Masses, Mass Shifts and Higgs

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1. Inertial and Gravitational Masses

In the SM and even SSSM, where masses come from, is just as obscure as were the nature of interactions prior to the notion of gauge invariance.

Given the current unsatisfactory understanding on the origin of masses, let us look at the problem from a different perspective:

Can general relativity give us a clue in delineating between various proposed mechanisms for masses ?

Inertial and gravitational masses are defined through the energy-pressure tensor $T_{\mu\nu}$. $T_{\mu\nu}$ has one time-like (the scalar energy density) and three space-like eigenvalues (related to the pressure).

The trace of the energy-pressure tensor

$$T = T^{\mu}_{\mu} = \epsilon - 3P \tag{1}$$

Inertial matter mass density $\rho_{inertial}$ may be equated to the energy density in the local Lorentz rest frame of the matter

$$\rho_{inertial} = \frac{\epsilon}{c^2} \tag{2}$$

The notion of gravitational mass-density is more subtle and requires the concept of a tidal force which is defined through

$$\Phi_{\lambda\sigma} = R_{\lambda\mu\sigma\nu} u^{\mu} u^{\nu}, \qquad (3)$$

where u^{μ} is the 4-velocity of the material system.

The trace of the tidal force may be computed through the Einstein equation

$$R_{\mu\nu} = \frac{8\pi G}{c^4}[T_{\mu\nu} - (1/2) g_{\mu\nu} T], \qquad (4)$$

giving

$$\Phi = \Phi^{\lambda}_{\lambda} = R_{\mu\nu} u^{\mu} u^{\nu} = \frac{8\pi G}{c^2}[\epsilon - (1/2)T] = \frac{4\pi G}{c^2}(\epsilon + 3P) \tag{5}$$

$$\Phi = \frac{4\pi G}{c^2}(\epsilon + 3P) = (4\pi G)\rho_{gravitational}. \tag{6}$$

Thus, the gravitational mass density is given by

$$\rho_{gravitational} = \frac{\epsilon + 3P}{c^2} = \rho_{inertial} + \frac{3P}{c^2}. \tag{7}$$

• 1. Eq.(6) is the general relativistic version of the Newtonian field equation $\nabla^2\phi = (4\pi G)\ \rho_{gravitational}$.

• 2. Eq.(7) generalizes Tolman's result derived for the special case of a spherically symmetric Schwartzschild metric.

• 3. General relativistic arguments on the stability of a material system require that $3P \leq \epsilon$.

Thus, we have the general relativistic inequality between the inertial and gravitational mass densities

$$\rho_{gravitational} \leq 2\rho_{inertial}. \qquad (8)$$

For any field-theoretic model with conformal symmetry

$$\rho_{gravitational} = 2\rho_{inertial}, \tag{9}$$

reflecting that therein

$$\epsilon = 3P \qquad and\ T = 0. \tag{10}$$

Examples:

(i). Eq.(10) is true for the Maxwell radiation field.

(ii) In the SM (zeroeth level; sans Higgs) with all fermions and gauge bosons massless, Eq.(10) is true.

(iii) glue balls: Suppose one were to make an inertial mass for glueballs made up of massless constituent gluons. But then due to the above general result, one would find that its gravitational mass would be twice its inertial mass. Thus, we can -perhaps not surprisingly- conclude that very little of matter around us could be due to glue balls.

Since all the matter which we see around us satisfies to a fantastic accuracy the equality

$$\rho_{gravitational} = \rho_{inertial}, \tag{11}$$

the growth of equal inertial and gravitational particle masses

$$M_{inertial}c^2 = \int \epsilon(dV), \tag{12}$$

and

$$M_{gravitational}c^2 = \int (\epsilon + 3P)(dV), \tag{13}$$

must be a requirement on any dynamical scheme of conformal symmetry breaking. Let us see how it works for the Higgs model.

2. Higgs Model

We all know that the conformal symmetry with massless fermions and gauge bosons is here broken by the Higgs field which grows masses for all of them. Writing

$$\phi = (1/\sqrt{2})[v + \sigma] \tag{14}$$

The Lagrange density for the Higgs coupling into the QCD degrees of freedom is via the quark fields

$$\mathcal{L}_{ind} = -\widehat{T}(\frac{\sigma}{v}) = -(\sum_j \bar{\Psi} m_j c^2 \Psi)(\frac{\sigma}{v}) \tag{15}$$

and here the equality between the inertial and gravitational masses however posits the Higgs field with a rather remarkable direct interaction with the gravitational field itself.

