String Basics

Sunil Mukhi, Tata Institute of Fundamental Research

Outline

Closed Bosonic Strings

- We define a string through its spacetime coordinates $X^{\mu}(\sigma, t)$.

Closed Bosonic Strings

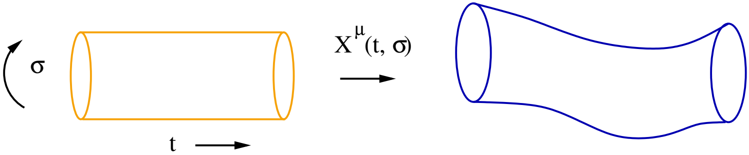
- We define a string through its spacetime coordinates $X^{\mu}(\sigma, t)$.
- To start with, assume it propagates in flat D-dimensional spacetime.

Closed Bosonic Strings

- We define a string through its spacetime coordinates $X^\mu(\sigma, t)$.
- To start with, assume it propagates in flat D-dimensional spacetime.
- σ is a coordinate along the string. Its range is $0 \leq \sigma \leq \pi$.

Closed Bosonic Strings

- We define a string through its spacetime coordinates $X^\mu(\sigma, t)$.
- To start with, assume it propagates in flat D-dimensional spacetime.
- σ is a coordinate along the string. Its range is $0 \leq \sigma \leq \pi$.
- t is the worldsheet time.



- We start with a worldsheet action generalising $\frac{1}{2}m(\dot{X}^i)^2$ for a free nonrelativistic particle:

$$S = -\frac{T}{2}\int d\sigma\, dt\; \partial_a X^\mu \partial^a X_\mu$$

- We start with a worldsheet action generalising $\frac{1}{2}m(\dot{X}^i)^2$ for a free nonrelativistic particle:

$$S = -\frac{T}{2}\int d\sigma\, dt\; \partial_a X^\mu \partial^a X_\mu$$

- The solutions of the equations of motion will be vibration modes of a free string along with the center of mass position/momentum mode of the string.

- We start with a worldsheet action generalising $\frac{1}{2}m(\dot{X}^i)^2$ for a free nonrelativistic particle:

$$S = -\frac{T}{2}\int d\sigma\, dt\; \partial_a X^\mu \partial^a X_\mu$$

- The solutions of the equations of motion will be vibration modes of a free string along with the center of mass position/momentum mode of the string.
- Strings can be closed or open.

- We start with a worldsheet action generalising $\frac{1}{2}m(\dot{X}^i)^2$ for a free nonrelativistic particle:

$$S = -\frac{T}{2} \int d\sigma \, dt \; \partial_a X^\mu \partial^a X_\mu$$

- The solutions of the equations of motion will be vibration modes of a free string along with the center of mass position/momentum mode of the string.
- Strings can be closed or open.
- We first discuss the simpler case of closed strings, defined by:

$$X^\mu(\sigma + \pi, t) = X^\mu(\sigma, t)$$

- In units where $\hbar = c = 1$, the constant T has dimensions of

$$\text{length}^{-2} \sim \text{mass}/\text{length}$$

It is called the string tension.

- In units where $\hbar = c = 1$, the constant T has dimensions of

$$\text{length}^{-2} \sim \text{mass}/\text{length}$$

  It is called the string tension.
- We often use a parameter α' of dimension length^2 defined by:

$$T = \frac{1}{2\pi\alpha'}$$

- In units where $\hbar = c = 1$, the constant T has dimensions of

$$\text{length}^{-2} \sim \text{mass}/\text{length}$$

It is called the string tension.

- We often use a parameter α' of dimension length^2 defined by:

$$T = \frac{1}{2\pi\alpha'}$$

- $\sqrt{\alpha'}$ is a length scale called the string length: the typical size of a string.

- The above action leads to the worldsheet equation of motion:

$$\partial_a \partial^a X^\mu = (\partial_t^2 - \partial_\sigma^2) X^\mu \sim \partial_- \partial_+ X^\mu = 0$$

where the light-cone coordinates are:

$$\xi^\pm = t \pm \sigma, \quad \partial_\pm = \frac{1}{2}(\partial_t \pm \partial_\sigma)$$

- The above action leads to the worldsheet equation of motion:

$$\partial_a \partial^a X^\mu = (\partial_t^2 - \partial_\sigma^2) X^\mu \sim \partial_- \partial_+ X^\mu = 0$$

where the light-cone coordinates are:

$$\xi^\pm = t \pm \sigma, \quad \partial_\pm = \tfrac{1}{2}(\partial_t \pm \partial_\sigma)$$

- The equations of motion are solved by:

$$X^\mu(\sigma, t) = X_L^\mu(t - \sigma) + X_R^\mu(t + \sigma)$$

where X_L, X_R are arbitrary functions of one argument, called left movers and right movers respectively.

- For closed strings X^μ must be periodic, which leads to the mode expansion:

$$
\begin{aligned}
X_L^\mu(t-\sigma) &= \tfrac{1}{2}x^\mu + \tfrac{1}{2}p^\mu\,(t-\sigma) + \tfrac{i}{2}\sum_{n\neq 0}\tfrac{1}{n}\,\alpha_n^\mu\,e^{-2in(t-\sigma)} \\
X_R^\mu(t+\sigma) &= \tfrac{1}{2}x^\mu + \tfrac{1}{2}p^\mu\,(t+\sigma) + \tfrac{i}{2}\sum_{n\neq 0}\tfrac{1}{n}\,\tilde{\alpha}_n^\mu\,e^{-2in(t+\sigma)}
\end{aligned}
$$

where we have put $\alpha' = \frac{1}{2}$ for convenience.

- For closed strings X^μ must be periodic, which leads to the mode expansion:

$$X_L^\mu(t-\sigma) = \frac{1}{2}x^\mu + \frac{1}{2}p^\mu(t-\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu\, e^{-2in(t-\sigma)}$$
$$X_R^\mu(t+\sigma) = \frac{1}{2}x^\mu + \frac{1}{2}p^\mu(t+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^\mu\, e^{-2in(t+\sigma)}$$

  where we have put $\alpha' = \frac{1}{2}$ for convenience.
- As promised, the modes $\alpha^\mu, \tilde{\alpha}^\mu$ are the vibrational modes of the string, while x^μ, p^μ are the position/momentum of the string centre-of-mass.

- For closed strings X^μ must be periodic, which leads to the mode expansion:

$$\begin{aligned} X^\mu_L(t-\sigma) &= \tfrac{1}{2}x^\mu + \tfrac{1}{2}p^\mu\,(t-\sigma) + \tfrac{i}{2}\sum_{n\neq 0}\tfrac{1}{n}\,\alpha^\mu_n\,e^{-2in(t-\sigma)} \\ X^\mu_R(t+\sigma) &= \tfrac{1}{2}x^\mu + \tfrac{1}{2}p^\mu\,(t+\sigma) + \tfrac{i}{2}\sum_{n\neq 0}\tfrac{1}{n}\,\tilde{\alpha}^\mu_n\,e^{-2in(t+\sigma)} \end{aligned}$$

  where we have put $\alpha' = \frac{1}{2}$ for convenience.
- As promised, the modes $\alpha^\mu, \tilde{\alpha}^\mu$ are the vibrational modes of the string, while x^μ, p^μ are the position/momentum of the string centre-of-mass.
- Reality of the coordinates implies that:

$$(\tilde{\alpha}^\mu_n)^* = \tilde{\alpha}^\mu_{-n}, \qquad (\alpha^\mu_n)^* = \alpha^\mu_{-n}$$

- To quantise the system we impose the natural commutation relations:

$$[\alpha^\mu_m, \alpha^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}, \quad [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}$$

on the oscillators, and

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}$$

on the zero modes.

- To quantise the system we impose the natural commutation relations:

$$[\alpha^\mu_m, \alpha^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}\,, \quad [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}$$

 on the oscillators, and

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}$$

 on the zero modes.
- Henceforth we focus only on the left-moving oscillators. It is understood that at the end, the states we construct must be combined with right-moving ones.

- The reality condition on the classical oscillators implies that the corresponding operators satisfy:

$$(\alpha_n^\mu)^\dagger = \alpha_{-n}^\mu$$

- The reality condition on the classical oscillators implies that the corresponding operators satisfy:

$$(\alpha_n^\mu)^\dagger = \alpha^\mu_{-n}$$

- Next one defines a ground state for each oscillator, and treats α_n^μ as creation operators for $n < 0$ and annihilation operators for $n > 0$.

- We can now construct the vibrational states of the string.

- We can now construct the vibrational states of the string.
- The normalised ground state $|0\rangle$ of the string is defined by

$$
\begin{aligned}
\alpha_n^\mu |0\rangle &= 0, \ n > 0 \\
\langle 0|0\rangle &= 1
\end{aligned}
$$

- We can now construct the vibrational states of the string.
- The normalised ground state $|0\rangle$ of the string is defined by

$$\alpha_n^\mu|0\rangle = 0, \ n > 0$$
$$\langle 0|0\rangle = 1$$

- Excited states of the string are then constructed as, for example,

$$\alpha_{-n}^\mu|0\rangle$$

and more generally

$$\alpha_{-m_1}^{\mu_1}\alpha_{-m_2}^{\mu_2}\cdots\alpha_{-m_M}^{\mu_M}|0\rangle$$

- We see right away that something is wrong.

- We see right away that something is wrong.
- Consider the excited state $\alpha^{\mu}_{-n}|0\rangle$. Its norm is:

$$\|\alpha^{\mu}_{-n}|0\rangle\|^2 = \langle 0|\alpha^{\mu}_{n}\,\alpha^{\mu}_{-n}|0\rangle = n\,\eta^{\mu\mu}$$

Thus for $\mu = 0$ (the time direction) we have negative-norm states, which are unacceptable in any physical theory.

- We see right away that something is wrong.
- Consider the excited state $\alpha^\mu_{-n}|0\rangle$. Its norm is:

$$\|\alpha^\mu_{-n}|0\rangle\|^2 = \langle 0|\alpha^\mu_n\, \alpha^\mu_{-n}|0\rangle = n\, \eta^{\mu\mu}$$

  Thus for $\mu = 0$ (the time direction) we have negative-norm states, which are unacceptable in any physical theory.
- This exemplifies a very general problem in relativistic physics. Degrees of freedom with spacetime indices always lead to negative-norm states, unless the theory has gauge constraints.

- Therefore we must modify the action to incorporate a suitable gauge symmetry, namely worldsheet general coordinate invariance:

$$(t,\sigma) \rightarrow (t'(t,\sigma), \sigma'(t,\sigma))$$

- Therefore we must modify the action to incorporate a suitable gauge symmetry, namely worldsheet general coordinate invariance:

$$(t,\sigma) \to (t'(t,\sigma), \sigma'(t,\sigma))$$

- After gauge fixing, we recover our original action:

$$S = -\frac{T}{2} \int d\xi^{+}\, d\xi^{-}\; \partial_{+}X^{\mu}\, \partial_{-}X_{\mu}$$

but now it is supplemented by the bilinear constraints:

$$\partial_{+}X^{\mu}\, \partial_{+}X_{\mu} = 0, \qquad \partial_{-}X^{\mu}\, \partial_{-}X_{\mu} = 0$$

- The worldsheet invariance also imposes two additional conditions:
  (i) for a closed string, the total number of left and right moving excitations must be equal.
  (ii) there is an anomaly proportional to $D - 26$.

