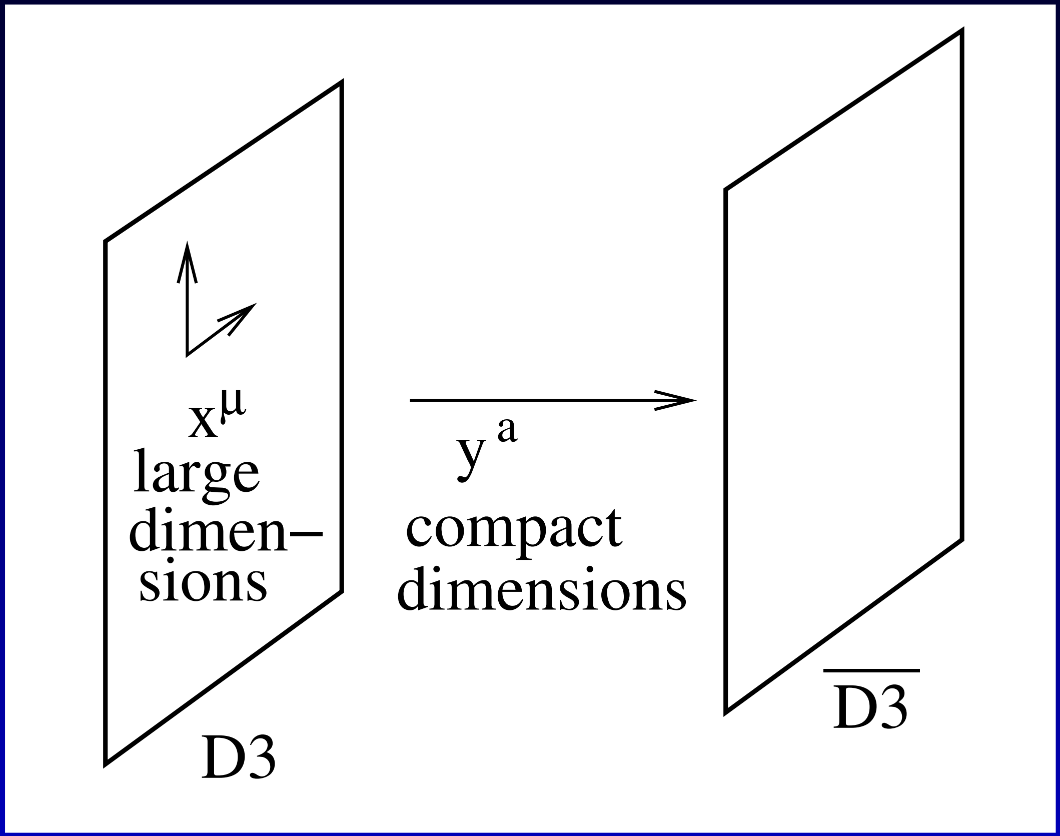
Brane-antibrane inflation



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From Strings to LHC, 8 Jan. 2007

Outline

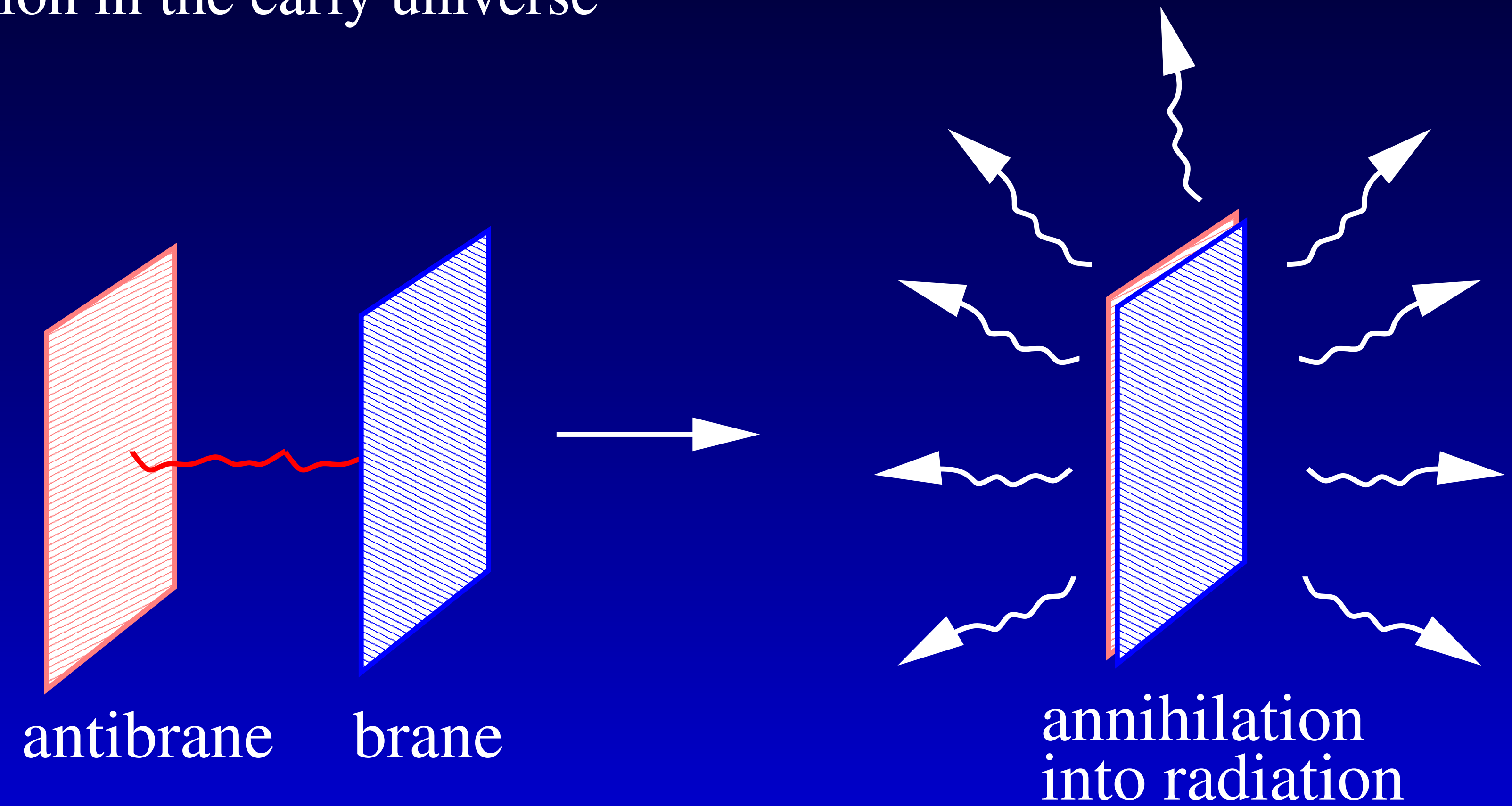
Brane-antibrane Inflation:

- Early attempts
- KKLMMT and the η problem
- Tuning with generic superpotential corrections
- Realistic superpotential corrections[1]

[1] C.P. Burgess, JC, K. Dasgupta, H. Firouzjahi, hep-th/0610320

Inflation from brane annhilation

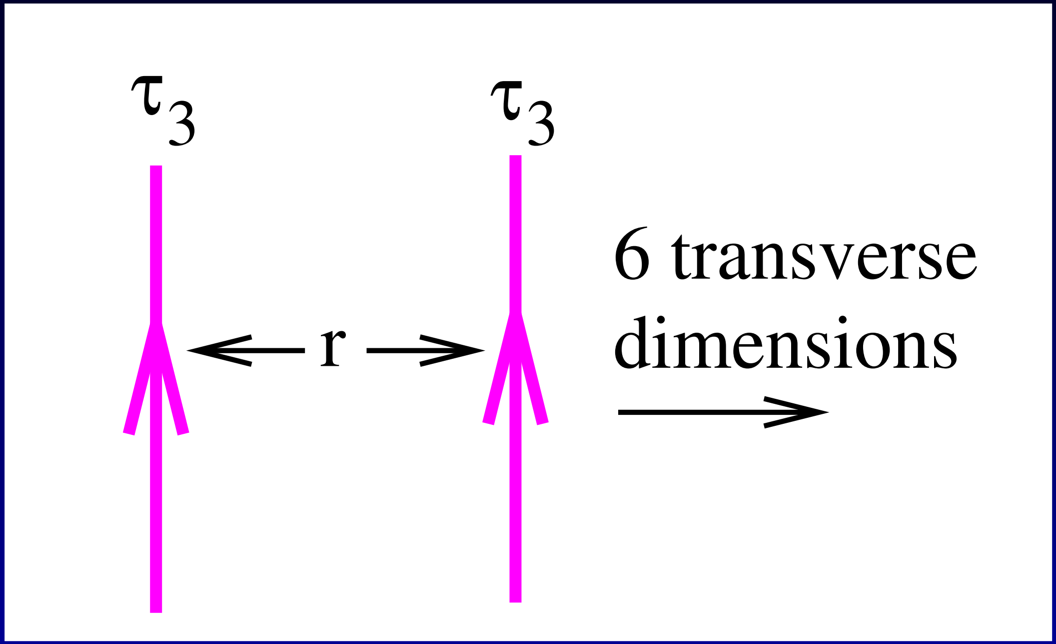
Interaction energy between brane and antibrane can give rise to inflation in the early universe



Lightest mode of stretched string between branes becomes tachyonic at critical separation, ending inflation

Brane-brane potential

Parallel BPS (supersymmetric) D3 branes exert no force on each other:



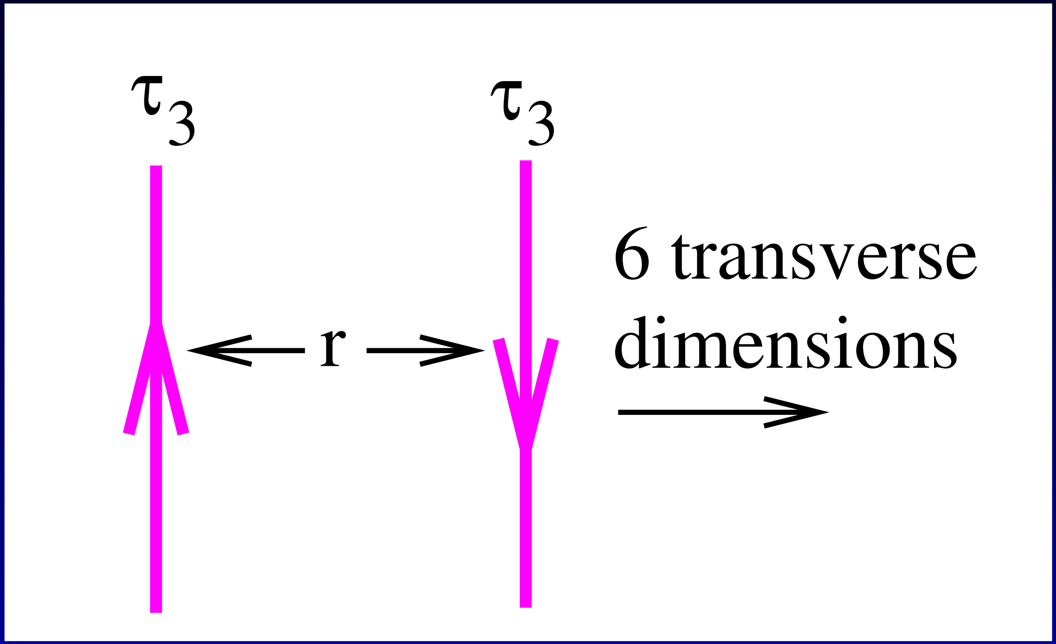
$$V_{\rm grav} = -\kappa_{10}^2 \frac{\tau_3^2}{r^4} \quad \text{gravitational attraction}$$

$$V_{\rm gauge} = +\kappa_{10}^2 \frac{\tau_3^2}{r^4} \quad \text{RR gauge field repulsion}$$

- Net brane-brane potential $V_{\rm tot} = 0$

Brane-antibrane potential

Antiparallel D3 branes $\equiv$ brane-antibrane pair, have an attractive force:



$$V_{\text{grav}} = -\kappa_{10}^2 \frac{\tau_3^2}{r^4} \quad \text{gravitational attraction}$$

