Effective Lagrangians on Domain Walls and Other Solitons

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Phys. Rev. **D73** (2006) 125008, [hep-th/0602289], References

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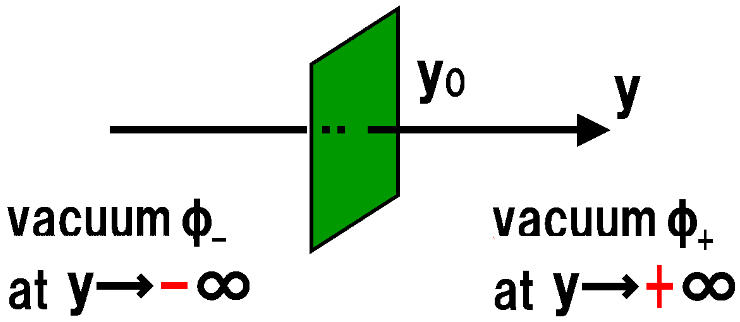

y: extra dimension

1 Introduction

Brane-World = Our world on a Topological Defect in higher dim. spacetime

Topological Defects : **Walls**, **Vortices**, ... are preferably **Solitons**

Part of Supersymmetry (SUSY) preserved $\longrightarrow$ **BPS** state

Solves the Equation of Motion

Soliton Dynamics: Important for **Nonperturbative Effects**

Supersymmetry (SUSY) helps

to obtain realistic unified models if four SUSY is preserved

to find Solitons (Walls, Junctions, $\cdots$) as BPS states

Parameters of the Solution = **Moduli**

$\longrightarrow$ **Massless fields** on the world volume vector-localization

Moduli dynamics = **Effective field theory** of massless fields

Weak dpendence on world volume coordinates = Slow-Move Approx.

Our purpose :

1. Wish a Systematic Method for Effective Lagrangian on BPS Background in SUSY Gauge Theories with Preserved SUSY Manifest
2. Domain Walls and Vortices in 8 SUSY $\boldsymbol{U(N_{\rm C})}$ Gauge Theories with $\boldsymbol{N_{\rm F}(\geq N_{\rm C})}$ Hypermultiplets in the Fundamental Rep.

Results :

1. Orders of Slow-Movement Parameter $\boldsymbol{\lambda}$ for Fields can be Specified
2. A Systematic Expansion of the Fundamental Lagrangian in $\boldsymbol{\lambda}$ Gives BPS Eqs.: $\boldsymbol{\lambda^0}$, (Superfield) Eqs. for Fluctuation Fields : $\boldsymbol{\lambda^1}$
3. Preserved SUSY Helps to Solve Obtaining Fluctuation Fields
4. Integration over the Extra Dimensions $\longrightarrow$ Effective Lagrangian

   Background Energy: $\boldsymbol{\lambda^0}$

   Lowest Nontrivial Order: $\boldsymbol{\lambda^2}$

   Higer Orders in $\boldsymbol{\lambda}$ Should Come out Systematically

5. Kähler Potential of NonLinear Sigma Models is Directly Obtained without Going through the Metric

6. Effective Lagrangian for Multi-Walls :
$\mathbf{1/2}$ BPS Lumps $\longrightarrow$ Energy of Boojum

2 Slow-Move Approximation for Walls

2.1 Component Formalism

SUSY $U(N_C)$ Gauge Theory with N_F Flavors

Vector multiplets : W_M Gauge field, Σ Real Scalar ($N_C \times N_C$ matrix)

Hypermultiplets : $(H^i)^{rA} \equiv H^{irA}$ Complex Scalar ($N_C \times N_F$ matrix)

($i = 1, 2$; Color $r = 1, \cdots, N_C$; Flavor $A = 1, \cdots, N_F$)