- The worldsheet invariance also imposes two additional conditions:
  (i) for a closed string, the total number of left and right moving excitations must be equal.
  (ii) there is an anomaly proportional to $D - 26$.
- To cancel the anomaly, we have to work in 26 dimensions.

- The constraints eliminate all negative-norm states.

- The constraints eliminate all negative-norm states.
- This is most conveniently seen in light-cone gauge where they simply remove two of the D components of the vector index, including the time component.

- The constraints eliminate all negative-norm states.
- This is most conveniently seen in light-cone gauge where they simply remove two of the D components of the vector index, including the time component.
- This is very much like the photon field $A_i, i = 1, 2, \ldots, D-2$ in light cone gauge, having only $D-2$ components.

- The constraints eliminate all negative-norm states.
- This is most conveniently seen in light-cone gauge where they simply remove two of the D components of the vector index, including the time component.
- This is very much like the photon field $A_i, i = 1, 2, \ldots, D-2$ in light cone gauge, having only $D-2$ components.
- Therefore in string theory we need only focus on the transverse oscillators $\alpha^i_{-n}, \tilde{\alpha}^j_{-n}$ with $i, j = 1, 2, \ldots, D-2$.

- Now the string excitations start to resemble familiar objects.

- Now the string excitations start to resemble familiar objects.
- We interpret the string in any given excitation state as an elementary particle whose quantum numbers can be read off from the state.

- Now the string excitations start to resemble familiar objects.
- We interpret the string in any given excitation state as an elementary particle whose quantum numbers can be read off from the state.
- The mass-squared of the particle is given by the weighted number operator counting the oscillator excitations:

$$M^2 = \frac{2}{\alpha'} \left(\sum_{m=1}^{\infty} \alpha^i_{-m} \, \alpha^i_m + \sum_{m=1}^{\infty} \tilde{\alpha}^i_{-m} \, \tilde{\alpha}^i_m - 2 \right)$$

- Now the string excitations start to resemble familiar objects.
- We interpret the string in any given excitation state as an elementary particle whose quantum numbers can be read off from the state.
- The mass-squared of the particle is given by the weighted number operator counting the oscillator excitations:

$$M^2 = \frac{2}{\alpha'}\left(\sum_{m=1}^{\infty} \alpha^i_{-m}\,\alpha^i_m + \sum_{m=1}^{\infty} \tilde{\alpha}^i_{-m}\,\tilde{\alpha}^i_m - 2\right)$$

- The -2 is determined by consistency, and has sinister consequences.

- The ground state $|0\rangle$ has no quantum numbers. So it is a scalar particle at zero transverse momentum.

- The ground state $|0\rangle$ has no quantum numbers. So it is a scalar particle at zero transverse momentum.
- By acting with a zero-mode operator $exp(ik \cdot X)$ this can be converted to a state with transverse momentum k_i.

- The ground state $|0\rangle$ has no quantum numbers. So it is a scalar particle at zero transverse momentum.
- By acting with a zero-mode operator $exp(ik \cdot X)$ this can be converted to a state with transverse momentum k_i.
- Henceforth we always keep the transverse momentum zero. It can be restored at the end.

- The ground state $|0\rangle$ has no quantum numbers. So it is a scalar particle at zero transverse momentum.
- By acting with a zero-mode operator $exp(ik \cdot X)$ this can be converted to a state with transverse momentum k_i.
- Henceforth we always keep the transverse momentum zero. It can be restored at the end.
- This particle has

$$M^2 = -\frac{4}{\alpha'}$$

and is therefore a tachyon.

- The ground state $|0\rangle$ has no quantum numbers. So it is a scalar particle at zero transverse momentum.
- By acting with a zero-mode operator $exp(ik \cdot X)$ this can be converted to a state with transverse momentum k_i.
- Henceforth we always keep the transverse momentum zero. It can be restored at the end.
- This particle has

$$M^2 = -\frac{4}{\alpha'}$$

  and is therefore a tachyon.
- Due to the left-right matching constraint, the first excited state is:

$$\zeta_{ij}\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0\rangle$$

  and this state is massless.

- The polarisation tensor ζ_{ij} decomposes into three irreducible parts: symmetric traceless, antisymmetric, and a trace part which is a singlet.

- The polarisation tensor ζ_{ij} decomposes into three irreducible parts: symmetric traceless, antisymmetric, and a trace part which is a singlet.
- Each one can be identified with the transverse components of a field:

$$\begin{aligned} \zeta_{(ij)}(k) - \tfrac{1}{D-2}\,\delta_{ij}\,\delta^{mn}\,\zeta_{mn}(k) &\rightarrow G_{ij}(x) \\ \zeta_{[ij]}(k) &\rightarrow B_{ij}(x) \\ \delta^{ij}\,\zeta_{ij}(k) &\rightarrow \Phi(x) \end{aligned}$$

- The polarisation tensor ζ_{ij} decomposes into three irreducible parts: symmetric traceless, antisymmetric, and a trace part which is a singlet.
- Each one can be identified with the transverse components of a field:

$$\begin{aligned} \zeta_{(ij)}(k) - \frac{1}{D-2}\,\delta_{ij}\,\delta^{mn}\,\zeta_{mn}(k) &\rightarrow G_{ij}(x) \\ \zeta_{[ij]}(k) &\rightarrow B_{ij}(x) \\ \delta^{ij}\,\zeta_{ij}(k) &\rightarrow \Phi(x) \end{aligned}$$

- These fields, in turn, are the transverse components of the massless fields $G_{\mu\nu}, B_{\mu\nu}, \Phi$ in ten dimensions.

- But it is a theorem that the only consistent action for a massless symmetric rank-2 tensor field is that of gravity.

- But it is a theorem that the only consistent action for a massless symmetric rank-2 tensor field is that of gravity.
- Therefore closed string theory is a theory of gravity!

- But it is a theorem that the only consistent action for a massless symmetric rank-2 tensor field is that of gravity.
- Therefore closed string theory is a theory of gravity!
- The other two particles are important too. The antisymmetric tensor $B_{\mu\nu}$ is responsible for the stability of the string. And in four dimensions it will be an axion.

- But it is a theorem that the only consistent action for a massless symmetric rank-2 tensor field is that of gravity.
- Therefore closed string theory is a theory of gravity!
- The other two particles are important too. The antisymmetric tensor $B_{\mu\nu}$ is responsible for the stability of the string. And in four dimensions it will be an axion.
- Finally, the scalar Φ is called the dilaton and governs the interaction strength of the string.

- String interactions are introduced by defining "vertex operators" for each excited state and computing their correlation functions on the worldsheet.

- String interactions are introduced by defining "vertex operators" for each excited state and computing their correlation functions on the worldsheet.
- This leads to unique answers for every amplitude.

- String interactions are introduced by defining "vertex operators" for each excited state and computing their correlation functions on the worldsheet.
- This leads to unique answers for every amplitude.
- From the amplitudes one can read off the tree-level low-energy effective action of the massless modes, to find:

$$S = \int d^{10}x \sqrt{-\|G\|} e^{-2\Phi} \left(R - \frac{1}{3!} \partial_{[\mu} B_{\nu\lambda]} \, \partial^{[\mu} B^{\nu\lambda]} - \frac{1}{2} \partial_{\mu} \Phi \, \partial^{\mu} \Phi \right)$$

- The existence of the parameter α' means this action can – and does – have corrections involving higher derivatives of fields along with higher powers of α'.

- The existence of the parameter α' means this action can – and does – have corrections involving higher derivatives of fields along with higher powers of α'.
- Also we see that the dilaton vev governs the string coupling:

$$S \sim e^{-2<\Phi>} \int \cdots \sim \frac{1}{g_s^2} \int \cdots \implies e^{<\Phi>} = g_s$$

- The existence of the parameter α' means this action can – and does – have corrections involving higher derivatives of fields along with higher powers of α'.
- Also we see that the dilaton vev governs the string coupling:

$$S \sim e^{-2<\Phi>} \int \cdots \sim \frac{1}{g_s^2} \int \cdots \implies e^{<\Phi>} = g_s$$

- From a particle physicist's point of view, string theory can most often be reduced to such a low energy effective action, with higher derivative and higher loop corrections.

- The existence of the parameter α' means this action can – and does – have corrections involving higher derivatives of fields along with higher powers of α'.
- Also we see that the dilaton vev governs the string coupling:

$$S \sim e^{-2<\Phi>} \int \cdots \sim \frac{1}{g_s^2} \int \cdots \implies e^{<\Phi>} = g_s$$

- From a particle physicist's point of view, string theory can most often be reduced to such a low energy effective action, with higher derivative and higher loop corrections.
- Importantly, given a spacetime background the effective action is unique and computable.

- Besides the tachyon and the massless states, the closed string has infinitely many excited states that are all massive and have increasing spin.

- Besides the tachyon and the massless states, the closed string has infinitely many excited states that are all massive and have increasing spin.
- The effective action for massless states should be thought of as the result of integrating them out.

Outline

Open Bosonic Strings

- For open strings, $X^\mu(\sigma, t)$ is no longer periodic in σ.

Open Bosonic Strings

- For open strings, $X^{\mu}(\sigma, t)$ is no longer periodic in σ.
- Instead, the string must end at $\sigma = 0, \pi$.

Open Bosonic Strings

- For open strings, $X^\mu(\sigma, t)$ is no longer periodic in σ.
- Instead, the string must end at $\sigma = 0, \pi$.
- At each end, we need to specify boundary conditions for the coordinate X^μ or its derivatives.

Open Bosonic Strings

- For open strings, $X^\mu(\sigma, t)$ is no longer periodic in σ.
- Instead, the string must end at $\sigma = 0, \pi$.
- At each end, we need to specify boundary conditions for the coordinate X^μ or its derivatives.
- These are restricted by demanding the absence of boundary terms when varying the worldsheet action.

- We have:

$$\delta S = T \int_0^{\pi} d\sigma \int dt\, \delta X^{\mu}\, \partial_a \partial^a X_{\mu} - T \int dt\, [\delta X^{\mu}\, \partial_{\sigma} X_{\mu}]_0^{\pi}$$

- We have:

$$\delta S = T \int_0^\pi d\sigma \int dt \; \delta X^\mu \, \partial_a \partial^a X_\mu - T \int dt \, [\delta X^\mu \, \partial_\sigma X_\mu]_0^\pi$$

- To make the second term vanish, we require:

$$\delta X^\mu(0,t) \, \partial_\sigma X_\mu(0,t) = \delta X^\mu(\pi,t) \, \partial_\sigma X_\mu(\pi,t) = 0$$

- We have:

$$\delta S = T \int_0^{\pi} d\sigma \int dt\ \delta X^{\mu}\, \partial_a \partial^a X_{\mu} - T \int dt\, [\delta X^{\mu}\, \partial_{\sigma} X_{\mu}]_0^{\pi}$$

- To make the second term vanish, we require:

$$\delta X^{\mu}(0,t)\, \partial_{\sigma} X_{\mu}(0,t) = \delta X^{\mu}(\pi,t)\, \partial_{\sigma} X_{\mu}(\pi,t) = 0$$

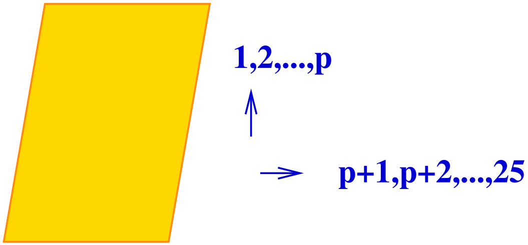
- Thus, at $\sigma = 0$, we can impose one of the following two boundary conditions on each of the spacetime coordinates X^{μ}:

$$\begin{aligned} \partial_{\sigma} X^{\mu}(0,t) &= 0 \qquad \text{(Neumann)} \\ X^{\mu}(0,t) &= c^{\mu} \qquad \text{(Dirichlet)} \end{aligned}$$

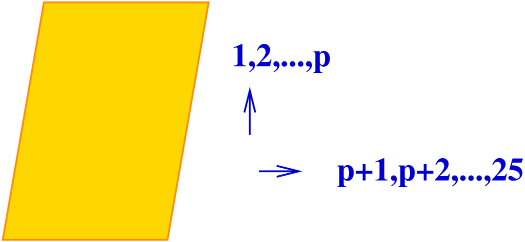
where c^{μ} is an arbitrary constant.