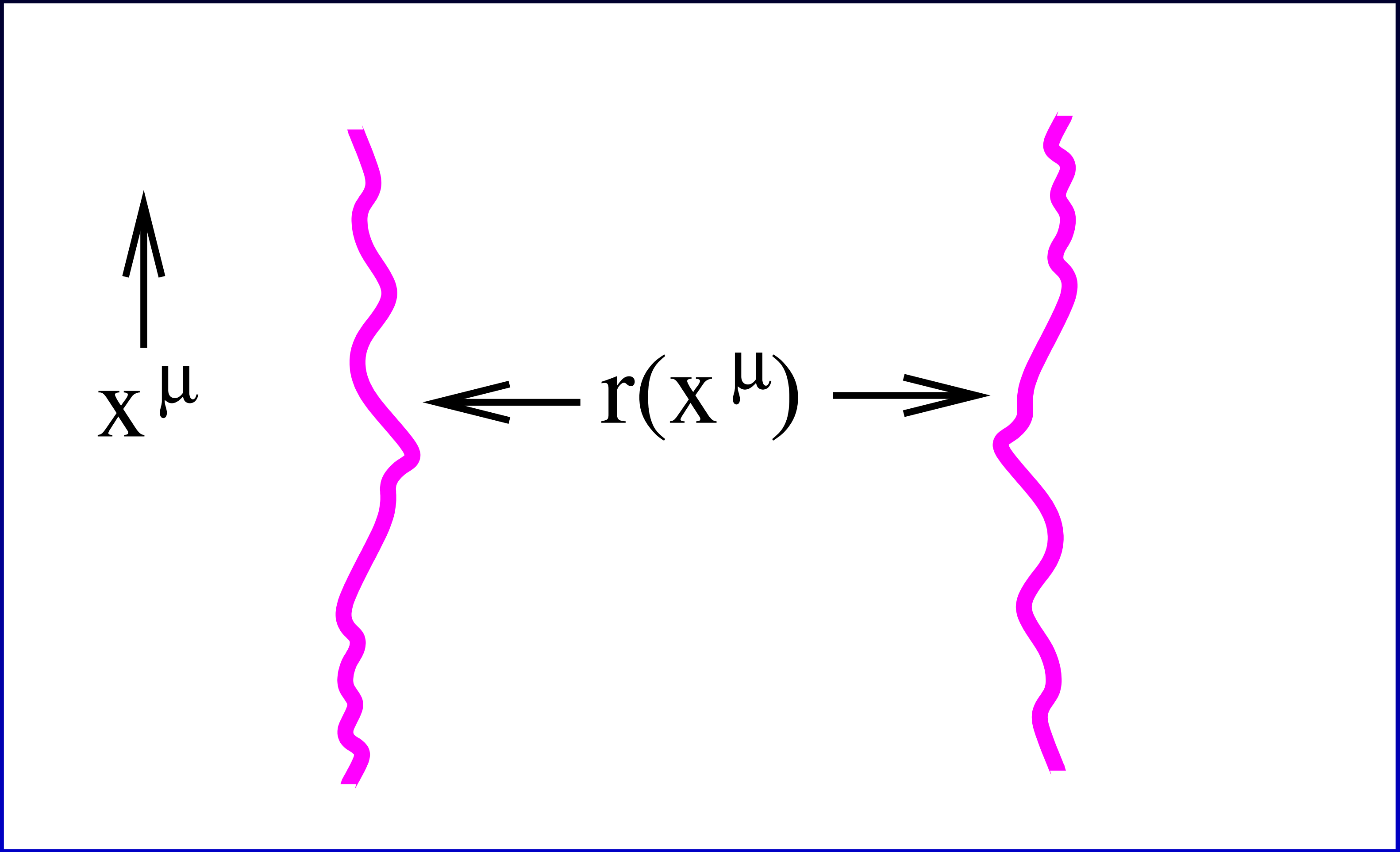
$$V_{\text{gauge}} = -\kappa_{10}^2 \frac{\tau_3^2}{r^4} \quad \text{RR gauge field attraction}$$

Total potential is

$$V_{\text{tot}} = -2\kappa_{10}^2 \frac{\tau_3^2}{r^4}$$

Inflaton field

$r(x^\mu)$ = relative distance between branes at x^μ:



Inflaton field: kinetic term

$r(x^\mu)$ = relative distance between branes at x^μ.

Kinetic energy comes from DBI action; for a single D3 or $\overline{\text{D3}}$,

$$S = -\tau_3 \int d^4x \sqrt{-G}$$

$$G_{\mu\nu} = \text{induced metric on brane}$$

$$= g_{AB} \frac{\partial X^A}{\partial x^\mu} \frac{\partial X^B}{\partial x^\nu} = \eta^{\mu\nu} + \frac{\partial \phi^I}{\partial x^\mu} \frac{\partial \phi^I}{\partial x^\nu}$$

(ϕ^I are transverse oscillations)

$$\det G = -1 + \left(\frac{\partial \phi}{\partial x}\right)^2 + \ldots$$

$$S = -\tau_3 \int d^4x \left(1 - \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \ldots\right)$$

Canonically normalized inflaton

Let $r^I = \phi^I - \bar{\phi}^I$ (brane minus antibrane position)

$$\mathcal{L} = -\frac{1}{2}\tau_3(\partial r)^2 - V(r)$$

Canonically normalized inflaton is

$$\varphi = \sqrt{\tau_3}\, r = \sqrt{\tau_3}\left(\sum_I (r^I)^2\right)^{1/2}$$

Potential is

$$V = 2\left(\tau_3 - \frac{c}{\varphi^4}\right), \qquad c = \kappa_{10}^2 \tau_3^4$$

10D gravitional coupling is

$$\kappa_{10}^2 = M_{10}^{-8} = M_p^{-2}\, L^6$$

in terms of compactification volume L^6

Flatness problem for potential

To get enough inflation, need slow-roll parameters to be small:

Size of extra dimensions

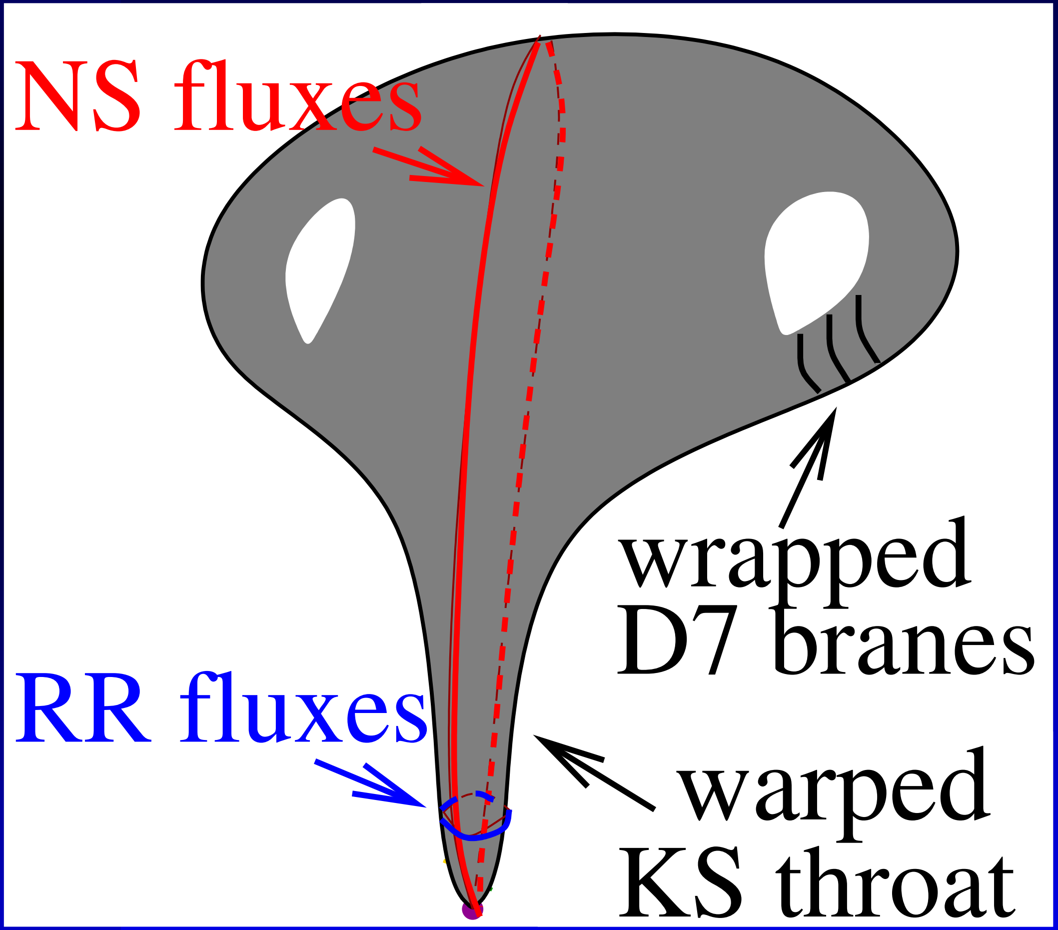
$$\eta \equiv M_p^2 \frac{V''}{V} \sim -\left(\frac{L}{r}\right)^6$$

Need $r \gg L$ to get small η — inconsistent!

A further problem: no convincing mechanism for stabilizing extra dimensions existed.

Stabilization using fluxes

Giddings, Kachru and Polchinski (GKP) stabilize dilaton and complex structure moduli of type IIB string theory with fluxes in 6D Calabi-Yau manifold:



Generates hierarchy with warping (*à la* Randall-Sundrum)

Can get exponentially large hierarchy from quantized fluxes

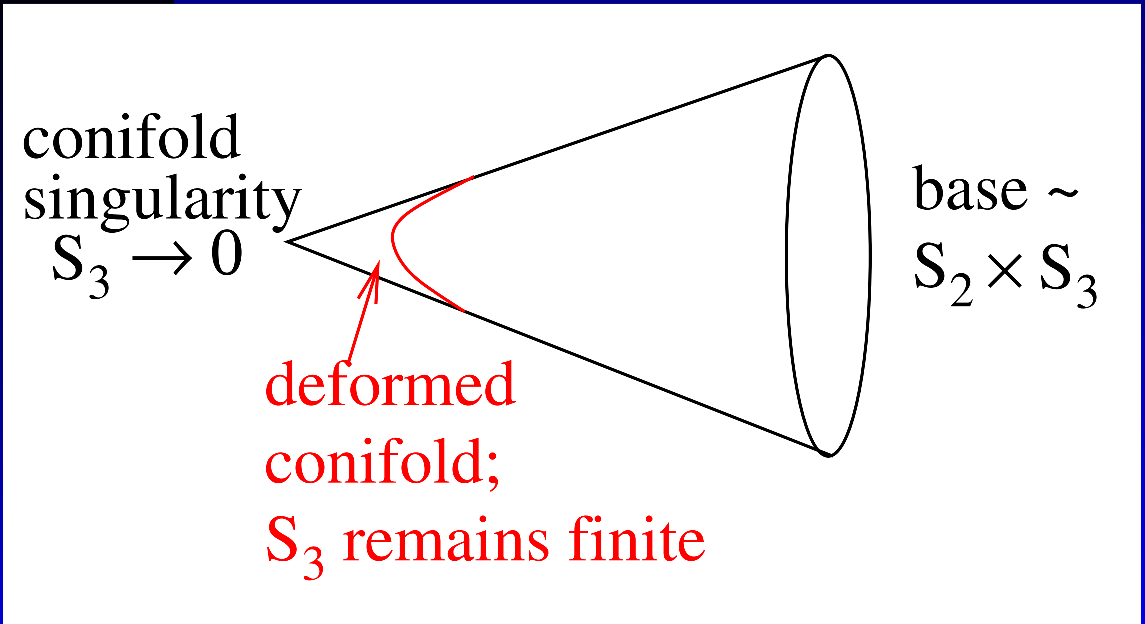
Conifold singularity at tip of throat breaks $\mathcal{N} = 4$ SUSY to $\mathcal{N} = 1$

Klebanov-Strassler Warped Throat

Generalizes RS to 10D; geometry is approximately $\mathrm{AdS}_5 \times T_{1,1}$

$$ds^2 = a^2(r)(-dt^2 + dx^2) + a^{-2}(r)(dr^2 + r^2 ds^2_{T_{1,1}})$$

$$a(r) \cong \frac{r}{R}, \quad R = \text{AdS curvature scale}$$



Singular conifold:

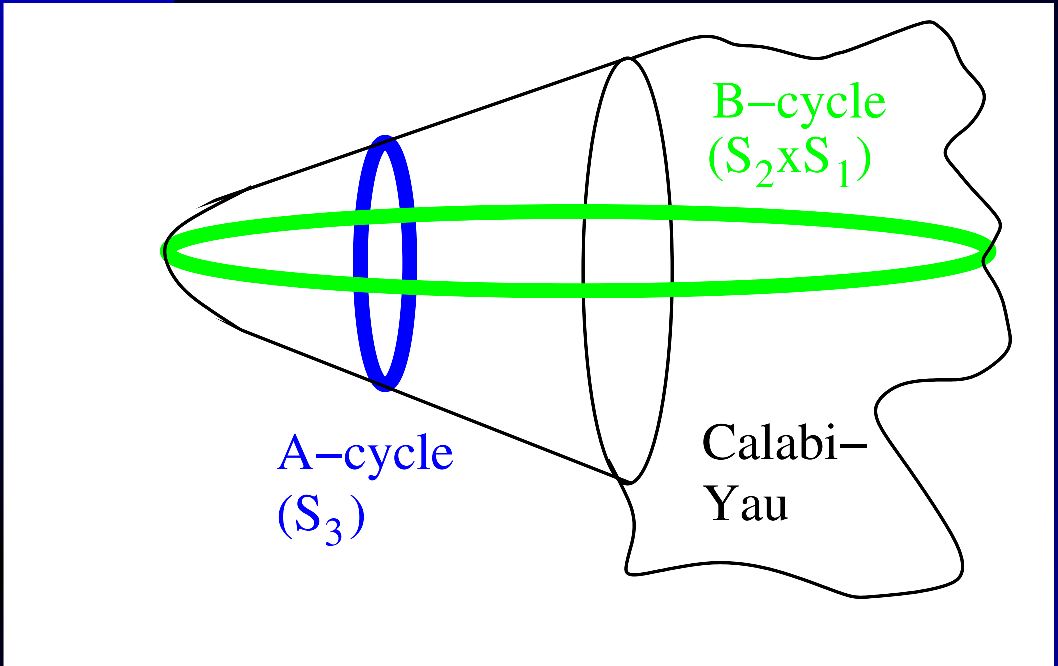
$$\sum_{i=1}^4 w_i^2 = 0$$

Deformed conifold:

$$\sum_{i=1}^4 w_i^2 = z \neq 0$$

result of turning on fluxes of $H_{(3)}$ (Kalb-Ramond field) and $F_{(3)}$ (field strength of RR 2-form $C_{(2)}$)

KS: effect of fluxes



flux quanta:

$$\left(\frac{M_s}{2\pi}\right)^2 \int_A F_3 = M$$

$$\left(\frac{M_s}{2\pi}\right)^2 \int_B H_3 = -K$$

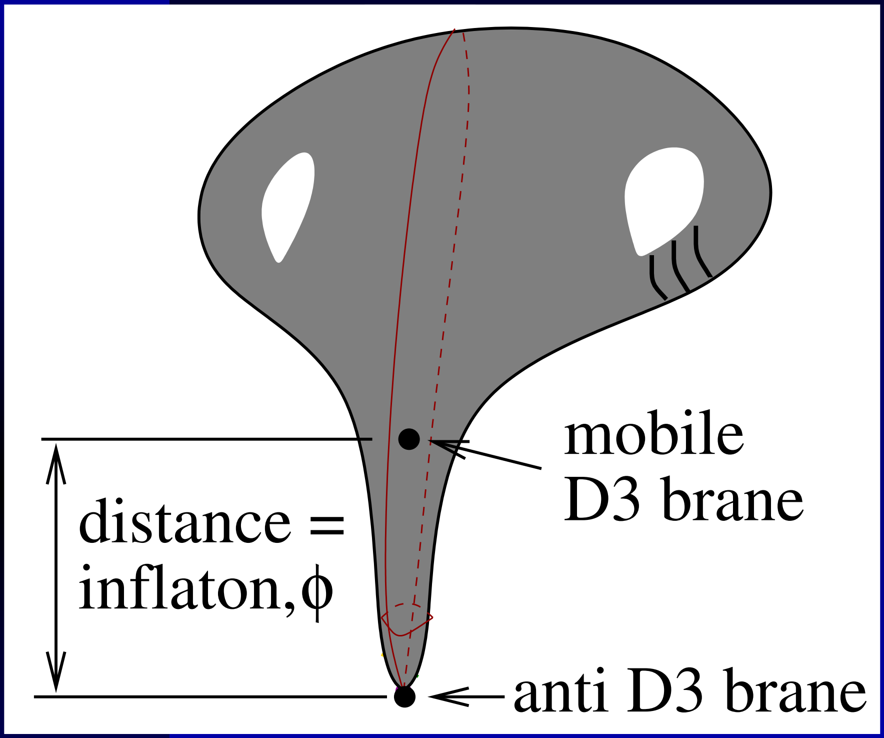
Fluxes fix complex structure modulus $z = \sum_i w_i^2$; tip of cone is at

$$z = e^{-2\pi K/g_s M} = a_0^3$$

a_0 = warp factor at bottom of throat

Getting Inflation: KKLMMT

KKLMMT put D3 and $\overline{\text{D3}}$ into throat:



$\overline{\text{D3}}$ sinks quickly to bottom

D3 is almost buoyant,

due to induced $F_{(5)}$ background:

$$dF_{(5)} \sim H_{(3)} \wedge F_{(3)}$$

Corresponds to

$$C_{(4)} = a^4(r)$$

DBI action for D3 or $\overline{\text{D3}}$ in warped background is

$$S = -\tau_3 \int d^4x \, \left(a^4(r)\sqrt{1 + a^{-4}(r)(\partial\phi^I)^2} \mp C_{(4)} \right)$$

$$= \tfrac{1}{2}\tau_3(\partial\phi^I)^2 + \begin{cases} 0, & \text{D3} \\ -2\tau_3\, a^4(r) \int d^4x, & \overline{\text{D3}} \end{cases}$$

Warped brane-antibrane potential

Action for a static antibrane at position $r = r_0$ in throat:

$$S = -\tau_3 \int d^4x \sqrt{g_4(r_0)} - \tau_3 \int d^4x\, C_{(4)}(r_0)$$

With no brane, $\sqrt{g_4} = C_{(4)} = a_0^4$, and $S = -2a_0^4\tau_3$.

Now add brane at r; it perturbs geometry

$$g^{(6)}_{\mu\nu} \to g^{(6)}_{\mu\nu} + \delta g^{(6)}_{\mu\nu}$$

Perturbation satisfies Poisson eq. in the 6 extra dimensions,

$$\nabla^2 \delta g^{(6)}_{\mu\nu} = C\,\eta_{\mu\nu}\,\delta^{(6)}(\vec{r}) \quad \Rightarrow \quad \delta g^{(6)}_{\mu\nu} \sim C\,\eta_{\mu\nu}\,(r - r_0)^{-4}$$

Substitute perturbed $g_4 \sim 1/g_6 \sim C^2_{(4)}$ back into S:

$$\boxed{V = \frac{2a_0^4\tau_3}{1 + a_0^4(r - r_0)^{-4}}}$$

Why is this V good for inflation?

$$V \cong 2\epsilon\tau \left(1 - \frac{\epsilon}{r^4}\right)$$

By taking $\epsilon \equiv a_0^4 \ll 1$, can make V very flat!