Bosonic Part of the Fundamental Lagrangian in $\mathbf{5}$ Dim

$$\mathcal{L} = \mathrm{Tr}\Big[-\frac{1}{2g^2}F_{MN}(W)F^{MN}(W) - \frac{1}{g^2}(\mathcal{D}_M\Sigma)^2 + \mathcal{D}^M H^i(\mathcal{D}_M H^i)^\dagger - V\Big]$$

$$\begin{aligned} V &= \mathrm{Tr}\left[(\Sigma H^i - H^i M)(\Sigma H^i - H^i M)^\dagger\right] \\ &+ \frac{g^2}{4}\mathrm{Tr}\left[\left(H^1 H^{1\dagger} - H^2 H^{2\dagger} - c\mathbf{1}_{N_C}\right)^2 + 4H^2 H^{1\dagger} H^1 H^{2\dagger}\right] \end{aligned}$$

Fayet-Iliopoulos (FI) parameter c, Hypermultiplet Mass $(M)^A{}_B \equiv m_A \delta^A{}_B$

Non-degenerate mass : $m_A > m_{A+1}$ $\longrightarrow$ Flavor symmetry : $U(1)_{\rm F}^{N_{\rm F}-1}$

Discrete SUSY Vacua : Color-Flavor Locking $\langle A_1 A_2 \cdots A_{N_{\rm C}} \rangle$

$$H^{1rA} = \sqrt{c}\,\delta^{A_r}{}_A, \quad H^{2rA} = 0, \quad \Sigma = {\rm diag}(m_{A_1}, \cdots, m_{A_{N_{\rm C}}})$$

Higgs Phase: Walls and Vortices as Elementary Solitons

1/2 BPS Equations

Dependence on $y \equiv x^4$, $\quad$ 4 D Poincaré Invariance $\longrightarrow$ $W_{M \neq y} = 0$

Bogomol'nyi Completion of Energy Density $\mathcal{E}$

$$\begin{aligned}\mathcal{E} &= {\rm Tr}|\mathcal{D}_y H^1 + \Sigma H^1 - H^1 M|^2 + {\rm Tr}|\mathcal{D}_y H^2 - \Sigma H^2 + H^2 M|^2 \\ &+ \frac{g^2}{4}{\rm Tr}\left[\left(\frac{2}{g^2}\mathcal{D}_y \Sigma + H^1 H^{1\dagger} - H^2 H^{2\dagger} - c\mathbf{1}_{N_{\rm C}}\right)^2 + 4 H^2 H^{1\dagger} H^1 H^{2\dagger}\right] \\ &+ c\partial_y {\rm Tr}\Sigma\end{aligned}$$

1/2 BPS Equations $\Longleftrightarrow$ Conserved SUSY: $\gamma^4 \varepsilon^i = -i(\sigma^3)^i{}_j \varepsilon^j$

$$\mathcal{D}_y H^1 = -\Sigma H^1 + H^1 M, \qquad \mathcal{D}_y H^2 = \Sigma H^2 - H^2 M$$

$$\mathcal{D}_y \Sigma = g^2 \left(c\mathbf{1}_{N_{\rm C}} - H^1 H^{1\dagger} + H^2 H^{2\dagger}\right)/2, \qquad 0 = g^2 H^1 H^{2\dagger}$$