- The D boundary condition states that the end of the string is stuck on a particular hypersurface, called a D-brane.

- The D boundary condition states that the end of the string is stuck on a particular hypersurface, called a D-brane.
- A D-brane has space dimension p if there are p coordinates with Neumann boundary conditions and $25 - p$ coordinates with Dirichlet boundary conditions:



- The D boundary condition states that the end of the string is stuck on a particular hypersurface, called a D-brane.
- A D-brane has space dimension p if there are p coordinates with Neumann boundary conditions and $25 - p$ coordinates with Dirichlet boundary conditions:



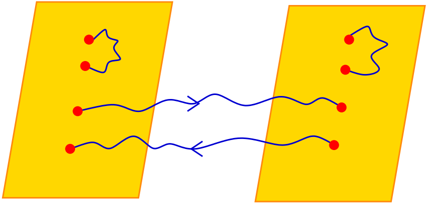
- One should remember that there is also time, so the worldvolume of a p-brane is a $p + 1$-dimensional spacetime.

- At the other end $\sigma = \pi$, we must also independently choose N or D boundary conditions.

- At the other end $\sigma = \pi$, we must also independently choose N or D boundary conditions.
- Thus an open string can be Neumann-Neumann (NN), Dirichlet-Dirichlet (DD) or Neumann-Dirichlet (ND), with respect to each of its spacetime coordinates.

- At the other end $\sigma = \pi$, we must also independently choose N or D boundary conditions.
- Thus an open string can be Neumann-Neumann (NN), Dirichlet-Dirichlet (DD) or Neumann-Dirichlet (ND), with respect to each of its spacetime coordinates.
- In the DD case, the two ends can be stuck at the same location c^μ or at two different locations c^μ, d^μ.

- At the other end $\sigma = \pi$, we must also independently choose N or D boundary conditions.
- Thus an open string can be Neumann-Neumann (NN), Dirichlet-Dirichlet (DD) or Neumann-Dirichlet (ND), with respect to each of its spacetime coordinates.
- In the DD case, the two ends can be stuck at the same location c^μ or at two different locations c^μ, d^μ.
- In one case, the string starts and ends on the same brane, while in the other, it stretches between two different branes.



- With open-string boundary conditions, a wave travelling one way on the string gets reflected back from the end. So there is only one set of vibrational modes rather than two.

- With open-string boundary conditions, a wave travelling one way on the string gets reflected back from the end. So there is only one set of vibrational modes rather than two.
- For NN boundary conditions, we find:

$$X^\mu(\sigma, t) = x^\mu + p^\mu\, t + i \sum_{n \neq 0} \frac{1}{n}\, \alpha_n^\mu\, e^{-int} \cos n\sigma$$

- With open-string boundary conditions, a wave travelling one way on the string gets reflected back from the end. So there is only one set of vibrational modes rather than two.
- For NN boundary conditions, we find:

$$X^\mu(\sigma, t) = x^\mu + p^\mu\, t + i \sum_{n\neq 0} \tfrac{1}{n}\, \alpha_n^\mu\, e^{-int} \cos n\sigma$$

- For DD boundary conditions the result is:

$$X^\mu(\sigma, t) = c^\mu \left(1 - \frac{\sigma}{\pi}\right) + d^\mu \frac{\sigma}{\pi} - \sum_{n\neq 0} \tfrac{1}{n}\, \alpha_n^\mu\, e^{-int} \sin n\sigma$$

- With open-string boundary conditions, a wave travelling one way on the string gets reflected back from the end. So there is only one set of vibrational modes rather than two.
- For NN boundary conditions, we find:

$$X^\mu(\sigma,t) = x^\mu + p^\mu\, t + i \sum_{n\neq 0} \tfrac{1}{n}\, \alpha_n^\mu\, e^{-int} \cos n\sigma$$

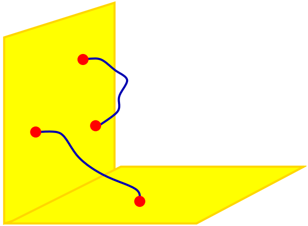
- For DD boundary conditions the result is:

$$X^\mu(\sigma,t) = c^\mu \left(1 - \frac{\sigma}{\pi}\right) + d^\mu \frac{\sigma}{\pi} - \sum_{n\neq 0} \tfrac{1}{n}\, \alpha_n^\mu\, e^{-int} \sin n\sigma$$

- Here c^μ, d^μ specify the locations of the D-branes on which the ends of the string are fixed. As one would expect, there are no translational zero modes x^μ, p^μ in this case.

- For DN and ND strings, the mode expansion involves half-integer modes, as one can easily check.

- For DN and ND strings, the mode expansion involves half-integer modes, as one can easily check.
- This case typically arises for strings connecting intersecting branes.



- In all the above cases, quantising the open string leads to the same problem as for the closed string: negative norm states.

- In all the above cases, quantising the open string leads to the same problem as for the closed string: negative norm states.
- The solution is also the same: these unphysical states are eliminated by gauge constraints on the worldsheet.

- In all the above cases, quantising the open string leads to the same problem as for the closed string: negative norm states.
- The solution is also the same: these unphysical states are eliminated by gauge constraints on the worldsheet.
- We are left with the modes transverse to two light-cone directions.

- Consider first the case of NN boundary conditions in all 25 directions. This defines a D25-brane, which coincides with all of spacetime.

- Consider first the case of NN boundary conditions in all 25 directions. This defines a D25-brane, which coincides with all of spacetime.
- Thus the ends of the strings can be anywhere in spacetime.

- Consider first the case of NN boundary conditions in all 25 directions. This defines a D25-brane, which coincides with all of spacetime.
- Thus the ends of the strings can be anywhere in spacetime.
- In light cone gauge the states of such a string are:

$$\alpha^{i_1}_{-n_1}\alpha^{i_2}_{-n_2}\cdots\alpha^{i_N}_{-n_N}|0\rangle$$

since there is only one type of oscillator.

- Consider first the case of NN boundary conditions in all 25 directions. This defines a D25-brane, which coincides with all of spacetime.
- Thus the ends of the strings can be anywhere in spacetime.
- In light cone gauge the states of such a string are:

$$\alpha^{i_1}_{-n_1}\alpha^{i_2}_{-n_2}\cdots\alpha^{i_N}_{-n_N}|0\rangle$$

since there is only one type of oscillator.

- The masses of these states are given by:

$$M^2 = \frac{1}{\alpha'}\left(\sum_{m=1}^{\infty}\alpha^i_{-m}\,\alpha^i_m - 1\right)$$

- Thus we again have a tachyon at the lowest level, the state $|0\rangle$ of $M^2 = -\frac{1}{\alpha'}$. However, this is the open-string tachyon, distinct from the closed string tachyon that we encountered earlier.

- Thus we again have a tachyon at the lowest level, the state $|0\rangle$ of $M^2 = -\frac{1}{\alpha'}$. However, this is the open-string tachyon, distinct from the closed string tachyon that we encountered earlier.
- The first excited state is a vector:

$$\alpha^i_{-1}|0\rangle$$

and we see that it is massless.

- Thus we again have a tachyon at the lowest level, the state $|0\rangle$ of $M^2 = -\frac{1}{\alpha'}$. However, this is the open-string tachyon, distinct from the closed string tachyon that we encountered earlier.
- The first excited state is a vector:

$$\alpha^i_{-1}|0\rangle$$

  and we see that it is massless.
- A massless vector field in field theory has to be a gauge field, and indeed it is so.

- Thus we again have a tachyon at the lowest level, the state $|0\rangle$ of $M^2 = -\frac{1}{\alpha'}$. However, this is the open-string tachyon, distinct from the closed string tachyon that we encountered earlier.
- The first excited state is a vector:

$$\alpha^i_{-1}|0\rangle$$

  and we see that it is massless.
- A massless vector field in field theory has to be a gauge field, and indeed it is so.
- Thus the massless spectrum of open strings on a D25-brane consists of a gauge field in spacetime.

- Next consider boundary conditions that are NN in p directions and DD in the remaining $25 - p$ directions.

- Next consider boundary conditions that are NN in p directions and DD in the remaining $25 - p$ directions.
- According to our notation this corresponds to a Dp-brane.

- Next consider boundary conditions that are NN in p directions and DD in the remaining $25 - p$ directions.
- According to our notation this corresponds to a Dp-brane.
- The boundary conditions break translation invariance in $25 - p$ directions. They also break rotation invariance:

$$SO(25,1) \rightarrow SO(25-p) \times SO(p,1)$$

- Next consider boundary conditions that are NN in p directions and DD in the remaining $25-p$ directions.
- According to our notation this corresponds to a Dp-brane.
- The boundary conditions break translation invariance in $25-p$ directions. They also break rotation invariance:

$$SO(25,1) \rightarrow SO(25-p) \times SO(p,1)$$

- Both these effects would be natural if we were dealing with an excited state of string theory that contained a physical object (like a soliton) stretching over p spatial dimensions.

- Next consider boundary conditions that are NN in p directions and DD in the remaining $25 - p$ directions.
- According to our notation this corresponds to a Dp-brane.
- The boundary conditions break translation invariance in $25 - p$ directions. They also break rotation invariance:

$$SO(25,1) \to SO(25-p) \times SO(p,1)$$

- Both these effects would be natural if we were dealing with an excited state of string theory that contained a physical object (like a soliton) stretching over p spatial dimensions.
- In other words, the D-brane can be interpreted as a physical excitation in string theory.

- What is the spectrum of string states in this case?

- What is the spectrum of string states in this case?
- As before we have the tachyon, but now it is confined to the brane, where the open strings end.

- What is the spectrum of string states in this case?
- As before we have the tachyon, but now it is confined to the brane, where the open strings end.
- Next there is a massless state:

$$\alpha^{i}_{-1}|0\rangle, \quad i = 1, 2, \ldots, 24$$

and this too is confined to the brane.

- What is the spectrum of string states in this case?
- As before we have the tachyon, but now it is confined to the brane, where the open strings end.
- Next there is a massless state:

$$\alpha^i_{-1}|0\rangle, \quad i = 1, 2, \ldots, 24$$

  and this too is confined to the brane.
- Because of this, we cannot think of the above as the transverse components of a 26-dimensional gauge field.