Slow roll parameter:

$$\eta = \frac{V''}{V} \cong -20\epsilon$$

No fine tuning needed.

But η strikes back:

We ignored overall volume (Kähler) modulus T.
Interaction of T with inflaton ϕ gives big mass to ϕ,

$$\delta V = \tfrac{1}{2}m^2\phi^2, \qquad m^2 \sim V_0 \sim H^2$$

Since $m \sim H$, inflation is spoiled:

$$\eta = \frac{V''}{V} \rightarrow \frac{2}{3}$$

Inflaton never rolls slowly!

Origin of η problem

T dependence of metric:

$$ds^2 = e^{-6u}\, a^4\, dx^2 + e^{2u}\, a^{-4}\, \tilde{g}^{(6)}_{ab}\, dy^a\, dy^b$$

$$e^{4u} = T + \overline{T} = L^4$$

Recompute DBI action of brane in this metric:

$$(\partial\varphi)^2 \to \frac{(\partial\varphi)^2}{T + \overline{T}}$$

Consistency with SUGRA description implies that Kähler potential for T gets modified:

$$K = -3\ln(T + \overline{T} - |\varphi|^2) \equiv -2\ln 2\sigma$$

Origin of η problem

Potential is also modified: $V \sim e^K$, so

$$V \to \frac{V}{(2\sigma)^2} = \frac{V}{(T + \overline{T} - |\varphi|^2)^2}$$

$$\mathcal{L} \sim -\frac{(\partial\varphi)^2}{T + \overline{T}} - \frac{V}{(T + \overline{T})^2}\left(1 + \frac{2|\varphi|^2}{T + \overline{T}}\right)$$

Canonically normalizing φ, see that

$$m^2 \sim \frac{V}{2\sigma} \sim H^2$$

Warp factor does not help to make η small

KKLMMT: How to fix it?

Can invoke additional source of φ-dependence to cancel unwanted dependence, *e.g.*, correction to superpotential,

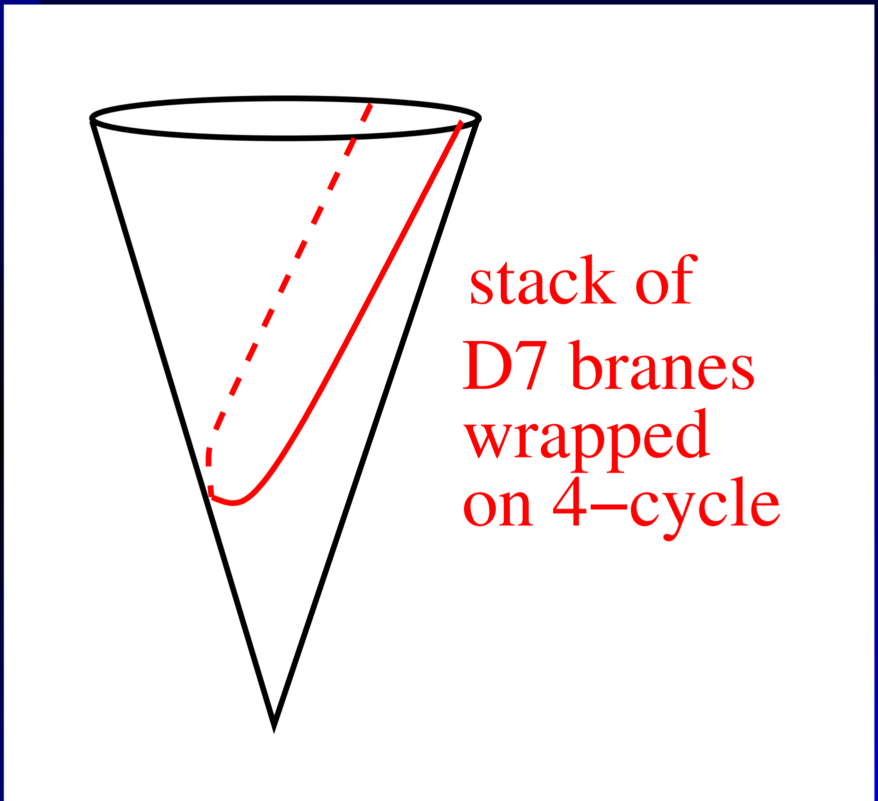
$$W \to W_0 + Ae^{-aT}(1 + \delta\varphi^2)$$

Must tune δ to 1 part in 100.

New development: form of superpotential corrections have been computed from string theory, Baumann *et al.*, hep-th/0607050

Superpotential corrections

Ignore Calabi-Yau and compute everything in throat.



Specify 4-cycle by

$$\prod_{i=1}^{4} w_i^{p_i} = \mu^P$$

p_i integers, $P = \sum p_i$;

preserves SUSY (Ouyang)

Baumann *et al.* show that

$$W = W_0 + Ae^{-aT}\left(1 - \frac{\prod_i w_i^{p_i}}{\mu^P}\right)^{1/N_{D7}}$$

F-term potential

Burgess, JC, Dasgupta, Firouzjahi, hep-th/0610320, compute V_F:
$(T \to \rho,\, 2\sigma \to R)$

$$\begin{aligned} V_F &= \frac{\kappa_4^2}{3R^2}\Big[(\rho+\bar\rho)|W_{,\rho}|^2 - 3(\overline{W}W_{,\rho} + \text{c.c.}) \\ &\qquad + \frac{3}{2}\left(\overline{W}_{,\bar\rho}w^j W_{,j} + \text{c.c.}\right) + \frac{1}{c}k^{\bar\imath j}\overline{W}_{,\bar\imath}W_{,j}\Big] \\ &= \frac{\kappa_4^2}{3R^2}\Big[\left[(\rho+\bar\rho)a^2 + 6a\right]|A|^2 e^{-2a(\rho+\bar\rho)} + 3aW_0(Ae^{-a\rho} + \bar A e^{-a\bar\rho}) \\ &\qquad \boxed{-\frac{3}{2}a\,e^{-a(\rho+\bar\rho)}\left(\bar A w^j A_{,j} + \text{c.c.}\right) + \frac{1}{c}k^{\bar\imath j}\overline{A}_{,\bar\imath}A_{,j}e^{-a(\rho+\bar\rho)}}\Big] \end{aligned}$$

new terms –– δV_F

F-term potential: does not work!