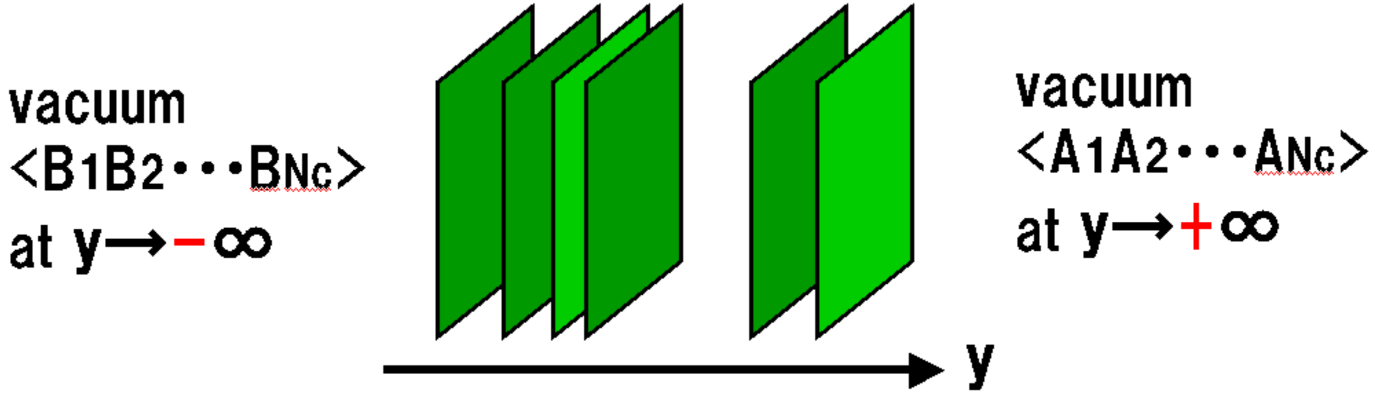

Figure 1: Multi-Wall connecting vacua $\langle A_1 A_2 \cdots A_{N_C}\rangle$ and $\langle B_1 B_2 \cdots B_{N_C}\rangle$

Boundary Condition: **Vacuum** at $y = -\infty$ and at $y = +\infty$

BPS bound for the Wall Tension is Saturated by BPS Walls

$$\int_{-\infty}^{+\infty} \mathcal{E} dy \geq c\Big[\mathrm{Tr}(\Sigma)\Big]_{-\infty}^{+\infty} = c\left(\sum_{k=1}^{N_C} m_{A_k} - \sum_{k=1}^{N_C} m_{B_k}\right)$$

Solving BPS Equations

$\Sigma + iW_y \equiv S^{-1}(y)\partial_y S(y)$ Defines

Complexified $U(N_C)$ Gauge Transformations $S(y) \in GL(N_C, \mathrm{C})$

BPS Eqs. for Hypermultiplet can be Solved by

$$H^1(y) = S^{-1}(y) H_0 e^{My}, \quad H^2(y) = 0$$

"**Moduli Matrix**" H_0 is a Complex $N_{\rm C} \times N_{\rm F}$ Constant Matrix

Vector Multiplet BPS Eq. $\longrightarrow$ Master Eq. for Gauge Invariant $\Omega \equiv S S^\dagger$

$$\partial_y \left(\Omega^{-1} \partial_y \Omega \right) = g^2 c \left(1_{\rm C} - \Omega^{-1} \Omega_0 \right), \qquad \Omega_0 \equiv c^{-1} H_0 e^{2My} H_0^\dagger$$

Moduli Matrix H_0 Contains All the Moduli of Solutions of BPS Eqs.

Slow-Move Approximation

Promote the moduli parameters to Fields ϕ^α on the World-Volume of walls

$$H_0(\phi^\alpha) \longrightarrow H_0(\phi^\alpha(x^\mu)), \qquad \mu = 0, 1, 2, 3$$

Manton, Phys.Lett.**B110** 54 (1982), $\cdots$

Slow-Movement Parameter $\lambda \ll$ Characteristic Mass Scales

$$\lambda \ll \min(\Delta m, g\sqrt{c})$$

Assigning the Order of λ

$$H^1 \sim \mathcal{O}(1), \quad \Sigma \sim \mathcal{O}(1)$$

$$\partial_\mu \sim \mathcal{O}(\lambda), \quad W_\mu \sim \mathcal{O}(\lambda), \quad H^2 \sim \mathcal{O}(\lambda)$$

$$\mathcal{D}_\mu H^1 \sim \mathcal{O}(\lambda), \quad \mathcal{D}_\mu \Sigma \sim \mathcal{O}(\lambda), \quad F_{\mu y}(W) \sim \mathcal{O}(\lambda)$$

