# *Running Neutrino Mass from 6D*

# *based on hep-ph/0511001 with E. Dudas and C. Grojean*

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- Distinct signatures at colliders, cosmology and other phenomenological studies have been conducted.
- Most studies have been concentrated on the 5 D models.

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- We are interested in studying the generalisation of neutrino mass generation from 5D to 6D with a brane localised mass term.
- Particularly, we are interested in studying some peculiar features associated with 6D field theories, namely, RG running of the brane localised mass term which happens at the classical level.

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- We find : the physical neutrino mass has a logarithmic cutoff divergence related to the zero-size limit of the brane in the transverse space; this is also true for the KK masses.
- This means an RG running for neutrino masses above the compactification scale (say, in the large radii limit $R \sim eV^{-1}$) without having any new

# Methodology

Easy to understand in the scalar case : (Goldberger and Wise 01)

$$S = \frac{1}{2} \int d^4x d^2y \, \left( (\partial^M \Phi)(\partial_M \Phi) - h_2 \, \Phi^2 \delta^2(\mathbf{y}) \right) \qquad (1)$$

leads to

$$\partial_M \partial^M \Phi + h_2 \Phi \, \delta^2(\mathbf{y}) = 0 \, . \qquad (2)$$

*KK Decomposition*

$$\Phi(x, \mathbf{y}) = \sum_{(k_1,k_2) \in \mathcal{I}} \frac{\phi_{(k_1,k_2)}(x)}{\sqrt{2\pi^2 R_1 R_2}} \frac{\cos\left( \frac{k_1}{R_1} y_1 + \frac{k_2}{R_2} y_2 \right)}{\sqrt{2^{\delta_{k_1 0} \, \delta_{k_2 0}}}}, \qquad (3)$$

# Methodology

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{mass} \tag{4}$$

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \sum_{(k_1,k_2),(p_1,p_2)\in\mathcal{I}} \phi_{(k_1,k_2)}\, \mathcal{M}^2_{(k_1,k_2),(p_1,p_2)}\, \phi_{(p_1,p_2)} \tag{5}$$

with the mass matrix given by

$$\mathcal{M}^2_{(k_1,k_2),(p_1,p_2)} = \frac{2\bar{m}^2}{\sqrt{2^{\delta_{k_1 0}}\, 2^{\delta_{k_2 0}}}} + \left(\frac{k_1^2}{R_1^2} + \frac{k_2^2}{R_2^2}\right) \delta_{k_1,p_1} \delta_{k_2,p_2} \ , \tag{6}$$

where

$$\bar{m}^2 \equiv \frac{h_2}{4\pi^2 R_1 R_2} \tag{7}$$

The characteristic equation of the above matrix is given by

$$\frac{1}{\bar{m}^2} = \sum_{k_1,k_2=-\infty}^{\infty} \frac{1}{m^2 - k_1^2/R_1^2 - k_2^2/R_2^2} \ . \tag{8}$$

In equal, large radii limit, the lightest eigenvalue is given as

$$\frac{1}{\bar{m}^2} = \frac{4\pi^2 R_c^2}{h_2} \simeq -\pi R_c^2 \ln(\Lambda^2 R_c^2) + \frac{1}{m^2} \ . \tag{9}$$

Same effect for heavier KK modes also.

# Methodology: Dirac Fermions

$$S \; = \; \int d^4x d^2y \; i\bar{\Psi}\Gamma^M \mathcal{D}_M \Psi \; . \tag{10}$$

Decomposing..

$$\Psi = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} , \quad \mathbb{Z}_2 \lambda_1(\mathbf{y}) = \lambda_1(-\mathbf{y}), \, \mathbb{Z}_2 \lambda_2(\mathbf{y}) = -\lambda_2(-\mathbf{y}) \; . \tag{11}$$

$$S_{mass} = \int d^4x \quad (h \, \bar{\nu}_L \lambda_1(\mathbf{y}=0) \, H + h.c.) \;\; , \tag{12}$$

The mass dimensions of $\Psi, \nu_L, H$ are respectively $5/2, 3/2$ and $1$.