- What is the spectrum of string states in this case?
- As before we have the tachyon, but now it is confined to the brane, where the open strings end.
- Next there is a massless state:

$$\alpha^i_{-1}|0\rangle, \quad i = 1, 2, \ldots, 24$$

  and this too is confined to the brane.
- Because of this, we cannot think of the above as the transverse components of a 26-dimensional gauge field.
- Rather, the components of the state with $i = 1, 2, \ldots, p-1$ are the transverse components of a p-dimensional gauge field confined to the brane.

- What is the spectrum of string states in this case?
- As before we have the tachyon, but now it is confined to the brane, where the open strings end.
- Next there is a massless state:

$$\alpha^i_{-1}|0\rangle, \quad i = 1, 2, \dots, 24$$

and this too is confined to the brane.
- Because of this, we cannot think of the above as the transverse components of a 26-dimensional gauge field.
- Rather, the components of the state with $i = 1, 2, \dots, p-1$ are the transverse components of a p-dimensional gauge field confined to the brane.
- The remaining components with $i = p, p+1, \dots 25$ are $25 - p$ massless scalar fields confined to the brane.

- The fact that translational invariance is broken in precisely $25 - p$ directions and we have found the same number of massless scalars can hardly be a coincidence.

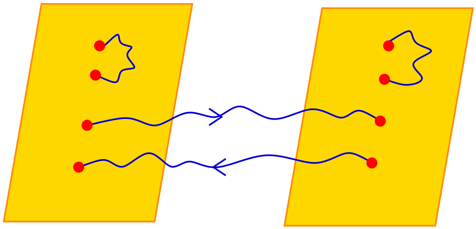
- The fact that translational invariance is broken in precisely $25 - p$ directions and we have found the same number of massless scalars can hardly be a coincidence.
- These are in fact the Goldstone bosons associated with spontaneously broken translation invariance in the state containing a Dp-brane.

- The fact that translational invariance is broken in precisely $25 - p$ directions and we have found the same number of massless scalars can hardly be a coincidence.
- These are in fact the Goldstone bosons associated with spontaneously broken translation invariance in the state containing a Dp-brane.
- In fact the VEV of these scalars is nothing but the position of the brane in the transverse space.

- To conclude our study of the bosonic string, let's notice that if we can have one Dp-brane, we can surely have N of them.

- To conclude our study of the bosonic string, let's notice that if we can have one Dp-brane, we can surely have N of them.
- In this case there are N open strings that start and end on the same brane.

- To conclude our study of the bosonic string, let's notice that if we can have one Dp-brane, we can surely have N of them.
- In this case there are N open strings that start and end on the same brane.
- But there are also $N(N-1)$ open strings stretching from one brane to another, as illustrated here for $N = 2$:



- First take all N Dp-branes to be coincident.

- First take all N Dp-branes to be coincident.
- Quantising the N^2 open strings, we find N^2 massless gauge fields, which can be conveniently encoded into an $N \times N$ Hermitian matrix.

- First take all N Dp-branes to be coincident.
- Quantising the N^2 open strings, we find N^2 massless gauge fields, which can be conveniently encoded into an $N \times N$ Hermitian matrix.
- And there are $25 - p$ scalars, also encoded in similar matrices.

- First take all N Dp-branes to be coincident.
- Quantising the N^2 open strings, we find N^2 massless gauge fields, which can be conveniently encoded into an $N \times N$ Hermitian matrix.
- And there are $25 - p$ scalars, also encoded in similar matrices.
- From the study of interactions among these gauge fields, we find that they are non-Abelian gauge fields for the group $U(N)$.

- First take all N Dp-branes to be coincident.
- Quantising the N^2 open strings, we find N^2 massless gauge fields, which can be conveniently encoded into an $N \times N$ Hermitian matrix.
- And there are $25 - p$ scalars, also encoded in similar matrices.
- From the study of interactions among these gauge fields, we find that they are non-Abelian gauge fields for the group $U(N)$.
- The scalars, being matrices, are in the adjoint representation of this group.

- A $U(1)$ factor can be associated with the overall centre-of-mass of the system of branes, and it decouples from the dynamics.

- A $U(1)$ factor can be associated with the overall centre-of-mass of the system of branes, and it decouples from the dynamics.
- Thus the gauge group is effectively $SU(N)$.

- A $U(1)$ factor can be associated with the overall centre-of-mass of the system of branes, and it decouples from the dynamics.
- Thus the gauge group is effectively $SU(N)$.
- So N coincident Dp-branes automatically realise $U(N)$ gauge fields as their massless states, along with an adjoint scalar for each transverse direction.

- Now let us separate the branes.

- Now let us separate the branes.
- The gauge fields from the “diagonal” strings continue to be massless, but the ones stretching from one brane to another become massive due to their length and tension.

- Now let us separate the branes.
- The gauge fields from the “diagonal” strings continue to be massless, but the ones stretching from one brane to another become massive due to their length and tension.
- This is just the Higgs mechanism:

$$U(N) \to U(1)^N$$

which is what we expect if the Higgsing is done by an adjoint scalar – namely, the one corresponding to the transverse direction along which the branes were separated.

- Now let us separate the branes.
- The gauge fields from the “diagonal” strings continue to be massless, but the ones stretching from one brane to another become massive due to their length and tension.
- This is just the Higgs mechanism:

$$U(N) \to U(1)^N$$

which is what we expect if the Higgsing is done by an adjoint scalar – namely, the one corresponding to the transverse direction along which the branes were separated.

- To summarise, N parallel Dp-branes describe the dynamics of a $p+1$-dimensional $U(N)$ non-Abelian gauge theory on their worldvolume.

- There is a nice way to extend this to other gauge groups.

- There is a nice way to extend this to other gauge groups.
- Let us quotient the string theory by a Z_2 symmetry that simultaneously acts as a geometrical inversion along $25-p$ directions and also inverts the orientation of the string:

$$\begin{aligned} X^{\mu}(t,\sigma) &= X^{\mu}(t,-\sigma), \quad \mu = 0,1,2,\cdots,p \\ X^{i}(t,\sigma) &= -X^{i}(t,-\sigma), \quad i = p+1, p+2, \cdots, 25 \end{aligned}$$

- There is a nice way to extend this to other gauge groups.
- Let us quotient the string theory by a Z_2 symmetry that simultaneously acts as a geometrical inversion along $25-p$ directions and also inverts the orientation of the string:

$$\begin{aligned} X^\mu(t,\sigma) &= X^\mu(t,-\sigma), \quad \mu = 0,1,2,\cdots,p \\ X^i(t,\sigma) &= -X^i(t,-\sigma), \quad i = p+1, p+2, \cdots, 25 \end{aligned}$$

- This quotient creates a fixed locus at $X^i = 0$ that stretches along the X^μ directions: an orientifold plane.

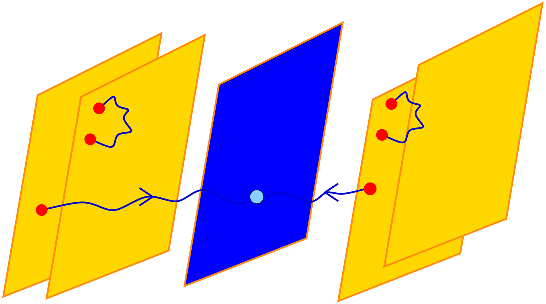
- There is a nice way to extend this to other gauge groups.
- Let us quotient the string theory by a Z_2 symmetry that simultaneously acts as a geometrical inversion along $25-p$ directions and also inverts the orientation of the string:

$$
\begin{aligned}
X^\mu(t,\sigma) &= X^\mu(t,-\sigma), \quad \mu = 0,1,2,\cdots,p \\
X^i(t,\sigma) &= -X^i(t,-\sigma), \quad i = p+1, p+2, \cdots, 25
\end{aligned}
$$

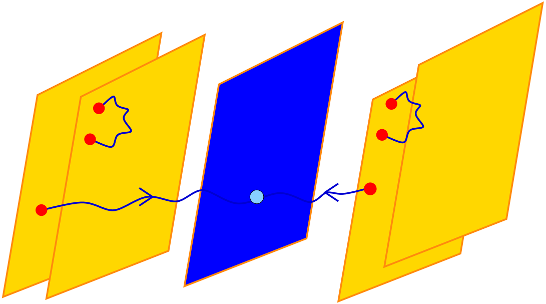
- This quotient creates a fixed locus at $X^i = 0$ that stretches along the X^μ directions: an orientifold plane.
- Strings reverse their orientation on passing through an orientifold plane.

- Now place N Dp-branes parallel to an orientifold p-plane. There is an equal number of images on the other side.

- Now place N Dp-branes parallel to an orientifold p-plane. There is an equal number of images on the other side.
- Open strings stretching between pairs of branes can be projected in or out of the spectrum by the orientifold action.



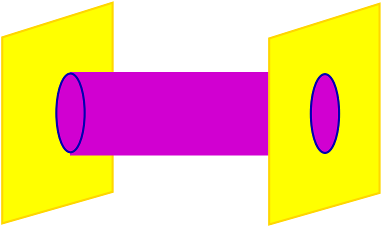
- Now place N Dp-branes parallel to an orientifold p-plane. There is an equal number of images on the other side.
- Open strings stretching between pairs of branes can be projected in or out of the spectrum by the orientifold action.



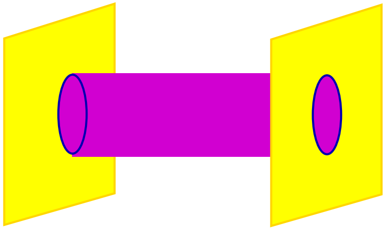
- Depending on the choice of orientifold action, this breaks $U(2N)$ to its subgroups $SO(2N)$ or $Sp(2N)$.

- We have noted that D-branes are dynamical objects that can be created or destroyed in string theory.

- We have noted that D-branes are dynamical objects that can be created or destroyed in string theory.
- They are characterised by a tension which can be found by doing a calculation involving open strings stretching between a pair of branes.



- We have noted that D-branes are dynamical objects that can be created or destroyed in string theory.
- They are characterised by a tension which can be found by doing a calculation involving open strings stretching between a pair of branes.



- The above picture can also be interpreted as closed-string exchange between a pair of branes. This includes graviton exchange and therefore measures the tension.

- The result is found to be:

$$\tau_p \sim \frac{1}{g_s} \frac{1}{(\alpha')^{\frac{p+1}{2}}}$$

- The result is found to be:

$$\tau_p \sim \frac{1}{g_s} \frac{1}{(\alpha')^{\frac{p+1}{2}}}$$

- The α' dependence can be found by dimensional analysis. More striking is the dependence on the string coupling g_s.

- The result is found to be:

$$\tau_p \sim \frac{1}{g_s} \frac{1}{(\alpha')^{\frac{p+1}{2}}}$$

- The α' dependence can be found by dimensional analysis. More striking is the dependence on the string coupling g_s.
- A similar calculation can be done for orientifold planes. They behave effectively like objects with negative tension.

- As we have seen, the closed bosonic string has a tachyon that propagates in spacetime.