Field Equations for Fluctuations should be Solved such as for W_μ

$$0 = \frac{1}{g^2}\mathcal{D}_y F_{\mu y} + \frac{i}{g^2}[\Sigma, \mathcal{D}_\mu \Sigma] + \frac{i}{2}\left(H^1 \mathcal{D}_\mu H^{1\dagger} - \mathcal{D}_\mu H^1 H^{1\dagger}\right)$$

2.2 Superfield Formalism of Slow-Move Approx.

Superfields with 4 SUSY Manifest

8 SUSY Vector Multiplet = **4** SUSY Vector + Chiral Multiplets

Vector Superfield with **4 SUSY Manifest** in the Wess-Zumino Gauge

$$\mathcal{V}\Big|_{\mathrm{WZ}} = -\theta\sigma^\mu\bar{\theta}W_\mu + i\theta^2\bar{\theta}\bar{\lambda}_+ - i\bar{\theta}^2\theta\lambda_+ + \frac{1}{2}\theta^2\bar{\theta}^2\mathcal{Y}^3, \qquad \mathcal{Y}^3 \equiv Y^3 - \mathcal{D}_y\Sigma,$$

Adjoint Chiral Superfield with **4 SUSY Manifest**

$$\Phi = \Sigma + iW_y + \sqrt{2}\theta(-i\sqrt{2}\lambda_-) + \theta^2(Y^1 + iY^2)$$

Hypermultiplet = **2** Chiral Multiplets with **4 SUSY Manifest**

$$\mathcal{H}^1 = H^1 + \sqrt{2}\theta\psi_+ + \theta^2\mathcal{F}^1, \qquad \mathcal{F}^1 \equiv F^1 + (\mathcal{D}_y - \Sigma)H^2 + H^2 M$$

$$\mathcal{H}^2 = H^2 + \sqrt{2}\bar{\theta}\bar{\psi}_- + \bar{\theta}^2\mathcal{F}^2, \qquad \mathcal{F}^2 \equiv -F^2 - (\mathcal{D}_y + \Sigma)H^1 + H^1 M$$

4 SUSY **Auxiliary Fields** $\mathcal{Y}^3, \mathcal{F}^i$ are Shifted by (Covariant) Divergences

Mirabelli and Peskin, Phys.Rev.**D58**, 065002 (1998); Arkani-Hamed, Gregoire and Wacker, JHEP **0203**, 055 (2002); Marti and Pomarol, Phys.Rev.**D64**, 105025 (2001); Hebecker, Nucl.Phys.**B632**, 101 (2002); Kakimoto and Sakai, Phys. Rev. **D68**, 065005 (2003); $\cdots$

Covariant Derivatives for the Complexified $U(N_{\rm C})$ in Extra Dim

$$\hat{D}_y\mathcal{H}^1 = (\partial_y + \Phi)\mathcal{H}^1, \quad \hat{D}_y e^{2\mathcal{V}} \equiv \partial_y e^{2\mathcal{V}} - \Phi^\dagger e^{2\mathcal{V}} - e^{2\mathcal{V}}\Phi$$

Fundamental Lagrangian with **4 SUSY Manifest**

$$\mathcal{L} = -\mathcal{E}_{\rm w}$$
$$+ \int d^4\theta \,{\rm Tr}\left[-2c\mathcal{V} + \frac{1}{2g^2}\left(e^{-2\mathcal{V}}\hat{D}_y e^{2\mathcal{V}}\right)^2 + e^{2\mathcal{V}}\mathcal{H}^1\mathcal{H}^{1\dagger} + e^{-2\mathcal{V}}\mathcal{H}^2\mathcal{H}^{2\dagger}\right]$$
$$+ \left(\int d^2\theta \,{\rm Tr}\left[\hat{D}_y\mathcal{H}^1\mathcal{H}^{2\dagger} - \mathcal{H}^1 M\mathcal{H}^{2\dagger} + \frac{1}{4g^2}\mathcal{W}^\alpha\mathcal{W}_\alpha\right] + {\rm h.c.}\right)$$

$$\mathcal{W}_\alpha \equiv -\frac{1}{8}\bar{D}\bar{D}e^{-2\mathcal{V}}D_\alpha e^{2\mathcal{V}}$$

