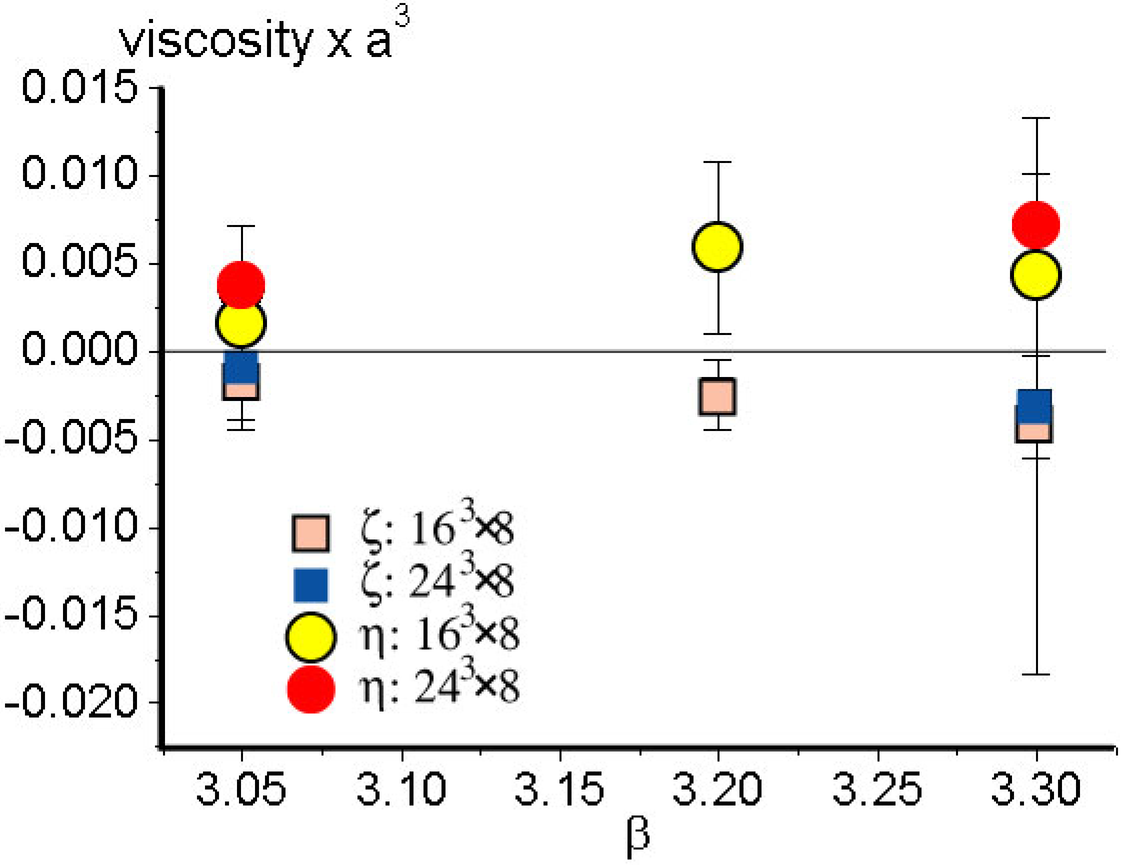
$\beta=3.30$: 1265500 sweeps

$$24^3 \times 8$$

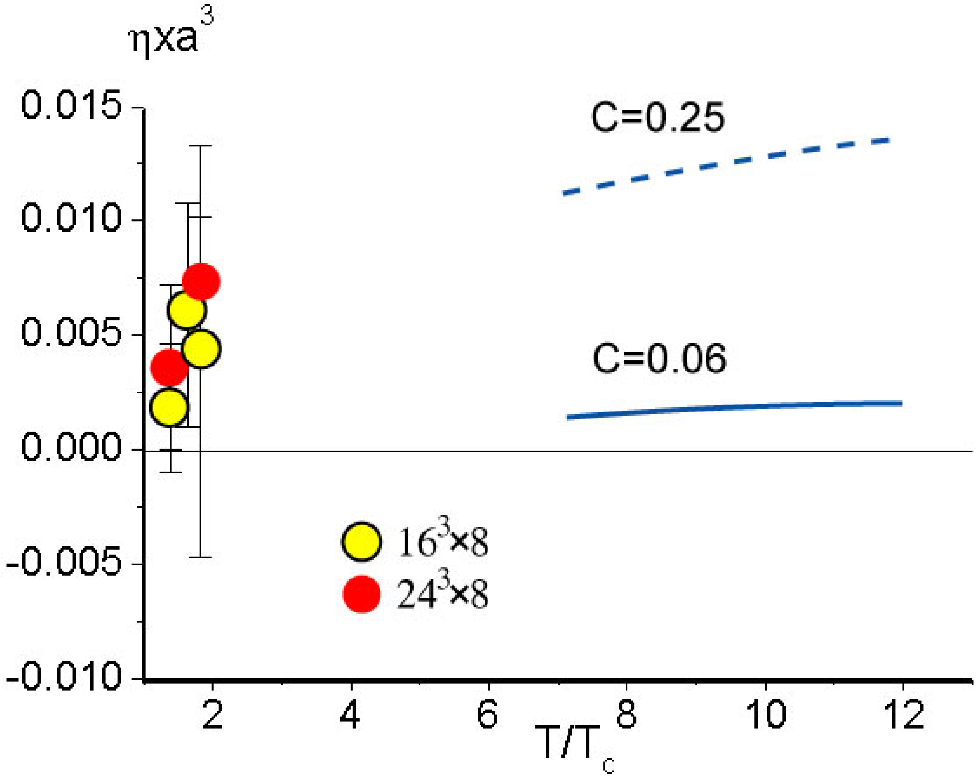
$\beta=3.05$: 61000 sweeps

$\beta=3.30$: 84000 sweeps

Results: Shear and Bulk Viscosities



Results: Shear and Bulk Viscosities together with Perturbation



Perturbation

$$\eta a^3 = \frac{C}{N_t^{\ 3} \alpha_s^{\ 2} \log \alpha_s}$$

Hosoya-Kajantie:C=0.06

Horsley-Shoenmaker:
C=0.08~0.25

$$\frac{\eta}{T^3} = \frac{\eta}{\left(\frac{1}{N_t a}\right)^3} = (8a)^3 \eta$$

Summary

- We have calculated Transport Coefficients on Nt=8 Lattice:
  - We can fit three parameters in the Spectral Function:

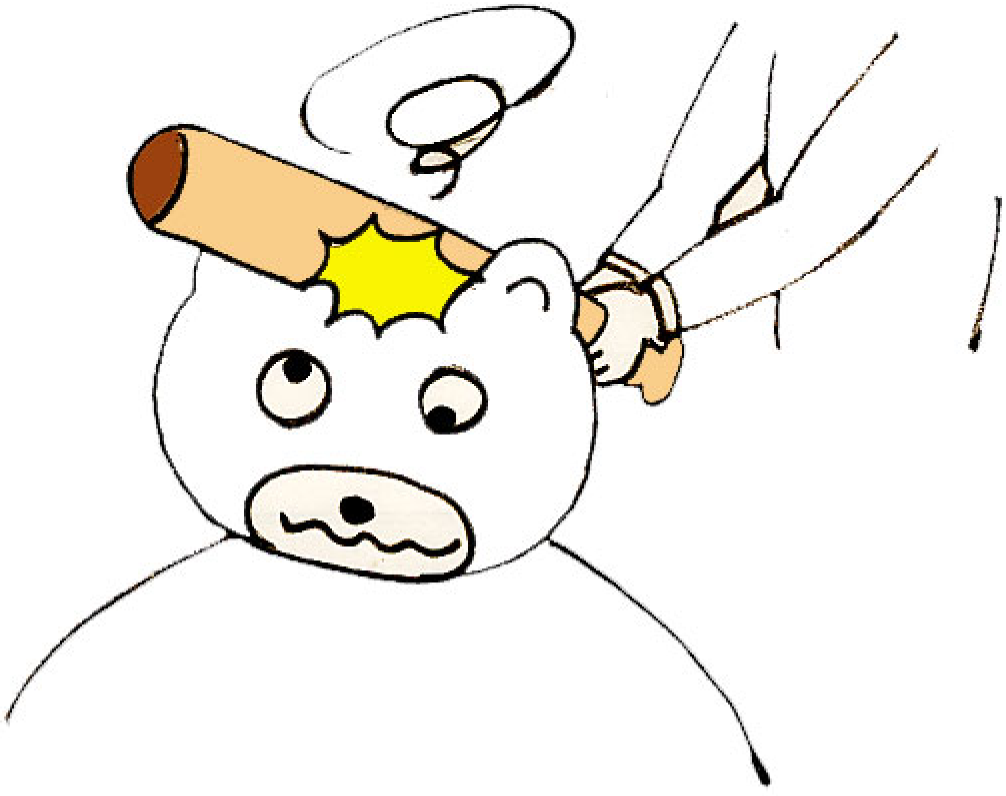
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2}+\frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

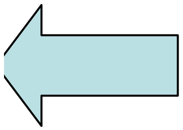
- Shear Viscosity
  - Positive
  - consistent with the extrapolation of the Perturbative calculation
- Bulk Viscosity ~ 0
- Heat Conductivity suffers from large Noise, and cannot be obtained.
- Improved Action works well to get good Signal/Noise ratio.
  - Coarse lattice results in $T_{\mu\nu}$ with noise ?

Future direction ?

- Anisotropic lattice has matured and will help us to get more data points to determine the spectral function.
- Then, we may improve the treatment of the spectral function.
  - More parameters ?
  - More sophisticated functional form ?
  - If Maximal Entropy Method works, we can determine the spectral function ?
  - But

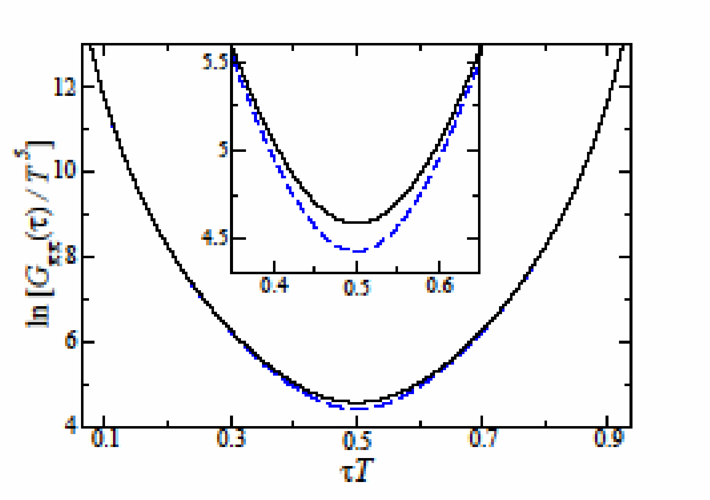
G.Aarts and J.M. Martinez Resco
"Transport coefficients from the lattice ?“
Talk at Lattice 2002 (Boston)





Gert Aarts

G(τ) is remarkably insensitive to details of ρ(ω) when ω<<T and we conclude that it is extremely difficult to extract transport coefficients in weakly-coupled field theories from the Euclidean lattice"



"this result is a potential problem for the Maximal Entropy Method when the reconstruction of the low-frequency parts of spectral functions is attempted."