- As we have seen, the closed bosonic string has a tachyon that propagates in spacetime.
- And the open bosonic string also has a tachyon, localised on the corresponding D-brane.

- As we have seen, the closed bosonic string has a tachyon that propagates in spacetime.
- And the open bosonic string also has a tachyon, localised on the corresponding D-brane.
- From the worldsheet point of view, a tachyon corresponds to negative energy. It is plausible that it can be eliminated by having worldsheet supersymmetry, due to which the worldsheet energy would be bounded below by zero.

- As we have seen, the closed bosonic string has a tachyon that propagates in spacetime.
- And the open bosonic string also has a tachyon, localised on the corresponding D-brane.
- From the worldsheet point of view, a tachyon corresponds to negative energy. It is plausible that it can be eliminated by having worldsheet supersymmetry, due to which the worldsheet energy would be bounded below by zero.
- This is the motivation to consider superstrings.

Outline

Closed superstrings

- The superstring can be defined in various different formalisms.

Closed superstrings

- The superstring can be defined in various different formalisms.
- We choose the Green-Schwarz formalism defined by adding fermionic coordinates $S_\alpha^A(\sigma, t)$ to the usual $X^\mu(\sigma, t)$ on the worldsheet.

Closed superstrings

- The superstring can be defined in various different formalisms.
- We choose the Green-Schwarz formalism defined by adding fermionic coordinates $S_\alpha^A(\sigma, t)$ to the usual $X^\mu(\sigma, t)$ on the worldsheet.
- This can be done consistently only in $3, 4, 6, 10$ spacetime dimensions. We anticipate that 10 will be the only consistent choice.

Closed superstrings

- The superstring can be defined in various different formalisms.
- We choose the Green-Schwarz formalism defined by adding fermionic coordinates $S^A_\alpha(\sigma, t)$ to the usual $X^\mu(\sigma, t)$ on the worldsheet.
- This can be done consistently only in $3, 4, 6, 10$ spacetime dimensions. We anticipate that 10 will be the only consistent choice.
- The S^A_α are both worldsheet fermions (via the index $\alpha = 1, 2$) and and spacetime fermions (via the index $A = 1, 2, \cdots, 8$ which makes a spinor of $SO(9, 1)$).

- The local reparametrisation symmetry on the worldsheet is now promoted to supersymmetry.

- The local reparametrisation symmetry on the worldsheet is now promoted to supersymmetry.
- After gauge-fixing and incorporating the constraints, one finds the light-cone action:

$$S = -\frac{T}{2} \int d\sigma\, dt \left(\partial_a X^i \partial_a X_i - i S_+^A \partial_- S_+^A - i \bar{S}_-^A \partial_+ \bar{S}_-^A \right)$$

- The local reparametrisation symmetry on the worldsheet is now promoted to supersymmetry.
- After gauge-fixing and incorporating the constraints, one finds the light-cone action:

$$S = -\frac{T}{2} \int d\sigma\, dt \left(\partial_a X^i \partial_a X_i - i S_+^A \partial_- S_+^A - i \bar{S}_-^A \partial_+ \bar{S}_-^A \right)$$

- This time too there is an anomaly, proportional to $D - 10$. Thus the superstring is consistent in 10 dimensions.

- The local reparametrisation symmetry on the worldsheet is now promoted to supersymmetry.
- After gauge-fixing and incorporating the constraints, one finds the light-cone action:

$$S = -\frac{T}{2}\int d\sigma\, dt \left(\partial_a X^i \partial_a X_i - iS^A_+ \partial_- S^A_+ - i\bar{S}^A_- \partial_+ \bar{S}^A_-\right)$$

- This time too there is an anomaly, proportional to $D - 10$. Thus the superstring is consistent in 10 dimensions.
- The equations of motion are the familiar Klein-Gordon and Dirac equations in two dimensions:

$$\partial_- \partial_+ X^\mu = 0, \qquad \partial_- S^A_+ = 0, \quad \partial_+ S^A_- = 0$$

- The mode expansion of the X^μ is as before. But now we would also like to make a mode expansion of the $S^A_\pm$.

- The mode expansion of the X^μ is as before. But now we would also like to make a mode expansion of the $S^A_\pm$.
- Impose closed string boundary conditions on the fermions:

$$S^A_\pm(\sigma+\pi,t)=S^A_\pm(\sigma,t)$$

- The mode expansion of the X^μ is as before. But now we would also like to make a mode expansion of the $S^A_\pm$.
- Impose closed string boundary conditions on the fermions:

$$S^A_\pm(\sigma+\pi,t) = S^A_\pm(\sigma,t)$$

- The mode expansions are then:

$$\begin{aligned} S^A_-(\sigma,t) &= \sum_{n\in\mathbb{Z}} S^A_n\, e^{-2in(t-\sigma)} \\ S^A_+(\sigma,t) &= \sum_{n\in\mathbb{Z}} \tilde{S}^A_n\, e^{-2in(t+\sigma)} \end{aligned}$$

and the fermion oscillators are quantised by anticommutators:

$$\{S^A_m,\, S^B_n\} = \delta_{m+n,0}\,\delta^{AB}$$

- To be economical with equations, we will again do everything in the left-moving sector first.
  The left-moving part of the mass operator is given by:

$$M^2 = \frac{2}{\alpha'}\left(\sum_{m=1}^{\infty} \alpha^i_{-m}\,\alpha^i_m + \sum_{n=1}^{\infty} n\,S^A_{-n}\,S^A_n\right)$$

- To be economical with equations, we will again do everything in the left-moving sector first.
  The left-moving part of the mass operator is given by:

$$M^2 = \frac{2}{\alpha'} \left(\sum_{m=1}^{\infty} \alpha^i_{-m}\, \alpha^i_m + \sum_{n=1}^{\infty} n\, S^A_{-n}\, S^A_n \right)$$

- As anticipated, supersymmetry has eliminated the additive constant.

- To be economical with equations, we will again do everything in the left-moving sector first.
  The left-moving part of the mass operator is given by:

$$M^2 = \frac{2}{\alpha'}\left(\sum_{m=1}^{\infty} \alpha^i_{-m}\,\alpha^i_m + \sum_{n=1}^{\infty} n\, S^A_{-n}\, S^A_n\right)$$

- As anticipated, supersymmetry has eliminated the additive constant.
- Therefore the ground state is massless and the theory is manifestly tachyon-free.

- However, due to zero modes of the periodic worldsheet fermions, the ground state is degenerate.

- However, due to zero modes of the periodic worldsheet fermions, the ground state is degenerate.
- This state is defined (as usual) by:

$$S_n^A|0\rangle = 0, \quad n > 0$$

and the operators $S_{-n}^A, \; n > 0$ are creation operators.

- However, due to zero modes of the periodic worldsheet fermions, the ground state is degenerate.
- This state is defined (as usual) by:

$$S_n^A|0\rangle = 0, \quad n > 0$$

and the operators $S_{-n}^A, \; n > 0$ are creation operators.
- However there are also zero-frequency modes S_0^A.

- These zero modes satisfy a Clifford algebra, much like gamma-matrices:

$$\{S_0^A, S_0^B\} = \delta^{AB}$$

- These zero modes satisfy a Clifford algebra, much like gamma-matrices:

$$\{S_0^A, S_0^B\} = \delta^{AB}$$

- There is a slight difference: gamma matrices are spacetime vectors while the S_0^A are spacetime spinors.

- These zero modes satisfy a Clifford algebra, much like gamma-matrices:
$$\{S_0^A, S_0^B\} = \delta^{AB}$$
- There is a slight difference: gamma matrices are spacetime vectors while the S_0^A are spacetime spinors.
- True gamma matrices in 8d would give rise to a 16-fold degeneracy corresponding to spinors.

- These zero modes satisfy a Clifford algebra, much like gamma-matrices:
$$\{S_0^A, S_0^B\} = \delta^{AB}$$
- There is a slight difference: gamma matrices are spacetime vectors while the S_0^A are spacetime spinors.
- True gamma matrices in 8d would give rise to a 16-fold degeneracy corresponding to spinors.
- Similarly the S_0^A give rise to a 16-fold degeneracy, but this time the degenerate state contains a spacetime vector and a spacetime spinor.

- There are two inequivalent spinor representations of the transverse Lorentz group $SO(8)$:

$$\begin{aligned} \text{spinor:} \qquad & |A\rangle \\ \text{conjugate spinor:} \qquad & |A'\rangle \end{aligned}$$

where $A, A' = 1, 2, \ldots 8$.

- There are two inequivalent spinor representations of the transverse Lorentz group $SO(8)$:

$$\begin{aligned} \text{spinor:} \quad & |A\rangle \\ \text{conjugate spinor:} \quad & |A'\rangle \end{aligned}$$

  where $A, A' = 1, 2, \ldots 8$.

- These correspond to spacetime chirality.

- There are two inequivalent spinor representations of the transverse Lorentz group $SO(8)$:

$$\begin{aligned} \text{spinor:} \quad & |A\rangle \\ \text{conjugate spinor:} \quad & |A'\rangle \end{aligned}$$

  where $A, A' = 1, 2, \ldots 8$.
- These correspond to spacetime chirality.
- By choosing a chirality for the S_{-}^{A}, we can determine the chirality of the ground state, namely spinor or conjugate spinor.

- Thus the massless spectrum of left movers is a vector (Neveu-Schwartz) and a spinor (Ramond):

$$|i\rangle, |A\rangle \qquad \text{or} \qquad |i\rangle, |A'\rangle$$

where we have assigned them certain historical names.

- Thus the massless spectrum of left movers is a vector (Neveu-Schwartz) and a spinor (Ramond):

$$|i\rangle, |A\rangle \qquad \text{or} \qquad |i\rangle, |A'\rangle$$

  where we have assigned them certain historical names.
- These manifestly form a supermultiplet of massless left-moving ground states.

- Thus the massless spectrum of left movers is a vector (Neveu-Schwartz) and a spinor (Ramond):

$$|i\rangle, |A\rangle \qquad \text{or} \qquad |i\rangle, |A'\rangle$$

where we have assigned them certain historical names.
- These manifestly form a supermultiplet of massless left-moving ground states.
- The (left-moving) excited states of the superstring are obtained by acting with $\alpha^i_{-n}, S^A_{-n}, n > 0$ on these ground states.

- Combining left and right movers, we have to make a choice between spinor and conjugate spinor for the Ramond state, independently for left-movers and right-movers.

- Combining left and right movers, we have to make a choice between spinor and conjugate spinor for the Ramond state, independently for left-movers and right-movers.
- The overall choice is a convention, but the relative sign between left and right movers is important.

- Combining left and right movers, we have to make a choice between spinor and conjugate spinor for the Ramond state, independently for left-movers and right-movers.
- The overall choice is a convention, but the relative sign between left and right movers is important.
- Thus we have the following possibilities for the massless states:

$$
\begin{array}{rcccl}
\text{NS-NS:} & |i\rangle & \otimes & |\tilde{j}\rangle & \\
\text{R-R:} & |A\rangle & \otimes & |\tilde{B}\rangle & \text{or } |\tilde{B}'\rangle \\
\text{NS-R:} & |i\rangle & \otimes & |\tilde{B}\rangle & \text{or } |\tilde{B}'\rangle \\
\text{R-NS:} & |A\rangle & \otimes & |\tilde{j}\rangle &
\end{array}
$$

- The NS-NS states, just as for the bosonic string, break up into a symmetric traceless, antisymmetric and trace part.