Difference of **4** and **8** SUSY Lagrangians = **Topological Charge** Density

$$\mathcal{E}_{\rm w} = \partial_y\Big[{\rm Tr}\big[c\Sigma - (\Sigma H^1H^{1\dagger} - H^1MH^{1\dagger}) + (\Sigma H^2H^{2\dagger} - H^2MH^{2\dagger})$$
$$- \frac{2}{g^2}\mathcal{Y}^3\Sigma + \mathcal{F}^1H^{2\dagger} + H^2\mathcal{F}^{1\dagger} + (\text{fermionic terms})\big]\Big]$$

Slow-Move Expansion in Superfield

Order of the **Slow-Move Parameter $\boldsymbol{\lambda}$** for **Superfields**

$$\left(\frac{\partial}{\partial\theta}\right)^2 \sim \partial_\mu \sim \lambda \to d\theta \sim \frac{\partial}{\partial\theta} \sim \mathcal{O}(\lambda^{\frac{1}{2}})$$

$$\mathcal{H}^1 \sim \mathcal{O}(1), \qquad \mathcal{H}^2 \sim \mathcal{O}(\lambda)$$

$$\Phi \sim \mathcal{O}(1), \quad \mathcal{V} \sim \mathcal{O}(1), \qquad (W_\mu \sim \mathcal{O}(\lambda))$$

Expansion of Fundamental Lagrangian in powers of $\boldsymbol{\lambda}$

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \cdots$$

$$\mathcal{L}^{(0)} = -\mathcal{E}_{\rm w} + \int d^4\theta \mathrm{Tr}\left[-2c\mathcal{V} + e^{2\mathcal{V}}\mathcal{H}^1\mathcal{H}^{1\dagger} + \frac{1}{2g^2}\left(e^{-2\mathcal{V}}\hat{D}_y e^{2\mathcal{V}}\right)^2\right]$$

$$\mathcal{L}^{(2)} = \left(\int d^2\theta \mathrm{Tr}\left[\hat{D}_y\mathcal{H}^1\mathcal{H}^{2\dagger} - \mathcal{H}^1 M\mathcal{H}^{2\dagger}\right] + \mathrm{h.c.}\right)$$

$\mathcal{H}^2, \mathcal{V}$ Become **Lagrange Multipliers** Giving **Constraints**

$$\hat{D}_y\mathcal{H}^1 = \mathcal{H}^1 M, \qquad g^2(c - \mathcal{H}^1\mathcal{H}^{1\dagger}e^{2\mathcal{V}}) = -\hat{D}_y\left(e^{-2\mathcal{V}}\hat{D}_y e^{2\mathcal{V}}\right)$$

Lowest Components in $\boldsymbol{\theta}$ of Constraints Give **BPS Equations**

BPS Equations as an **Automatic Consequence** of $\boldsymbol{\lambda}$ Expansion

Solving Superfield Eqs.: Complexified Gauge Transformation Superfield $\mathcal{S}$

$$\Phi = \mathcal{S}^{-1}\partial_y \mathcal{S}$$

Hypermultiplet Constraint Equation is Solved by

$$\partial_y(\mathrm{SH}^1) = \mathrm{SH}^1 M \to \mathcal{H}^1(x,\theta,\bar{\theta},y) = \mathcal{S}^{-1}(x,\theta,\bar{\theta},y)\mathcal{H}_0(x,\theta,\bar{\theta})e^{My}$$

