SERC SCHOOL ON STATISTICAL PHYSICS, 16-28 FEB., 2004

COURSE OUTLINES

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Course 1: "General theory and rigorous results"

Lecturer: Prof. Deepak Dhar

Tentative Outline of Lectures:

Lecture 1 : What do we study in Statistical Physics?
Importance of the thermodynamic limit
Proof of existence of thermodynamic limit in classical and quantum systems

Lecture 2 : Stable interactions for classical continuum systems
Correlation functions, Ursell expansion
Convergence of Mayer expansion
Other weak and strong coupling expansions

Lecture 3 : Lee-Yang theory of mechanism of phase transitions
Lee-Yang circle theorem
Extensions to other systems: dimers, extended hard-core lattice gases

Lecture 4 : Spontaneous symmetry breaking, Elitzur's theorem
Absence of phase transitions in 1 dimension
Mermin-Wagner theorem
Peierls' argument and generalizations
Infrared bounds for continuous-spin systems

Lecture 5 : Ferromagnetic inequalties of Griffiths-Kelly Sherman
Inequalities of Fortuin-Kasteleyn-Griffiths
Coupling and inequalities in non-equilibrium

Lecture 6 : Recapitulation

References : The material discussed is mostly selected from
R. B. Griffths article in Domb and Green "Phase Transitions and Critical Phenomena" Vol. I.
and D. Ruelle " Statisical Mechanics".

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Course 2: "Elements of Time-dependent Statistical Mechanics"

Lecturer: Prof. S. Ramaswamy

Tentative Outline of Lectures :

Lecture 1: (a) What do we measure and what must we calculate?
Experimental probes of the dynamics of many-particle systems,
Scattering vs dynamical susceptibility,
Local vs collective probes

(b) Random walks, Brownian motion and the Langevin equation,
Correlation and response of a damped Brownian oscillator

(c) General properties of correlation and response functions,
Linear response theory,
The fluctuation-dissipation theorem,
The Kubo formulae

Lecture 2: (a) Microscopic nonequilibrium statistical mechanics: a short discussion,
The BBGKY hierarchy,
The Boltzmann Equation and the H-theorem

(b) Stochastic dynamics,
Slow and fast variables,
Langevin Equations in general: ‘‘derivation’’ and properties,
Examples from critical dynamics and elsewhere

Lecture 3: Mesoscale dynamics of macroscopic systems: an introduction to models,

Conserved, critical and broken-symmetry modes,

Liquids, liquid crystals, and solids,

The isotropic ferro- and antiferromagnets: spin waves and damping,

Dynamics of dilute polymer solutions

Lectures 4 and 5 : Calculational methods and applications,

Functional integrals for statistical dynamics,

Dynamical Renormalization Group,

Randomly stirred fluids, a hint of turbulence

Lecture 6 : Unfinished business, current topics;

Conclusion, summary, problems

Much of what I cover is in papers, not books. However, useful REFERENCE TEXTS for various parts of the course include:

Resibois and de Leener (can't remember the title)

Chaikin P M and Lubensky T C: PRINCIPLES OF CONDENSED MATTER PHYSICS, Cambridge Univ Press, New Delhi 1998

Bhattacharjee J K: STATISTICAL PHYSICS: EQUILIBRIUM AND NONEQUILIBRIUM ASPECTS, Allied Publishers, New Delhi 2000

Forster D: HYDRODYNAMIC FLUCTUATIONS, BROKEN SYMMETRY AND CORRELATION FUNCTIONS, Addison-Wesley, Reading, Mass. 1983.

Landau L D and Lifshitz E M: PHYSICAL KINETICS (Pergamon, New York)

Boon J-P and Yip S: MOLECULAR HYDRODYNAMICS (Dover, New York 1980)

de Groot S R and Mazur P: NONEQUILIBRIUM THERMODYNAMICS (Dover, New York 1984)

van Kampen N G: STOCHASTIC PROCESSES IN PHYSICS AND CHEMISTRY (North-Holland 1992)

Risken H: THE FOKKER-PLANCK EQUATION (Springer-Verlag 1985)

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Course 3: "Scaling and Critical Phenomena"

Lecturer: Prof. Mustansir Barma

Tentative Outline of Lectures:

Lecture 1: Phenomenology of phase transitions,
The Ising model and the lattice gas,
Mean field theory,
Landau theory

Lecture 2: Correlation functions; Ornstein-Zernike theory,
The Heisenberg, XY and O(n) models,
Critical behaviour: scaling and universality,
Upper and lower critical dimensions

