Cancellation of global anomalies in spontaneously broken gauge theories

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Anomalies, I

in gauge theories and gravity

local

gauge & gravity & mixed

$\delta_\alpha \Gamma \neq 0$

$$\delta_\alpha \Gamma = -\int d^D x; \alpha(x)^a \mathcal{G}(x)^a$$

$$\mathcal{G}(x) = D_\mu J^\mu(x)$$

global

gauge & gravity

$\pi_D(G) \neq 0$

$$\mathcal{M}^D \to \mathcal{S}^D \quad U : \mathcal{M}^D \to G$$

ex: $\pi_4(SU(2)) = Z_2$

`Witten, PLB (1982) 117`

Anomalies, II

model building contraints

- global $\longrightarrow$ chiral field content
- local $\left\{ \begin{array}{l} \text{irreducible} \longrightarrow \text{chiral field content} \\ \text{reducible} \longrightarrow \text{Green-Schwarz mechanism} \end{array} \right.$

Example, I

Standard model in $D = 6$ dimensions

[Dobrescu & Poppitz, PRL 87 (2001) 031801]

	Chirality	$U(1)_Y$	$SU(2)_L$	$SU(3)_c$
Q	$+$	$1/6$	2	3
U	$-$	$2/3$	1	3
D	$-$	$-1/3$	1	3
L	$+$	$-1/2$	2	1
E	$-$	-1	1	1
N	$-$	0	1	1

- irreducible (gauge & gravity) anomalies canceled by chiral field content
- reducible anomalies canceled by Green-Schwarz mechanism $\rightarrow$ 4 axions: 2 decoupled + 1 (photon) + 1 (gluon & photon) $\rightarrow$ bounds: $1/R > 10^6$ TeV ($M_f > 10^{11}$ TeV)

Example, II

Standard model in $D = 6$ dimensions

$SU(2)_L$ global anomaly cancellation

$$\pi_6(SU(2)) = Z_{12}$$

number of doublets of chirality $\pm$

$$N(2_+) - N(2_-) = 0 \bmod 6 = N_g \times (3+1)$$

$$\boxed{N_g = 0 \bmod 3}$$

chiral field content constraint

The Wess-Zumino action

Anomaly matching

in spontaneously broken gauge theories $G \to H$

U Goldstone bosons

$$\Gamma_{WZ}(A,U) \to \delta_\alpha \Gamma_{WZ} \neq 0$$

Anomaly cancellation

$$\delta_\alpha \Gamma_{WZ} + \delta_\alpha \Gamma = 0$$

The simplest example: $G \to 1$

- local anomaly: $\delta_\alpha \Gamma[A] = -\int d^D x \, \mathcal{G}^\alpha[A(x)]$
- no homotopy problems: $\pi_D(G) = 0$
- Goldstone bosons ξ^A: $U(x) = e^{i\xi^A(x)T^A}$
- WZ action: $\Gamma_{WZ}(A, \xi) = i \int_0^1 dt \int d^D x \, \xi^A(x) \mathcal{G}^\alpha[A_t]$
  $(A_t = e^{-it\xi} A e^{it\xi} + i e^{-it\xi} d e^{it\xi})$
- Abelian case: $\Gamma_{WZ} \simeq \int \xi F \tilde{F}$

Canceling global and local anomalies

[MF, Percacci, Piai, Serone, PRD 66 (2002) 105028]

1. the coset space G/H is reductive: $[T^i, T^\alpha] = i f_{i\alpha\beta} T^\beta$;
2. the fermion representations are free of local anomalies when restricted to the group H;
3. the fermion representations are free of global anomalies when restricted to the group H;
4. G can be embedded in a group K such that its homotopy group $\pi_d(K) = 0$ and the fermion representations can be extended to K without generating further anomalies of G.

Example, III

Standard model in $D = 6$ dimensions

$$K = SU(4)_c \times SU(4)_L \times U(1)_Y$$

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$H = SU(3)_c \times U(1)_{e.m.}$$

- Reducible (local) anomalies canceled without additional fields $\longrightarrow$ no axions
- Global anomalies canceled $\longrightarrow$ no constraint on families