$U(N_{\mathrm{C}})$ Gauge Invariant Vector Superfield Ω

$$\Omega \equiv \mathcal{S}e^{-2\mathcal{V}}\mathcal{S}^\dagger$$

Vector Multiplet Constraint Eq. becomes Superfield Master Equation

$$\partial_y\left(\Omega^{-1}\partial_y\Omega\right) = g^2 c\left(1-\Omega^{-1}\Omega_0\right), \quad \Omega_0 \equiv c^{-1}\mathcal{H}_0 e^{2My}\mathcal{H}_0^\dagger$$

Solution of Superfield Master Equation

Solution $\Omega_{\mathrm{sol}}(H_0(x), H_0^\dagger(x), y)$ of **Bosonic Master Equation**

Can be Promoted to **Vector Superfield** $\Omega_{\mathrm{sol}}(\mathcal{H}_0(x,\theta), \mathcal{H}_0^\dagger(x,\theta), y)$

Can be **Gauge-Transformed** to **Wess-Zumino Gauge** $\mathcal{V}_{\mathrm{sol}}$

by (Anti-)Chiral Superfield $\mathcal{S}_{\mathrm{sol}}$ ($\mathcal{S}_{\mathrm{sol}}^\dagger$) as

$$\Omega_{\mathrm{sol}}(\mathcal{H}_0(x,\theta), \mathcal{H}_0^\dagger(x,\theta), y) = \mathcal{S}_{\mathrm{sol}} e^{-2\mathcal{V}_{\mathrm{sol}}}\mathcal{S}_{\mathrm{sol}}^\dagger$$

$$= S_{\mathrm{sol}}S_{\mathrm{sol}}^\dagger + \theta\sigma^\mu\bar{\theta}\left(i(\partial_\mu S_{\mathrm{sol}})S_{\mathrm{sol}}^\dagger - iS_{\mathrm{sol}}(\partial_\mu S_{\mathrm{sol}}^\dagger) + 2S_{\mathrm{sol}}W_\mu^{\mathrm{sol}}S_{\mathrm{sol}}^\dagger\right) + \cdots$$

Expansion in powers of Grassmann spinor $\boldsymbol{\theta}$

$$\boldsymbol{\Omega}_{\rm sol}(\mathcal{H}_0, \mathcal{H}_0^\dagger, y) = \boldsymbol{\Omega}_{\rm sol} + \theta\sigma^\mu\bar{\theta}\left(i(\delta_\mu - \delta_\mu^\dagger)\boldsymbol{\Omega}_{\rm sol}\right) + \cdots$$

$$\delta_\mu \equiv \sum_i \partial_\mu \phi^i \frac{\delta}{\delta\phi^i}, \quad \delta_\mu^\dagger \equiv \sum_i \partial_\mu \phi^{i*} \frac{\delta}{\delta\phi^{i*}}, \quad \partial_\mu = \delta_\mu + \delta_\mu^\dagger$$

Fluctuations are Obtained as Solutions of Constraint Eqs.

$\boldsymbol{S}_{\rm sol}$ must depend on both ϕ and ϕ^*: $\quad \boldsymbol{S}_{\rm sol}\boldsymbol{S}_{\rm sol}^\dagger = \boldsymbol{\Omega}_{\rm sol}$

$$-iW_\mu^{\rm sol} = S_{\rm sol}^{-1}\delta_\mu^\dagger S_{\rm sol} + S_{\rm sol}^\dagger \delta_\mu S_{\rm sol}^{\dagger -1} + (\text{bi-linear terms of fermions})$$

$$\boldsymbol{\Phi}_{\rm sol} = \mathcal{S}_{\rm sol}^{-1}\partial_y \mathcal{S}_{\rm sol} \quad \rightarrow \quad \Sigma^{\rm sol} + iW_y^{\rm sol} = S_{\rm sol}^{-1}\partial_y S_{\rm sol}$$