Lecture 3: The Potts model and the cluster representation,
Renormalization group: basic idea, flows, fixed points,
Decimation for the 1-d Potts model,
Approximate real space RG for 2-d Potts models

Lecture 4: Multicriticality and crossover,
Corrections to scaling,
Momentum space RG; the Gaussian model,
Perturbation theory and the epsilon expansion

Lecture 5: RG near the lower critical dimension (Migdal approximation),
The 2-d XY model, Coulomb gas, and solid-on-solid model,
The Kosterlitz-Thouless transition; RG treatment,
2-d melting

Lecture 6: Review of first 5 lectures
Brief discussion of (i) Quantum critical points
(ii) Conformal invariance

Reference:

'Scaling and renormalization in statistical physics'

by J. Cardy (Cambridge Univ. Press, 1996)

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Course 4: "Exactly solved models"

Lecturer: Prof. Indrani Bose

Tentative Outline of Lectures :

Exactly-solved models are of significant interest in statistical mechanics as they provide an accurate description of the properties of systems described by the models. Approximate methods sometimes give wrong answers or miss out on essential features. Though exactly solved models are of special construction, their utility is three-fold: (1) many real systems can be described by these models. In most cases, the models related to real systems are not exactly-solvable but share common features with such models, (2) exact solutions provide important insight on general issues and (3) they provide testing grounds for approximate theories and numerical solutions. A model in equilibrium statistical mechanics is exactly solved if the partition function can be determined in an exact manner. Knowing the partition function, one can determine the free energy. The thermodynamic functions of a system, which are different derivatives of the free energy, can also be calculated in an exact manner. In particular, one can determine properties in the vicinity of phase transitions. The first and most well-known exactly solved model in statistical mechanics is the Ising model which is the simplest model of interacting spins. In the course of lectures, the transfer matrix (TM) technique of determining the exact partition function will be introduced. As an illustration, the Ising model in one dimension (1d) will be considered. In this model, phase transition from a disordered to an ordered phase of spins is not possible at a finite temperature. The ordered phase is obtained only at T = 0. The thermodynamic quantities of interest will be calculated and their nature in the vicinity of T = 0 discussed. Calculation of critical exponents and testing of the scaling hypothesis (an introduction to these concepts will be given in Prof. M. Barma's lectures) will be carried out. References: K. Huang, Statistical Mechanics
C. J. Thompson, Classical Equilibrium Statistical Mechancs

The next model to be considered is the transverse Ising model (TIM) in 1d. The model is quantum mechanical in nature and the Hamiltonian can be diagonalised exactly. The diagonalization technique involving Jordan-Wigner and Bogolyubov transformations will be introduced. The model exhibits a T = 0 quantum phase transition at a critical value of the transverse field which defines the critical point of the system. The partition function of the Ising model in 2d in the absence of a magnetic field (Onsager's celebrated solution) can be determined exactly using transformations similar to those employed in diagonalizing the TIM Hamiltonian. In the case of the 2d Ising model, phase transition from the disordered to the ordered phase occurs at a finite temperature which defines the critical point of the model. There is an equivalence between the T = 0 critical point properties of the TIM in 1d and the finite-T critical point properties of the Ising model in 2d. The relationship between the two models will be discussed using simple arguments.

Refs. :

J. B. Kogut, Rev. Mod. Phys. 51, 659 (1979)

D. C. Mattis, The Theory of Magnetism II

B. K. Chakrabarti, A. Dutta and P. Sen, Quantum Ising Phases and Transitions in Transverse Ising Models (Springer Lecture Notes in Physics)

The six vertex (6v) and zero-field eight vertex (8v) models will be introduced. These are classical lattice statistical models in 2d for which the partition function can be determined exactly using the transfer matrix technique. The connection between the 8v and generalized Ising models will be worked out. Related models like the hard hexagon and three-spin models will be briefly described. An outline of the Algebraic Bethe Ansatz method for obtaining the eigenvalues of the transfer matrix will be given. The connection with quantum spin chains will be pointed out. The vertex models have radically transformed the scope and content of research on exactly-solved models. A rich arsenal of techniques is now available and important information on other statistical mechanical problems (like polymers, lattice animals, crystal growth, traffic etc.) can be obtained by mapping these models onto exactly solved ones.

Refs. :

R. J. Baxter, Exactly solved models in statistical mechanics

C. J. Thompson, Classical Equilibrium Statistical Mechanics

E. H. Lieb and F. Y. Wu in Phase Transitions and Critical

Phenomena, vol. 1 ed. by C. Domb and M. S. Green
I. Bose in Models and Techniques of Statistical Physics ed. by
S. M. Bhattacharjee (Narosa)

An introduction will be given to the powerful matrix product method for determining the nonequilibrium steady state properties of model systems in an exact manner. As an illustration, the problem of asymmetric exclusion processes in 1d with open boundary conditions will be discussed.