- The NS-NS states, just as for the bosonic string, break up into a symmetric traceless, antisymmetric and trace part.
- In covariant language these are represented by massless fields propagating in 10 spacetime dimensions:

$$G_{\mu\nu}(x), B_{\mu\nu}(x), \Phi(x)$$

- In the R-R sector we have two physically inequivalent choices:

$$|A\rangle \otimes |\tilde{B}\rangle \quad \text{or} \quad |A\rangle \otimes |\tilde{B}'\rangle$$

- In the R-R sector we have two physically inequivalent choices:

$$|A\rangle \otimes |\tilde{B}\rangle \quad \text{or} \quad |A\rangle \otimes |\tilde{B}'\rangle$$

- The product of two spinorial representations of the Lorentz group is a tensorial representation. Thus in both cases, the R-R sector contains only bosons.

- In the R-R sector we have two physically inequivalent choices:

$$|A\rangle \otimes |\tilde{B}\rangle \quad \text{or} \quad |A\rangle \otimes |\tilde{B}'\rangle$$

- The product of two spinorial representations of the Lorentz group is a tensorial representation. Thus in both cases, the R-R sector contains only bosons.
- Introduce the notation:

$$C^{(r)}_{\mu_1,\mu_2,\ldots,\mu_r}$$

for a totally antisymmetric tensor field of rank r.

- A bit of group theory tells us that

$$|A\rangle \otimes |\tilde{B}'\rangle \;\rightarrow\; C^{(1)}_{\mu}(x),\;\; C^{(3)}_{\mu\nu\lambda}(x)$$

while

$$|A\rangle \otimes |\tilde{B}\rangle \;\rightarrow\; C^{(0)}(x),\;\; C^{(2)}_{\mu\nu}(x),\;\; C^{(4)}_{\mu\nu\lambda\rho}(x)$$

- A bit of group theory tells us that

$$|A\rangle \otimes |\tilde{B}'\rangle \;\rightarrow\; C^{(1)}_{\mu}(x),\;\; C^{(3)}_{\mu\nu\lambda}(x)$$

while

$$|A\rangle \otimes |\tilde{B}\rangle \;\rightarrow\; C^{(0)}(x),\;\; C^{(2)}_{\mu\nu}(x),\;\; C^{(4)}_{\mu\nu\lambda\rho}(x)$$

- These are inequivalent sets of bosonic fields in 10 dimensions.

- A bit of group theory tells us that

$$|A\rangle \otimes |\tilde{B}'\rangle \;\rightarrow\; C^{(1)}_{\mu}(x),\;\; C^{(3)}_{\mu\nu\lambda}(x)$$

while

$$|A\rangle \otimes |\tilde{B}\rangle \;\rightarrow\; C^{(0)}(x),\;\; C^{(2)}_{\mu\nu}(x),\;\; C^{(4)}_{\mu\nu\lambda\rho}(x)$$

- These are inequivalent sets of bosonic fields in 10 dimensions.
- A small technical point: the 4th rank tensor $C^{(4)}$ satisfies a self-duality condition.

- Finally we look at the NS-R and R-NS sectors. In each case, we are combining a tensor and spinor representation, so the result is spinorial.

- Finally we look at the NS-R and R-NS sectors. In each case, we are combining a tensor and spinor representation, so the result is spinorial.
- Therefore these sectors contain spacetime fermions.

- Finally we look at the NS-R and R-NS sectors. In each case, we are combining a tensor and spinor representation, so the result is spinorial.
- Therefore these sectors contain spacetime fermions.
- At the massless level, each of these sectors gives a gravitino and another fermion.

- Finally we look at the NS-R and R-NS sectors. In each case, we are combining a tensor and spinor representation, so the result is spinorial.
- Therefore these sectors contain spacetime fermions.
- At the massless level, each of these sectors gives a gravitino and another fermion.
- The two gravitinos have opposite chiralities for type IIA and the same chirality for type IIB. Therefore the latter theory is parity violating in 10 dimensions.

- The resulting string theory has spacetime supersymmetry.

- The resulting string theory has spacetime supersymmetry.
- Its massless fields are in one-to-one correspondence with those of type IIA and type IIB supergravity.

- The resulting string theory has spacetime supersymmetry.
- Its massless fields are in one-to-one correspondence with those of type IIA and type IIB supergravity.
- It follows that the low-energy effective action of ten-dimensional type IIA/IIB string theory is ten-dimensional type IIA/IIB supergravity.

- The resulting string theory has spacetime supersymmetry.
- Its massless fields are in one-to-one correspondence with those of type IIA and type IIB supergravity.
- It follows that the low-energy effective action of ten-dimensional type IIA/IIB string theory is ten-dimensional type IIA/IIB supergravity.
- But this is only to leading order in α'. The effective action has calculable derivative corrections that come with higher powers of α'.

- To summarise, the massless field contents are as follows:

Type IIA	bosons :	$G_{\mu\nu},\ B_{\mu\nu},\ \Phi$	(NS-NS)
		$C^{(1)}_{\mu},\ C^{(3)}_{\mu\nu\lambda}$	(R-R)
	fermions :	$\chi^{(L)}_{\mu,\alpha},\ \lambda^{(R)}_{\alpha}$	(R-NS)
		$\hat{\chi}^{(R)}_{\mu,\alpha},\ \hat{\lambda}^{(L)}_{\alpha}$	(NS-R)
Type IIB	bosons :	$G_{\mu\nu},\ B_{\mu\nu},\ \Phi$	(NS-NS)
		$C^{(0)},\ C^{(2)}_{\mu\nu},\ C^{(4)}_{\mu\nu\lambda\rho}$	(R-R)
	fermions :	$\chi^{(L)}_{\mu,\alpha},\ \lambda^{(R)}_{\alpha}$	(R-NS)
		$\hat{\chi}^{(L)}_{\mu,\alpha},\ \hat{\lambda}^{(R)}_{\alpha}$	(NS-R)

- To conclude this section, some comments:

- To conclude this section, some comments:
- (i) The RR fields enter only through their field strengths:

$$F^{(n+1)}_{\mu_1\mu_2\cdots\mu_{n+1}} = \partial_{[mu_1} C^{(n)}_{\mu_2\mu_3\cdots\mu_{n+1}]}$$

where the indices are totally antisymmetrised.

- To conclude this section, some comments:
- (i) The RR fields enter only through their field strengths:

$$F^{(n+1)}_{\mu_1\mu_2\cdots\mu_{n+1}} = \partial_{[mu_1} C^{(n)}_{\mu_2\mu_3\cdots\mu_{n+1}]}$$

where the indices are totally antisymmetrised.
- (ii) Therefore we have:

IIA: Even field strengths	$F^{(2)}, F^{(4)}$
	$F^{(6)} = {}^*F^{(4)}, F^{(8)} = {}^*F^{(2)}$
IIB: Odd field strengths	$F^{(1)}, F^{(3)}, F^{(5)} = {}^*F^{(5)}$
	$F^{(7)} = {}^*F^{(3)}, F^{(9)} = {}^*F^{(1)}$

- (iii) In type IIB, the dilaton Φ naturally combines with the RR scalar $C^{(0)}$ to make the axiodilaton:

$$\tau = C^{(0)} + ie^{-\Phi}$$

- (iii) In type IIB, the dilaton Φ naturally combines with the RR scalar $C^{(0)}$ to make the axiodilaton:

$$\tau = C^{(0)} + ie^{-\Phi}$$

- (iv) At tree level, the bosonic part of the effective action can be written as:

$$S_{eff} = \int d^{10}x \, \sqrt{-\|G\|} \, \left[e^{-2\Phi} (\text{NS-NS terms}) + (\text{R-R terms}) \right]$$

- (iii) In type IIB, the dilaton Φ naturally combines with the RR scalar $C^{(0)}$ to make the axiodilaton:

$$\tau = C^{(0)} + ie^{-\Phi}$$

- (iv) At tree level, the bosonic part of the effective action can be written as:

$$S_{eff} = \int d^{10}x \sqrt{-\|G\|} \left[e^{-2\Phi}(\text{NS-NS terms}) + (\text{R-R terms}) \right]$$

- So the scaling with coupling constant of the tree-level R-R terms is different from the NS-NS terms.

Outline

Open superstrings

- For the open superstring, the boundary conditions in the variation of the fermionic part of the action are easily seen to be:

$$\int dt \left[\delta S_+^A S_+^A - \delta S_-^A S_-^A \right]_0^\pi = 0$$

Open superstrings

- For the open superstring, the boundary conditions in the variation of the fermionic part of the action are easily seen to be:

$$\int dt \left[\delta S_+^A\, S_+^A - \delta S_-^A\, S_-^A\right]_0^\pi = 0$$

- The solution of these conditions is:

$$\begin{aligned} S_-^A(0,t) &= \eta_1\, S_+^A(0,t) \\ S_-^A(\pi,t) &= \eta_2\, S_+^A(\pi,t) \end{aligned}$$

where $\eta_1, \eta_2 = \pm 1$.

Open superstrings

- For the open superstring, the boundary conditions in the variation of the fermionic part of the action are easily seen to be:

$$\int dt \left[\delta S_+^A S_+^A - \delta S_-^A S_-^A\right]_0^\pi = 0$$

- The solution of these conditions is:

$$\begin{aligned} S_-^A(0,t) &= \eta_1 S_+^A(0,t) \\ S_-^A(\pi,t) &= \eta_2 S_+^A(\pi,t) \end{aligned}$$

  where $\eta_1, \eta_2 = \pm 1$.

- The physics only depends on the relative sign. The supersymmetry-preserving choice is $\eta_1 = \eta_2$.

- For the bosonic coordinates, the mode expansion depends on whether we have NN,DD,ND or DN boundary conditions, just as for the open bosonic string.

- For the bosonic coordinates, the mode expansion depends on whether we have NN,DD,ND or DN boundary conditions, just as for the open bosonic string.
- For the moment we assume NN conditions on all 9 directions, which amounts to having a D9-brane filling spacetime.

- For the bosonic coordinates, the mode expansion depends on whether we have NN,DD,ND or DN boundary conditions, just as for the open bosonic string.
- For the moment we assume NN conditions on all 9 directions, which amounts to having a D9-brane filling spacetime.
- Again there are worldsheet (super) gauge constraints, which leave only the coordinates with transverse indices.

- With the above boundary conditions, the fermions have integer modes:

$$\begin{aligned} S_-^A(\sigma,t) &= \sum_{n\in\mathbb{Z}} S_n^A\, e^{-in(t-\sigma)} \\ S_+^A(\sigma,t) &= \sum_{n\in\mathbb{Z}} S_n^A\, e^{-in(t+\sigma)} \end{aligned}$$

and we see again that there is only one set of oscillators.

- With the above boundary conditions, the fermions have integer modes:

$$S_-^A(\sigma,t) = \sum_{n\in\mathbb{Z}} S_n^A\, e^{-in(t-\sigma)}$$
$$S_+^A(\sigma,t) = \sum_{n\in\mathbb{Z}} S_n^A\, e^{-in(t+\sigma)}$$

and we see again that there is only one set of oscillators.