Other Components are Similarly Determined

Solution $\boldsymbol{\Omega}_{\rm sol}(y, \phi, \phi^*)$ is Substituted into Fundamental Lagrangian

$\rightarrow$ **Effective Lagrangian** on the BPS Walls

$$\mathcal{L} = -T_{\rm w} + \int d^4\theta K(\phi, \phi^*) + \textbf{higher derivatives } (\mathcal{O}(\lambda^4))$$

$T_{\rm w}$: Energy Density of Wall, K: **Kähler potential** of moduli fields ϕ, ϕ^*

$$K(\phi, \phi^*) = \int dy \left[c \log \det\Omega + c\,{\rm Tr}\left(\Omega_0 \Omega^{-1}\right) + \frac{1}{2g^2}{\rm Tr}\left(\Omega^{-1}\partial_y \Omega\right)^2\right]\Big|_{\Omega=\Omega_{\rm sol}}$$

Kähler Potential is Obtained without Going through Kähler Metric

Kähler Potential serves as the Action for the Master Equation of $\boldsymbol{\Omega}$

$g \to \infty$ Limit (Non-Linear Sigma Model)

$$K_0(\phi, \phi^*) = c \int dy \log \det \Omega_0$$

3 Effective Lagrangian of Multi-Wall System

3.1 Exact Result at Finite Coupling $g^2 c = m^2$

Exact Solutions of Double Walls have been Obtained

at $g^2 c = m^2$ with $M = \mathrm{diag}(2m, m, 0)$

Parametrization of Moduli Matrix

$$H_0 = \sqrt{c}\,(1, \tau^2, \tau^3) = \sqrt{c}\left(1, e^{\frac{\phi_+ + \phi_-}{2}}, e^{\phi_+}\right)$$

Position of Wall Interpolating A-th and $(A+1)$-th Vacua

$$y_A \approx -\frac{1}{m}\mathrm{Re}\frac{\log \tau^A - \log \tau^{A+1}}{n_A - n_{A+1}}$$

Gauge Invariant Ω for Kähler Potential Density

$$\Omega = |e^{\frac{\phi_+}{2}}|^2 e^{2my} \left(e^{my} |e^{-\frac{\phi_+}{2}}| + e^{-my} |e^{\frac{\phi_+}{2}}| + \sqrt{6 + |e^{\phi_-}|} \right)^2$$

Kähler Potential is Given by

$$K(\phi, \phi^*) = \frac{2c}{m} \left\{ \frac{1}{4} (\mathrm{Re}(\phi_+))^2 + \left(\log \left(\frac{\sqrt{6 + |e^{\phi_-}|} + \sqrt{2 + |e^{\phi_-}|}}{2} \right) \right)^2 - 2\sqrt{\frac{6 + |e^{\phi_-}|}{2 + |e^{\phi_-}|}} \log \left(\frac{\sqrt{6 + |e^{\phi_-}|} + \sqrt{2 + |e^{\phi_-}|}}{2} \right) \right\}$$

3.2 Boojums as a Solution of Effective Lagrangian

1/2 BPS Lump Solution of the Double-Wall Effective Lagrangian

$$\bar{\partial} \phi_- = 0, \qquad z = x^1 + i x^2$$

Gives **1/4** BPS Composite State of Lump Stretched between Two Walls

Energy of **1/2** BPS Lump on the Double Wall

$$\mathcal{E} = \frac{\partial^2 K(\phi, \phi^*)}{\partial \phi^i \partial \phi^{j*}} \partial_\mu \phi^i \partial^\mu \phi^{j*} \Big|_{\mathrm{BPS}} = 2 \partial \bar{\partial} K(\phi, \phi^*)|_{\mathrm{BPS}} = \frac{1}{2} \partial^2_{(2)} K(\phi, \phi^*)|_{\mathrm{BPS}}$$