# Methodology:Dirac Fermions

$$\begin{aligned}\mathcal{L} &= -i\lambda_1\,\sigma^\mu\,\partial_\mu\bar{\lambda}_1 - i\lambda_2\,\sigma^\mu\,\partial_\mu\bar{\lambda}_2 + \lambda_1\left(\partial_5 + i\partial_6\right)\lambda_2 \\ &\quad -\bar{\lambda}_2\left(\partial_5 - i\partial_6\right)\bar{\lambda}_1 + g_2(\nu_L\lambda_1 + \bar{\nu}_L\bar{\lambda}_1)\delta^2(\mathbf{y})\ , \end{aligned} \quad (13)$$

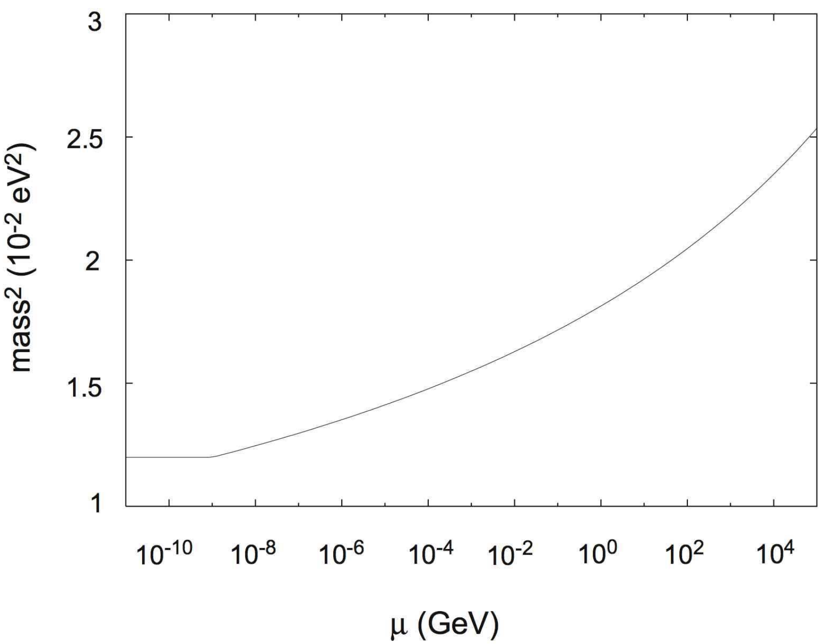
in terms of 6D KG equation..

$$\left(\partial_\mu\partial^\mu - \partial_5^2 - \partial_6^2\right)\ \lambda_1(x,\mathbf{y}) + g_2^2\ \lambda_1(x,\mathbf{y})\ \delta^2(\mathbf{y}) = 0 \quad (14)$$

Same characteristic equation

$$\frac{1}{\bar{m}^2}\ = \sum_{k_1,k_2=-\infty}^{\infty}\frac{1}{m^2 - k_1^2/R_1^2 - k_2^2/R_2^2}\ . \quad (15)$$

# *Phenomenology:Dirac Neutrinos; 1 generation*




Running of the neutrino squared masses with the energy. $\Lambda = 10^5$ GeV, $g_2 = 1$, and $R_1 = R_2 = 1$ eV$^{-1}$.

# *Methodology:Majorana Fermions*

Lepton Number violating term on the brane

$$\int d^4x \; d^2y \; (M_0 \; \lambda_1\lambda_1 + h.c) \; \delta^2(\mathbf{y}) \; . \qquad (16)$$

The characteristic equation is given by :

$$\frac{1}{\bar{m}^2 + mM_0/(4\pi^2 R_1 R_2)} = \sum_{k_1,k_2=-\infty}^{\infty} \frac{1}{m^2 - k_1^2/R_1^2 - k_2^2/R_2^2} \; . \qquad (17)$$

# Methodology: Neutrino Oscillations

$$P_{\nu_L \to \nu_L}(t) = \left| \sum_{(k_1,k_2)\in\mathcal{I}} \frac{2}{N_{(k_1,k_2)}} \; e^{i\frac{m^2_{(k_1,k_2)}\,t}{2p}} \right|^2 \tag{18}$$

$$\begin{aligned} N_i &= 1 + \bar{m}^2 \sum_{k_1,k_2=-\infty}^{\infty} \frac{m_i^2 + k_1^2/R_1^2 + k_2^2/R_2^2}{\left(m_i^2 - k_1^2/R_1^2 - k_2^2/R_2^2\right)^2} \\ &= 2\bar{m}^2 m_i^2 \sum_{k_1,k_2=-\infty}^{\infty} \frac{1}{\left(m_i^2 - k_1^2/R_1^2 - k_2^2/R_2^2\right)^2} \end{aligned} \tag{19}$$

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- Can be generalised to three generations; no running in mixing
- Also generalised to Majorana case, but 6D lorentz invariance should be broken
- Effect could possibly seen in future neutrinoless double beta decay experiments
- Neutrino oscillation formulae are studied.