3. Higgs into Graviton Decays

Through the Einstein equation of motion, we have the general result that

$$T|PhysicalState>=-(\frac{c^4}{8\pi G})R|PhysicalState> \tag{16}$$

In the Higgs model, it is the Higgs field which produces $T \neq 0$ and $R \neq 0$. The Higgs field effective action is

$$S_{eff} = \frac{1}{cv} \int [\sqrt{-g}(d^4x)] \sigma T, \qquad (17)$$

which also reads

$$S_{eff} = -\frac{c^3}{8\pi G v} \int [\sqrt{-g}(d^4x)] \sigma R, \qquad (18)$$

The above two equations ensure that the Higgs couples equivalently to inertial and gravitational masses. But then we can relate it to $\mathcal{L}_g$, the Lagrange density of the gravitational field

$$S_g = -\frac{c^3}{16\pi G}\int[\sqrt{-g}(d^4x)]R = (1/c)\int[\sqrt{-g}(d^4x)]\mathcal{L}_g. \tag{19}$$

Finally

$$S_{eff} = -(\frac{2}{cv})\int[\sqrt{-g}(d^4x)]\sigma\mathcal{L}_g \tag{20}$$

This is a remarkable result because in the Higgs coupling to the gravitons, Newton's constant G has vanished. Using standard techniques, we may compute the Higgs decay into two gravitons

$$\Gamma(H \to gg) = (\frac{\sqrt{2}}{16\pi})[\frac{G_F M_H^2}{\hbar c}](\frac{M_H c^2}{\hbar}). \quad (21)$$

For $M_H < 2M_{top}$, this is larger than $H \to q\bar{q}$ jets.

4. Higgs Induced Mass Shifts

In the SM there is no dimensional mass parameter and we all know that what we call the mass term for a fermion or a boson is simply the first term in the Yukawa coupling between the particle-antiparticle pair and the vacuum expectation value of the Higgs. What this implies is that a further coupling to the Higgs still remains which is the larger the higher the mass. Thus, it is to be expected that if we produce a heavy particle (such as the W, Z or a t, or heavier still in the SS model), the effective mass of the pair produced particle would not quite agree with the mass of the same particle when produced singly. This mass shift would depend upon the mass and upon how long the effect lasts that is upon the life- time of the "disturbing particle".

It is difficult (at least for us) to compute this effect using standard Feynman rules where mass along with the spin of a particle is considered a' priori fixed and the total life time of a particle does not enter the calculation. We have considered the problem semi-classically -similar to Feynman-Wheeler formulation of QED-in which one assumes that the path of the two produced particles is unchanged. In Feynman-Wheeler, it is the photon propagator $D_{\mu\nu}(x-y)$ coupling into the four-velocities of the two charged particles (at space-time points x and y) whereas here it is the scalar Higgs propagator $D_H(x-y)$ coupling into the two massive particles at the space-time points x and y.

The shift in the mass (ΔM) is related to the real part of the effective action

$$\Re e S_{eff} \approx -\frac{\Delta M c^2}{\Gamma} \tag{22}$$

The effect should be the largest when the two produced particles are at the same space-time point. Then we may employ the light-cone singularity of the Higgs propagator which is independent of the Higgs mass. In this approximation (which should be accurate for low mass Higgs), we find that the (energy dependent) mass shift (for a particle-antiparticle pair) is given by

$$\Delta M(s) = -\Gamma(\frac{\sqrt{2}G_F M^4}{2\pi\sqrt{s}}) \times \sqrt{\frac{1}{s-4M^2} ln(\frac{M}{\Gamma})}. \tag{23}$$

In the limit of $M >> M_H$, the mass shift is independent of the Higgs mass and it is largest near threshold.

It is a pity that at LEP while from singly produced Z^o, the Z^o mass was measured with extreme precision, in the reaction $e^+e- \rightarrow Z^oZ^o$ also measured at LEP, the Z^o mass was assumed to be the same. Ironically, in the radiative reaction $e^{+}e- \rightarrow\ Z^o\gamma$, the Z^o mass was determined quite accurately and agreed within errors with the singly produced Z^o mass. Of course, in this case there is no mass shift predicted due to the zero mass of the photon and its infinite life-time.

It would be desirable at the LHC that mass be left as a free parameter so that comparisions can be made between singly and doubly produced heavy masses. At present, the data from Fermilab on single top and pair produced tops lack the required mass measurement precision to investigate the effect.

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