- The mass is given by:

$$M^2 = \frac{1}{\alpha'}\left(\sum_{m=1}^{\infty} \alpha_{-m}^i\, \alpha_m^i + \sum_{n=1}^{\infty} n\, S_{-n}^A\, S_n^A\right)$$

- With the above boundary conditions, the fermions have integer modes:

$$
\begin{aligned}
S_-^A(\sigma, t) &= \sum_{n \in \mathbb{Z}} S_n^A \, e^{-in(t-\sigma)} \\
S_+^A(\sigma, t) &= \sum_{n \in \mathbb{Z}} S_n^A \, e^{-in(t+\sigma)}
\end{aligned}
$$

  and we see again that there is only one set of oscillators.

- The mass is given by:

$$
M^2 = \frac{1}{\alpha'} \left(\sum_{m=1}^{\infty} \alpha_{-m}^i \, \alpha_m^i + \sum_{n=1}^{\infty} n \, S_{-n}^A \, S_n^A \right)
$$

- Again there is no tachyon, but we have the now-familiar ground-state degeneracy.

- Thus the massless spectrum is:

$$\begin{array}{rll} \text{bosons:} & A_\mu & \text{(NS)} \\ \text{fermions:} & \lambda_A & \text{(R)} \end{array}$$

- Thus the massless spectrum is:

$$\begin{aligned} \text{bosons:} \quad & A_\mu \qquad \text{(NS)} \\ \text{fermions:} \quad & \lambda_A \qquad \text{(R)} \end{aligned}$$

- This is the field content of $\mathcal{N} = 1$ supersymmetric gauge theory in 10 dimensions.

- Thus the massless spectrum is:

$$\begin{array}{rll} \text{bosons:} & A_\mu & \text{(NS)} \\ \text{fermions:} & \lambda_A & \text{(R)} \end{array}$$

- This is the field content of $\mathcal{N} = 1$ supersymmetric gauge theory in 10 dimensions.
- We see that a D9-brane supports a supersymmetric gauge theory on its worldvolume.

- If we choose N coincident D9-branes then A_μ, λ_A are promoted to matrices and we get $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in 10D.

- If we choose N coincident D9-branes then A_μ, λ_A are promoted to matrices and we get $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in 10D.
- Instead of all NN boundary conditions, if we choose p NN and $9 - p$ DD boundary conditions, we find a Dp-brane instead of a D9-brane.

- If we choose N coincident D9-branes then A_μ, λ_A are promoted to matrices and we get $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in 10D.
- Instead of all NN boundary conditions, if we choose p NN and $9-p$ DD boundary conditions, we find a Dp-brane instead of a D9-brane.
- In this case, the massless fields are a photon A_μ in $p+1$ spacetime dimensions, as well as $9-p$ scalar fields ϕ_i.

- If we choose N coincident D9-branes then A_μ, λ_A are promoted to matrices and we get $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in 10D.
- Instead of all NN boundary conditions, if we choose p NN and $9 - p$ DD boundary conditions, we find a Dp-brane instead of a D9-brane.
- In this case, the massless fields are a photon A_μ in $p + 1$ spacetime dimensions, as well as $9 - p$ scalar fields ϕ_i.
- The fermions also decompose suitably, depending on the dimension p of the brane.

- The result is always the maximally supersymmetric gauge theory in that dimension.

- The result is always the maximally supersymmetric gauge theory in that dimension.
- It can be simply obtained as the dimensional reduction of the 10d supersymmetric gauge theory.

- The result is always the maximally supersymmetric gauge theory in that dimension.
- It can be simply obtained as the dimensional reduction of the 10d supersymmetric gauge theory.
- A classic example is a D3-brane. We find a gauge field A_μ, six scalars ϕ^K and four fermions λ^a. This is the spectrum of $\mathcal{N} = 4$ supersymmetric gauge theory in 3+1 dimensions.

- We mentioned earlier that D-branes are dynamical objects in string theory.

- We mentioned earlier that D-branes are dynamical objects in string theory.
- One way to verify this is to notice that objects with the same properties exist as stable solitonic solutions of the supergravity equations of motion.

- We mentioned earlier that D-branes are dynamical objects in string theory.
- One way to verify this is to notice that objects with the same properties exist as stable solitonic solutions of the supergravity equations of motion.
- The tension of these objects can be computed from this solution and compared with that obtained for D-branes from open-string scattering amplitudes. In both cases, we find:

$$T_p = \frac{1}{g_s} \frac{1}{(2\pi)^p} \frac{1}{(\alpha')^{\frac{p+1}{2}}}$$

- We mentioned earlier that D-branes are dynamical objects in string theory.
- One way to verify this is to notice that objects with the same properties exist as stable solitonic solutions of the supergravity equations of motion.
- The tension of these objects can be computed from this solution and compared with that obtained for D-branes from open-string scattering amplitudes. In both cases, we find:

$$T_p = \frac{1}{g_s} \frac{1}{(2\pi)^p} \frac{1}{(\alpha')^{\frac{p+1}{2}}}$$

- This shows that we are dealing with two different descriptions of the same object.

- It can also be shown using supergravity that the solitonic brane solutions are accompanied by a flux of some Ramond-Ramond (R-R) tensor gauge field.

- It can also be shown using supergravity that the solitonic brane solutions are accompanied by a flux of some Ramond-Ramond (R-R) tensor gauge field.
- This means they carry the corresponding R-R charge.

- It can also be shown using supergravity that the solitonic brane solutions are accompanied by a flux of some Ramond-Ramond (R-R) tensor gauge field.
- This means they carry the corresponding R-R charge.
- One finds the same result from open-string scattering amplitudes: D-branes carry Ramond-Ramond charge.

- It can also be shown using supergravity that the solitonic brane solutions are accompanied by a flux of some Ramond-Ramond (R-R) tensor gauge field.
- This means they carry the corresponding R-R charge.
- One finds the same result from open-string scattering amplitudes: D-branes carry Ramond-Ramond charge.
- Thus they can only be pair-produced, due to conservation of charge.

- Moreover, they are the lightest objects with the minimum quantum of charge. This is how we know that they are stable.

- Moreover, they are the lightest objects with the minimum quantum of charge. This is how we know that they are stable.
- The mechanism is similar to that which makes the string stable, except that it uses R-R rather than NS-NS gauge fields.

- Moreover, they are the lightest objects with the minimum quantum of charge. This is how we know that they are stable.
- The mechanism is similar to that which makes the string stable, except that it uses R-R rather than NS-NS gauge fields.
- Since they minimise their tension for a given charge, these D-branes saturate a bound called the Bogomolny-Prasad-Sommerfeld bound:

$$\text{tension} \geq \text{charge}$$

- Moreover, they are the lightest objects with the minimum quantum of charge. This is how we know that they are stable.
- The mechanism is similar to that which makes the string stable, except that it uses R-R rather than NS-NS gauge fields.
- Since they minimise their tension for a given charge, these D-branes saturate a bound called the Bogomolny-Prasad-Sommerfeld bound:

$$\text{tension} \geq \text{charge}$$

- Hence they are called BPS branes.

- As an explicit example, consider type IIA string theory, which has an RR gauge field $C_\mu^{(1)}$ with a field strength:

$$F_{\mu\nu}^{(2)} = \partial_\mu C_\nu^{(1)} - \partial_\nu C_\mu^{(1)}$$

- As an explicit example, consider type IIA string theory, which has an RR gauge field $C^{(1)}_\mu$ with a field strength:

$$F^{(2)}_{\mu\nu} = \partial_\mu C^{(1)}_\nu - \partial_\nu C^{(1)}_\mu$$

- In the presence of a D0-brane, this field strength satisfies the equation:

$$\sum_{j=1}^{9} \partial^j F^{(2)}_{0j} \sim q\,\delta^9(x)$$

with $q = \pm 1$

- As an explicit example, consider type IIA string theory, which has an RR gauge field $C^{(1)}_\mu$ with a field strength:

$$F^{(2)}_{\mu\nu} = \partial_\mu C^{(1)}_\nu - \partial_\nu C^{(1)}_\mu$$

- In the presence of a D0-brane, this field strength satisfies the equation:

$$\sum_{j=1}^{9} \partial^j F^{(2)}_{0j} \sim q\,\delta^9(x)$$

  with $q = \pm 1$

- Thus we can have both D0 and anti-D0 branes, which behave like electrically charged point particles.

- In the same type IIA superstring theory, there are also D2-branes.

- In the same type IIA superstring theory, there are also D2-branes.
- These are charged under the third rank R-R tensor gauge field $C^{(3)}_{\mu\nu\lambda}$. They are sources for the field strength $F^{(4)}$.

- In the same type IIA superstring theory, there are also D2-branes.
- These are charged under the third rank R-R tensor gauge field $C^{(3)}_{\mu\nu\lambda}$. They are sources for the field strength $F^{(4)}$.
- The charged, BPS branes are as follows:

$$\begin{aligned} \text{type IIA}: \quad & D0, D2, D4, D6, D8 \\ \text{type IIB}: \quad & D1, D3, D5, D7, D9 \end{aligned}$$

- In the same type IIA superstring theory, there are also D2-branes.
- These are charged under the third rank R-R tensor gauge field $C^{(3)}_{\mu\nu\lambda}$. They are sources for the field strength $F^{(4)}$.
- The charged, BPS branes are as follows:

$$\begin{aligned} \text{type IIA}: &\quad D0, D2, D4, D6, D8 \\ \text{type IIB}: &\quad D1, D3, D5, D7, D9 \end{aligned}$$

- There are other, uncharged, branes in superstring theory:

$$\begin{aligned} \text{type IIA}: &\quad D1, D3, D5, D7, D9 \\ \text{type IIB}: &\quad D0, D2, D4, D6, D8 \end{aligned}$$

which have tachyons on their world-volume.

- In the same type IIA superstring theory, there are also D2-branes.
- These are charged under the third rank R-R tensor gauge field $C^{(3)}_{\mu\nu\lambda}$. They are sources for the field strength $F^{(4)}$.
- The charged, BPS branes are as follows:

$$\begin{aligned} \text{type IIA}: &\quad D0, D2, D4, D6, D8 \\ \text{type IIB}: &\quad D1, D3, D5, D7, D9 \end{aligned}$$

- There are other, uncharged, branes in superstring theory:

$$\begin{aligned} \text{type IIA}: &\quad D1, D3, D5, D7, D9 \\ \text{type IIB}: &\quad D0, D2, D4, D6, D8 \end{aligned}$$

  which have tachyons on their world-volume.
- As Sen has explained, these tachyons are manifestations of the brane's instability.

Outline

Compactification

- Type IIA/B superstring theories in 10 dimensions are consistent theories that reduce to $\mathcal{N} = 2$ supergravity with derivative corrections.

Compactification

- Type IIA/B superstring theories in 10 dimensions are consistent theories that reduce to $\mathcal{N} = 2$ supergravity with derivative corrections.
- What does this have to do with 4-dimensional physics?

Compactification

- Type IIA/B superstring theories in 10 dimensions are consistent theories that reduce to $\mathcal{N} = 2$ supergravity with derivative corrections.
- What does this have to do with 4-dimensional physics?
- Our quantisation of the theory in 10 flat, extended spacetime dimensions has perhaps been slightly misleading. We could have chosen to have the string propagate in any 10-dimensional spacetime.