Burgess, JC, Dasgupta, Firouzjahi, hep-th/0610320, compute V_F:

Get explicit expression in terms of coordinates of $T_{1,1}$:

$$\begin{aligned}
w_1 &= r^{3/2}\, e^{\frac{i}{2}(\psi-\phi_1-\phi_2)} \sin\frac{\theta_1}{2}\, \sin\frac{\theta_2}{2} \\
w_2 &= r^{3/2}\, e^{\frac{i}{2}(\psi+\phi_1+\phi_2)} \cos\frac{\theta_1}{2}\, \cos\frac{\theta_2}{2} \\
w_3 &= r^{3/2}\, e^{\frac{i}{2}(\psi+\phi_1-\phi_2)} \cos\frac{\theta_1}{2}\, \sin\frac{\theta_2}{2} \\
w_4 &= r^{3/2}\, e^{\frac{i}{2}(\psi-\phi_1+\phi_2)} \sin\frac{\theta_1}{2}\, \cos\frac{\theta_2}{2}
\end{aligned}$$

New contribution to V_F does not help inflation: it is minimized at $\delta V_F = 0$ for $\theta_1 = \theta_2 = 0$ in $T_{1,1}$: $(\tilde{\psi} = \psi - \phi_1 - \phi_2)$

$$\delta V_F = M_{11}(\theta_1^2 + \theta_2^2) + M_{12} \cos\left(\tfrac{1}{2}\tilde{\psi}\right) \theta_1\theta_2 + \ldots$$

Competing effect: dilaton shift

Ouyang showed there is another new contribution to V, from back-reaction of D7 on dilaton (and nonprimitive G_3 fluxes)

$$e^{-\Phi} = \frac{1}{g_s} - \frac{N_{D7}}{2\pi} \log\left(\frac{r^{3/2}}{\mu} \sin\frac{\theta_1}{2} \sin\frac{\theta_1}{2}\right)$$

Leads to extra contribution to potential

$$\delta V_O = -\frac{\delta N(\epsilon)}{2\pi} \frac{T_3 \xi_0^4}{R^2} \left(\frac{r}{r_0}\right)^4 \log\left(\frac{r^{3/2}}{\mu} \sin\frac{\theta_1}{2} \sin\frac{\theta_1}{2}\right) + \mathcal{O}(\epsilon^2)$$

Prevents $\theta_i \to 0$; $\delta V_{\rm tot}$ is minimized at nontrivial value.

Can we tune $\delta V_{\rm tot}$ against $m^2\phi^2$ of KKLMMT to get flat potential for inflation?

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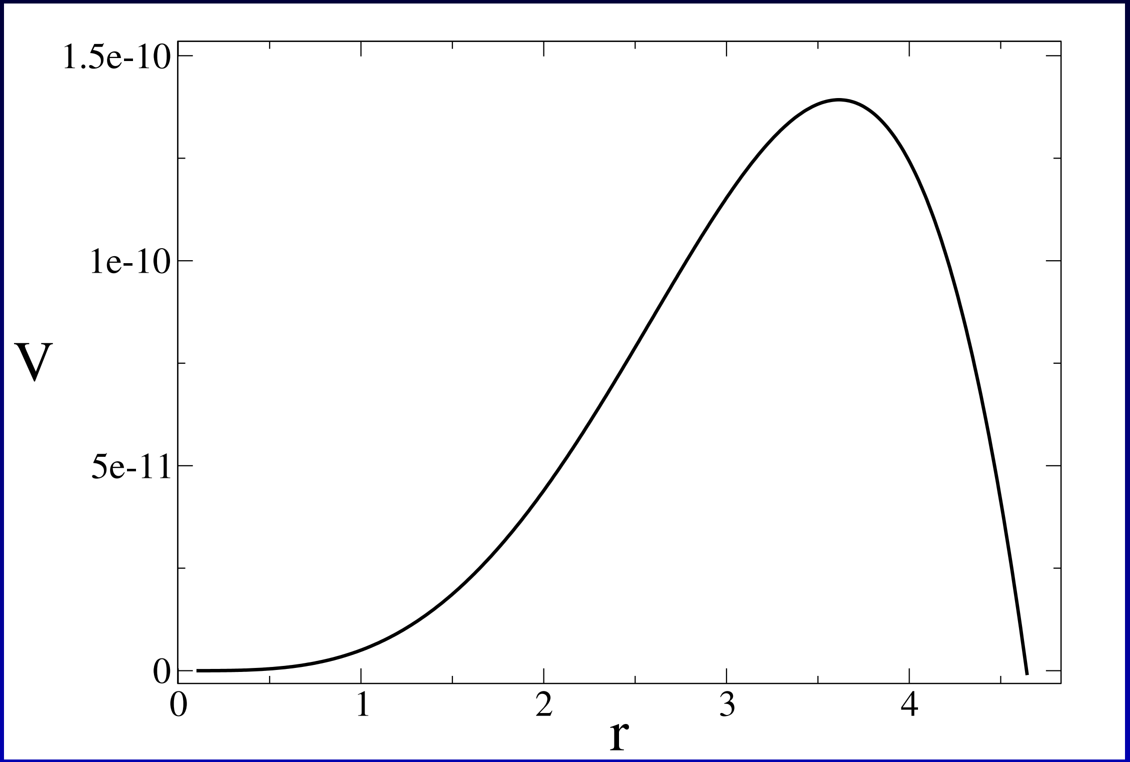
Prevents $\theta_i \to 0$; δV_{tot} is minimized at nontrivial value.

Can we tune δV_{tot} against $m^2\phi^2$ of KKLMMT to get flat potential for inflation?

No: correction has wrong sign!

New contribution to potential

Curvature of δV_{tot} is positive for small r, same as KKLMMT contribution.