Refs. : B. Derrida et al., J. Phys. A 26, 1493 (1993)
B. Derrida, Physics Reports 301, 65 (1998)

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Course 5: "Computational Physics"

Lecturer: Prof. Debashish Chowdhury

Tentative Outline of Lectures :

Part I: Equilibrium Statistical Physics.

Lecture 1. General concepts, tricks and methods

1. A few prototype models for the purpose of illustration:
    Random Walk
Percolation
    Ising model

2. General concepts and tricks:
    Use of look-up tables
    Avoiding IF staments
    Multi-spin coding and logical operations
    Boundary conditions: rigid, periodic, helical
    Updating schemes: Sequential, random-sequential, parallel

3. Simple sampling:

Random Walk, Self-avoiding Random Walk
Percolation: cluster counting and Hosen-Kopelman algorithm

Lecture 2. Importance sampling: illustration with Ising model

1. Importance sampling:
   - Single-spin-flip Ising model:
     - The algorithm
     - Thermodynamic quantities (M,E, etc.)
     - correlation function
     - Response Function
     - Finite-size effects
     - Finite sampling time effects
   - Errors: Systematic and statistical
     - Non-Self-averaging systems
     - Critical point and critical exponents

Lecture 3. Other algirithms and techniques for equilibrium stat.mech.

1. Finite-size scaling
2. Reweighting methods: Histogram method
3. Cluster-flipping methods: How to avoid critical-slowing down,
   - Fortuin-Kasteleyn theorem,
     - Swendsen-Wang algorithm,
     - Wolff algorithm
4. Microcanonical ensemble: Creutz's demon algorithm, Q2R
5. Multi-grid algorithms

Part II: Non-equilibrium Statistical Mechanics

Lecture 4. Fluctuations around equilibrium

1. Molecular Dynamics:

2. Brownian dynamics:

3. Critical dynamics:

How to compute correlation time,
DIM as an unusual problem

Lecture 5. Nucleation and Spinodal decomposition

How to compute domain size
Glauber single-spin-flip dynamics
Kawasaki Spin-exchange dynamics
Cell-dynamical approach

Lecture 6. Non-equilibrium Driven Systems

1. Driven and open systems far from equilibrium
   Cellular-automata and Lattice gas
2. CA hydrodynamics
3. Lattice Boltzmann approach

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Course 6: "Nonequilibrium Processes"

Lecturer: Prof. Satya Majumdar

Tentative Outline of Lectures:

Lecture 1: Introduction to nonequilibrium systems. I will mostly discuss two types of nonequilibrium systems:

(i) Systems Relaxing to their Thermal Equilibrium: Examples include phase ordering kinetics of spin systems, glassy dynamics etc.

(ii) Driven Systems with a nonequlibrium steady state: Examples include asymmetric simple exclusion process, traffic models, aggragation models, turbulence, sandpiles, reaction-diffusion systems etc.

Lecture 2 +3 : I will start with a specific simple example of a single particle connected by a spring to a hard wall and discuss in detail how this system relaxes to its thermal equilibrium. This example will also illustrate some key concepts of nonequlibrium processes. For example, we will see how to model thermal noise and diffusion processes. We will also discuss the Langevin and the Fokker-Planck equation and see how one computes the spectrum of relaxation times to the equibrium state.

Lecture 4: We will extend the basic idea developed for a single particle system to that of a many body system with interaction. We will discuss the Master Equation, the priciple of detailed balance and work out in detail the relaxational dynamics (known as the Glauber dynamics) of the one dimensional Ising model.

Lecture 5: We will discuss the simple aggregation model with diffusion and injection of masses. This is a prototypical example of class (ii) types in the Introduction, where the system is driven to a nonequilibrium steady state.

Lecture 6: A summary of the previous lectures, few problems and general questions.

References:

Lecture 1: For a general introduction, ‘‘Nonequilibrium Statistical Mechanics in One Dimension" Ed. by V. Privman (Cambridge University Press, 1997).

Lecture 2+3: ‘Stochastic Processes in Physics and Chemistry", N.G. van Kampen, (North Holland, Amsterdam, 1981).

Lecture 4 : R.G. Glauber, ‘‘Time-Dependent Statistics of the Ising Model’, J. Math. Phys. vol-4, page 294 (1963).

Lecture 5: H. Takayasu, I. Nishikawa, and H. Tasaki, Phys. Rev. A, 37, 3110 (1988).

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