Compactification

- Type IIA/B superstring theories in 10 dimensions are consistent theories that reduce to $\mathcal{N} = 2$ supergravity with derivative corrections.
- What does this have to do with 4-dimensional physics?
- Our quantisation of the theory in 10 flat, extended spacetime dimensions has perhaps been slightly misleading. We could have chosen to have the string propagate in any 10-dimensional spacetime.
- All such choices need not be consistent. But there is one very simple choice that is always consistent.

Compactification

- Type IIA/B superstring theories in 10 dimensions are consistent theories that reduce to $\mathcal{N} = 2$ supergravity with derivative corrections.
- What does this have to do with 4-dimensional physics?
- Our quantisation of the theory in 10 flat, extended spacetime dimensions has perhaps been slightly misleading. We could have chosen to have the string propagate in any 10-dimensional spacetime.
- All such choices need not be consistent. But there is one very simple choice that is always consistent.
- This is to compactify the spacetime on a product of circles.

- Let us use new labels for the spacetime directions:

$$
\begin{aligned}
0,1,2,3 &\quad \rightarrow \mu, \nu \cdots \\
4,5,6,7,8,9 &\quad \rightarrow i, j, \cdots
\end{aligned}
$$

- Let us use new labels for the spacetime directions:

$$\begin{aligned} 0,1,2,3 &\quad \rightarrow \mu, \nu \cdots \\ 4,5,6,7,8,9 &\quad \rightarrow i, j, \cdots \end{aligned}$$

- Now suppose that the six coordinates X^i are periodic:

$$X^i \sim X^i + 2\pi R^i$$

- Let us use new labels for the spacetime directions:

$$\begin{aligned} 0,1,2,3 &\quad \rightarrow \mu,\nu\cdots \\ 4,5,6,7,8,9 &\quad \rightarrow i,j,\cdots \end{aligned}$$

- Now suppose that the six coordinates X^i are periodic:

$$X^i \sim X^i + 2\pi R^i$$

- This has nothing to do with worldsheet boundary conditions! It says that some directions of physical space are curled up:

- Let us use new labels for the spacetime directions:

$$\begin{aligned} 0,1,2,3 \quad &\rightarrow \mu,\nu\cdots \\ 4,5,6,7,8,9 \quad &\rightarrow i,j,\cdots \end{aligned}$$

- Now suppose that the six coordinates X^i are periodic:

$$X^i \sim X^i + 2\pi R^i$$

- This has nothing to do with worldsheet boundary conditions! It says that some directions of physical space are curled up:
- If we probe such a world through experiments whose available energy E satisfies:

$$E \ll \frac{1}{R_i} \quad \text{for all } i$$

then this world will not appear 10-dimensional, but rather 4-dimensional.

- This is because, for its Fourier modes to fit into the compact dimension, an elementary particle needs an energy of order the inverse radius.

- This is because, for its Fourier modes to fit into the compact dimension, an elementary particle needs an energy of order the inverse radius.
- What would change if we formulated superstring theory in this kind of "toroidally compactified" spacetime?

- This is because, for its Fourier modes to fit into the compact dimension, an elementary particle needs an energy of order the inverse radius.
- What would change if we formulated superstring theory in this kind of "toroidally compactified" spacetime?
- (i) The periodicity of the six X^i's breaks the Lorentz group:

$$SO(9,1) \to SO(3,1)$$

This is, of course, a good thing!

- This is because, for its Fourier modes to fit into the compact dimension, an elementary particle needs an energy of order the inverse radius.
- What would change if we formulated superstring theory in this kind of "toroidally compactified" spacetime?
- (i) The periodicity of the six X^i's breaks the Lorentz group:

$$SO(9,1) \to SO(3,1)$$

  This is, of course, a good thing!
- (ii) The mode expansion of the closed string changes and we get additional modes. Instead of

$$X^i = x^i + 2\alpha' p^i t + \text{oscillators}$$

  we now have

$$X^i = x^i + 2\alpha' p^i t + 2L^i \sigma + \text{oscillators}$$

  where L^i are quantised winding modes.

- The impact of toroidal compactification is as follows.

- The impact of toroidal compactification is as follows.
- The massless modes in 10 dimensions must be decomposed into four-dimensional modes whose coefficients can vary over the compact directions:

$$f(x^0, x^1, \cdots, x^9) = \sum_i g_i(x^4, x^5, \cdots, x^9) h_i(x^0, x^1, x^2, x^3)$$

- The impact of toroidal compactification is as follows.
- The massless modes in 10 dimensions must be decomposed into four-dimensional modes whose coefficients can vary over the compact directions:

$$f(x^0, x^1, \cdots, x^9) = \sum_i g_i(x^4, x^5, \cdots, x^9) h_i(x^0, x^1, x^2, x^3)$$

- If g_i is harmonic ("massless") in the internal directions then h_i is a massless field, in fact a flat direction or modulus.

- The number of moduli depends on the type of 10d field we are considering, and the geometry of the compactification space.

- The number of moduli depends on the type of 10d field we are considering, and the geometry of the compactification space.
- In particular, if the 10d field is the metric then we generate one modulus field for every geometric deformation of the torus (its lengths and angles).

- Unfortunately, plain toroidal compactification is extremely unrealistic.

- Unfortunately, plain toroidal compactification is extremely unrealistic.
- It does not break any symmetries other than rotational invariance. In particular, we get the same supersymmetry as $\mathcal{N} = 2$ in 10 dimensions, which is $N = 8$ in 4 dimensions.

- Unfortunately, plain toroidal compactification is extremely unrealistic.
- It does not break any symmetries other than rotational invariance. In particular, we get the same supersymmetry as $\mathcal{N} = 2$ in 10 dimensions, which is $N = 8$ in 4 dimensions.
- And there are many moduli (36 for a 6-torus).

- A better way to compactify is to choose a 6-manifold that is not flat but nontrivially solves the supergravity equations of motion.

- A better way to compactify is to choose a 6-manifold that is not flat but nontrivially solves the supergravity equations of motion.
- First, note that one has to solve the vacuum Einstein equation:

$$R_{ij} = 0$$

in the internal space. Therefore the space is "Ricci flat".

- A better way to compactify is to choose a 6-manifold that is not flat but nontrivially solves the supergravity equations of motion.
- First, note that one has to solve the vacuum Einstein equation:

$$R_{ij} = 0$$

in the internal space. Therefore the space is "Ricci flat".
- Next, to preserve some supersymmetry after compactification one has to make the RHS of the gravitino variation vanish:

$$\delta\chi_{\mu,\alpha} = (\nabla_\mu \epsilon)_\alpha = 0$$

which puts a further constraint on the internal space.

- A better way to compactify is to choose a 6-manifold that is not flat but nontrivially solves the supergravity equations of motion.
- First, note that one has to solve the vacuum Einstein equation:

$$R_{ij} = 0$$

  in the internal space. Therefore the space is "Ricci flat".
- Next, to preserve some supersymmetry after compactification one has to make the RHS of the gravitino variation vanish:

$$\delta\chi_{\mu,\alpha} = (\nabla_\mu \epsilon)_\alpha = 0$$

  which puts a further constraint on the internal space.
- Additional conditions come from SUSY variations of the other fermions in 10d. These also restrict the possible values of RR fluxes along the compact directions.

- The simplest solution is to put all RR fluxes to zero.

- The simplest solution is to put all RR fluxes to zero.
- The 6-manifold then has to be a particular kind of Ricci-flat complex manifold satisfying other topological conditions, called a Calabi-Yau space.

- The simplest solution is to put all RR fluxes to zero.
- The 6-manifold then has to be a particular kind of Ricci-flat complex manifold satisfying other topological conditions, called a Calabi-Yau space.
- These spaces break supersymmetry down to $\mathcal{N} = 2$ in 4d, which is not really good enough since we would like $\mathcal{N} = 1$.

- The simplest solution is to put all RR fluxes to zero.
- The 6-manifold then has to be a particular kind of Ricci-flat complex manifold satisfying other topological conditions, called a Calabi-Yau space.
- These spaces break supersymmetry down to $\mathcal{N} = 2$ in 4d, which is not really good enough since we would like $\mathcal{N} = 1$.
- Moreover, there is a family of geometrical deformations for any given Calabi-Yau, that are known to mathematicians as moduli.

- The simplest solution is to put all RR fluxes to zero.
- The 6-manifold then has to be a particular kind of Ricci-flat complex manifold satisfying other topological conditions, called a Calabi-Yau space.
- These spaces break supersymmetry down to $\mathcal{N} = 2$ in 4d, which is not really good enough since we would like $\mathcal{N} = 1$.
- Moreover, there is a family of geometrical deformations for any given Calabi-Yau, that are known to mathematicians as moduli.
- Each CY modulus corresponds to a scalar field with an exactly flat potential in 4d. As we saw, these are called moduli fields.

- Thus, Calabi-Yau compactifications where the RR fluxes are set to zero fail on two counts:
  (i) too much supersymmetry.
  (ii) too many moduli fields.

- Thus, Calabi-Yau compactifications where the RR fluxes are set to zero fail on two counts:
  (i) too much supersymmetry.
  (ii) too many moduli fields.
- However, this happened because we chose the solutions with vanishing RR flux in the internal directions.

- Thus, Calabi-Yau compactifications where the RR fluxes are set to zero fail on two counts:
  (i) too much supersymmetry.
  (ii) too many moduli fields.
- However, this happened because we chose the solutions with vanishing RR flux in the internal directions.
- If we give up that assumption, things are much better.

- Thus, Calabi-Yau compactifications where the RR fluxes are set to zero fail on two counts:
  (i) too much supersymmetry.
  (ii) too many moduli fields.
- However, this happened because we chose the solutions with vanishing RR flux in the internal directions.
- If we give up that assumption, things are much better.
- We find that RR fluxes generically stabilise moduli.

- Example: on the torus, consider a 2-form field strength F_{12} along the x^1, x^2 directions. The flux quantisation condition is:

$$F_{12} L_1 L_2 = N$$

- Example: on the torus, consider a 2-form field strength F_{12} along the x^1, x^2 directions. The flux quantisation condition is:

$$F_{12} L_1 L_2 = N$$

- Then the energy of this configuration is:

$$E \sim F_{12}^2 L_1 L_2 = \frac{N^2}{L_1 L_2}$$

which is minimised by having the lengths of the torus expand.

- Example: on the torus, consider a 2-form field strength F_{12} along the x^1, x^2 directions. The flux quantisation condition is:

$$F_{12} L_1 L_2 = N$$

- Then the energy of this configuration is:

$$E \sim F_{12}^2 L_1 L_2 = \frac{N^2}{L_1 L_2}$$

  which is minimised by having the lengths of the torus expand.
- By considering more complicated configurations of fluxes, and manifolds more general than the torus, one can generate potentials that freeze the moduli at finite values.

- Example: on the torus, consider a 2-form field strength F_{12} along the x^1, x^2 directions. The flux quantisation condition is:

$$F_{12} L_1 L_2 = N$$

- Then the energy of this configuration is:

$$E \sim F_{12}^2 L_1 L_2 = \frac{N^2}{L_1 L_2}$$

  which is minimised by having the lengths of the torus expand.
- By considering more complicated configurations of fluxes, and manifolds more general than the torus, one can generate potentials that freeze the moduli at finite values.
- This is the basis of flux compactifications.