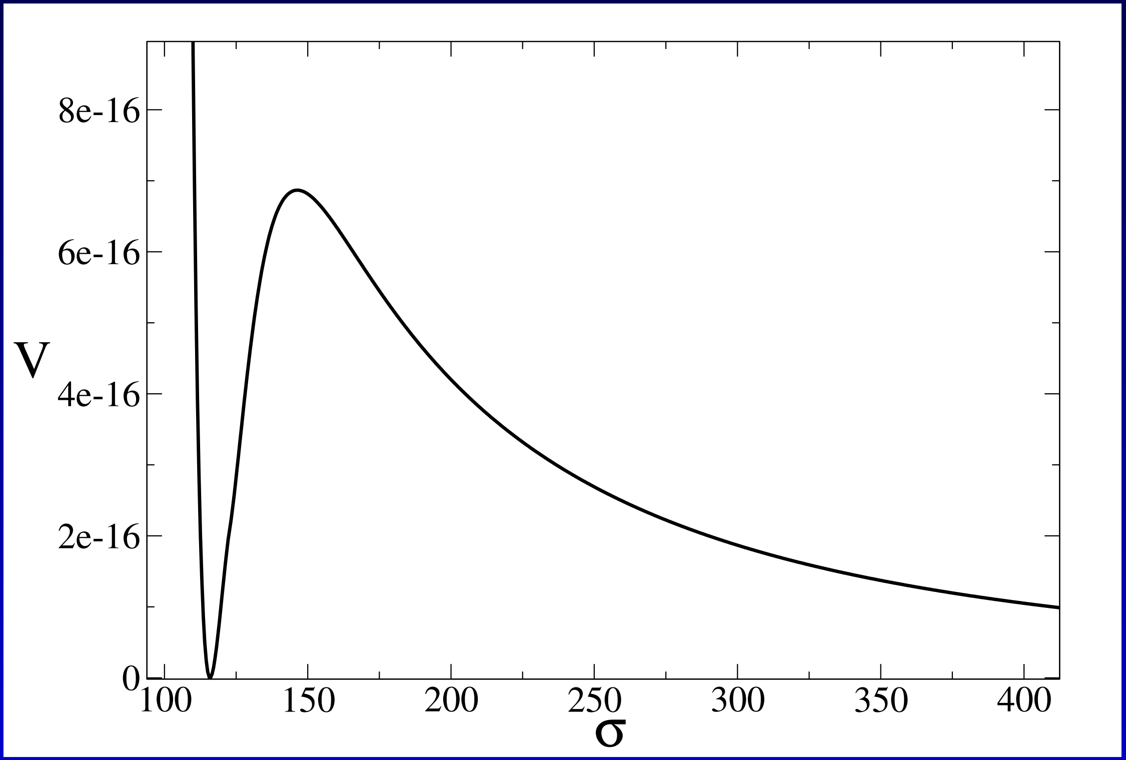


Near maximum of δV_{tot}, curvature is negative, but much too large. Contribution to η parameter:

$$\eta \sim \frac{(T+\overline{T})M_p^2}{\tau_3\, r_{\max}^2} \sim (T+\overline{T})g_s(2\pi)^3\left(\frac{M_p}{M_s}\right)^4 \gg 1$$

On the brighter side: uplifting

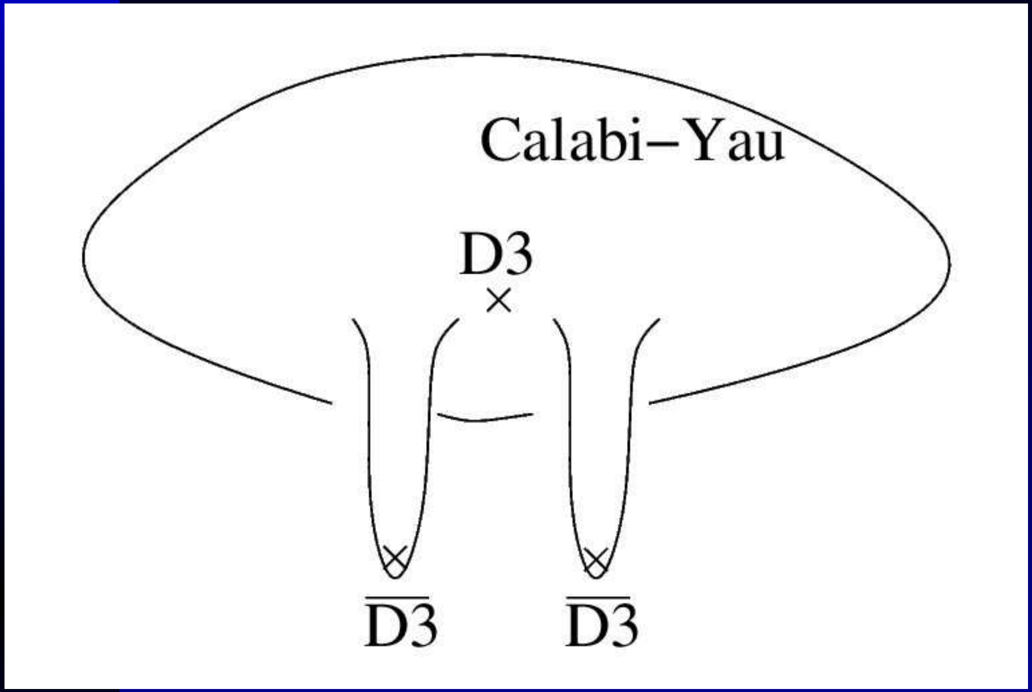
New potential $\delta V_{\rm tot}$ is > 0: can uplift potential for Kähler modulus to Minkowski or de Sitter vacuum, like KKLT, but without explicit SUSY-breaking $\overline{\rm D3}$.



Notice similarity to $\overline{\rm D3}$; no suprise that it works:

$$\delta V_{\overline{D3}} = \frac{c}{\sigma^2}, \qquad \delta V_{\rm tot} = \frac{c}{\sigma^2} \ln(f(\sigma))$$

An inflationary model that works



Iizuka and Trivedi, hep-th/0403203: two symmetrically placed throats, have unstable equilibrium at midpoint:

Gives potential with $V'' < 0$ at $r = 0$ (symmetric point):

$$V^I = -2T_3^2 Z^8 \frac{1}{2\pi^3 M_{10}^8} \left(\frac{1}{|\vec{r} - \vec{r_1}|^4} + \frac{1}{|\vec{r} + \vec{r_1}|^4} \right)$$

Can tune V'' against KKLMMT contribution to get $\eta \ll 1$:

$$r_1 \sim a_0^{2/3} L$$

Conclusions

- Brane-antibrane inflation: a beautiful idea, which does not work
- Maybe two-throat models are okay — should check that superpotential corrections don't ruin this model
- A number of improvements/fixes have been investigated. . .