Lump at $z = 0$ with **Vorticity** k and Size (and Phase) z_0

$$e^{\frac{\phi_-}{2}} = (z/z_0)^k, \quad e^{\phi_+} = \mathrm{const.}$$

A Lump With Increasingly Large Radius $\longrightarrow$ Cut-off at $r = \Lambda$

$$\begin{aligned} E_k(\Lambda) &= \frac{1}{2}\int_{r=0}^{r=\Lambda} dx^2 \partial_{(2)}^2 K = \pi \Big[r K' \Big]_{r=0}^{r=\Lambda} \\ &= \frac{4\pi c k}{m} \left\{ k \log \frac{\Lambda}{r_0} - 1 + \mathcal{O}\left(\left(\frac{r_0}{\Lambda} \right)^{2k} \right) \right\} \\ &= 2\pi c\, k L_k - \frac{4\pi\, m\, k}{g^2} + \mathcal{O}\left(\left(\frac{r_0}{\Lambda} \right)^{2k} \right) \end{aligned}$$

First Term = Energy of the Lump

Lump length $L_k = \frac{2k}{m} \log \frac{\Lambda}{r_0}$ with Size Moduli $r_0 \equiv |z_0|$

Second Term = Energy of **Boojum** (Binding Energy of Lump and Wall)

4 Conclusion

1. **Systematic Method** to Obtain the **Effective Lagrangian** on the $\mathbf{1/2}$ BPS Background (Wall, Vortex) is Worked out

2. Orders of Slow-Movement Parameter $\boldsymbol{\lambda}$ for Various Fields are Specified
3. A Systematic Expansion of the Fundamental Lagrangian in $\boldsymbol{\lambda}$ $\longrightarrow$
   Fields Appearing only as Fluctuations become Lagrange Multiplier
   $\longrightarrow$ **Superfield Constraint Eq.** $\longrightarrow$
   $\boldsymbol{\lambda}^{\mathbf{0}}$: BPS Eqs. $\boldsymbol{\lambda}^{\mathbf{1}}$: Superfield Eqs. for Fluctuation Fields
4. Integration over Extra Dimensions Gives **Effective Lagrangian**
   Background Energy: $\boldsymbol{\lambda}^{\mathbf{0}}$, Lowest Nontrivial Order: $\boldsymbol{\lambda}^{\mathbf{2}}$
5. **Kähler Potential** of NonLinear Sigma Models is **Directly Obtained** without Going through the Metric
6. **1/2** BPS Lumps of Wall Effective Lagrangian:
   $\longrightarrow$ **1/4** Composite Soliton (Boojum)

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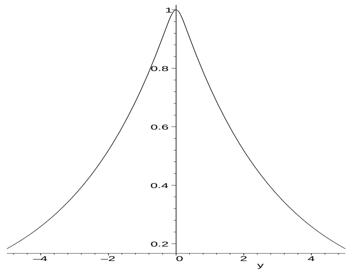
introduction



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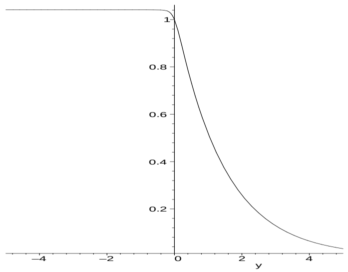
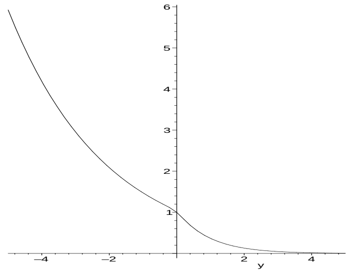


(b) a=1



(c) a=2

Other results related to the Brane-World

1. Walls in 5D SUGRA $\longrightarrow$ Warped metric models

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3. $\boldsymbol{U(1)}$ gauge field Localization on walls with tensor multiplet

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