Physics in the light flavoured sector

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- Mass comes from binding, but quark masses are much smaller

Feynman paper

Proton–Neutron Mass Difference

R. P. Feynman and G. Speisman
California Institute of Technology, Pasadena, California
(Received February 23, 1954)

SUPPOSE all deviations from isotopic spin symmetry are due solely to electromagnetic effects. Then such things as the mass difference of charged and neutral π mesons, and the neutron-proton mass difference would have to be just electrodynamic.

We have investigated this point and have found that it is a reasonable possibility. For particles of zero spin, like π mesons, assumed to be elementary particles, the self-energy is quadratically divergent. If the photon propagation function[1] $1/k^2$ is cut off by a convergence factor $C(k^2)=[\Lambda^2/(\Lambda^2-k^2)]^2$, the resulting energy is about $3e^2\Lambda^2/8\pi m$, where Λ is the cut-off energy and m is the π-meson mass, and we assumed $\Lambda\gg m$. This gives the observed mass difference of about 11 electron masses for a cut-off Λ of about 1.0 proton masses.[2]

It is usually assumed that the negative value of the proton-neutron mass difference speaks against an ultimate electromagnetic explanation. The following calculation shows that this is an unwarranted assumption. The particles are not simple and there is uncertainty as to the correct law of coupling to the electromagnetic field. But for low energy, the proton can be represented by the Dirac equation with an additional Pauli term to represent the anomalous moment. Since we do not know to how high an energy this may be a reasonable approximation, we have tried providing the moment coupling term with a cut-off factor of its own. We write for the proton self-energy

$$\Delta M=(e^2/\pi i)\int\left[\gamma_\mu-\frac{\mu}{4M}(\gamma_\mu \boldsymbol{k}-\boldsymbol{k}\gamma_\mu)G(k)\right](\boldsymbol{p}-\boldsymbol{k}-M)^{-1}$$
$$\times\left[\gamma_\mu+\frac{\mu}{4M}(\gamma_\mu \boldsymbol{k}-\boldsymbol{k}\gamma_\mu)G(k)\right]k^{-2}d^4kC(k)$$

in the notation of reference 1. We used $G(k)=-\lambda^2(k^2-\lambda^2)^{-1}$ to cut the moment coupling off at energies about λ, and $C(k)=-\Lambda^2(k^2-\Lambda^2)^{-1}$ to cut off the photon propagation function at energy Λ. The expression for the neutron is the same, except that the γ_μ coupling terms are omitted and the value of μ, the anomalous moment in nuclear magnetons, is -1.91 instead of 1.79 for the proton. M is the nucleon mass.

For the proton the term for $\mu=0$, representing coupling of current with current, is positive, as is also the term in μ^2. But the cross term, linear in μ, is negative and quite large if the moment is not cut off too soon. Thus the proton-neutron mass difference can easily turn out negative. For example, if Λ and λ are both taken at $1.4M$, the experimental value of -2.5 electron masses results for this difference (in this case for the neutron ΔM is roughly 1.5, for the proton -1.0 electron masses). No small difference of large numbers is involved. If the cut-off λ is reduced below about $0.75M$, a negative mass difference cannot be obtained. For $\lambda=1.0M$, $\Lambda=4M$ gives the experimental result.

The high cutoff for the anomalous moment implies that the charge responsible for the moment must be spread over only a small distance (of order $\hbar/Mc$). This is also suggested by the relatively small changes that the nucleon moments undergo when nucleons form nuclei.

The cutoff for the propagation function may be interpreted in two ways. Firstly, electrodynamics may fail at high energies, the failure being represented in a crude way by the cutoff. If this is so we could guess from our results that the failure occurs at energies in the neighborhood of the nucleon mass. Another possibility is that the electrodynamics is correct, but the cutoff represents, roughly, the error committed in assuming that the particles are elementary. For example, in the case of the π meson, we have assumed the π meson in virtual states acts as a simple particle. But for energies as high as M, strongly coupled virtual nucleon pairs may be formed. They, rather than failure of electrodynamics, may provide the convergence at energies of order M. In a like manner, the complex of virtual mesons presumed to be associated with nucleons may have the effect that at sufficiently high energy the electromagnetic coupling of neutron and proton may be nearly the same, so that the integral representing the difference of their masses may converge without modification of electrodynamics. In this way, the presumed convergence of the mass differences might tell us something about the character of coupling with the electromagnetic field at high energy.

We conclude that all of the deviations from isotopic spin symmetry could be due solely to coupling with the electromagnetic field.

[1] R. P. Feynman, Phys. Rev. **76**, 769 (1949). We use the notation in this reference.
[2] This result was given by one of us (RPF) at the International Conference in Theoretical Physics, Paris, 1950 (unpublished).

Polarization of Elastically Scattered Nucleons from Nuclei*

Warren Heckrotte and Joseph V. Lepore
Radiation Laboratory, University of California, Berkeley, California
(Received February 23, 1954)

NUCLEONS of low or moderate energy which are elastically scattered from nuclei should be partially polarized[1] by the strong spin-orbit potential underlying the predictions of the shell model of the nucleus. This spin-orbit potential is a consequence of the collective action of many nucleons on the particular nucleon. Thus for incident nucleons whose wavelength is greater than the nuclear spacing ($E\gtrsim 50$ Mev), it would be expected that the spin-orbit potential of the shell model would make itself felt. For progressively higher energies the incident nucleon begins to see only one nucleon at a time and while a spin dependence of the elastic scattering can still be expected, it would be more a reflection of the individual nucleon-nucleon interactions than of the spin-orbit potential of the shell model. It will be supposed that even at these higher energies the spin dependence has the form of the usual spin-orbit potential. In either case this spin dependence of the elastic scattering can be investigated phenomenologically by treating the interior of the nucleus in terms of a spin-dependent complex index of refraction[2]—an obvious generalization of the optical model of the nucleus.[3]

For low or moderate energies there is no suitable approximate method for treating the elastic scattering—a phase-shift analysis is necessary. Also, at high energies any polarization calculations using conventional approximation methods[4] are made uncertain by the direct dependence of the polarization on the phase of the scattered wave. A phase-shift analysis for various energies is therefore being undertaken on the Univac at the University of California Radiation Laboratory at Livermore in collaboration with S. Fernbach.

An estimate for small angles of scattering, though rough at best, may be readily obtained by making several simplifying assumptions. The magnitude of the polarization is given by

$$P=\left(\frac{AB^*+A^*B}{d\sigma/d\Omega}\right)\sin\theta. \qquad (1)$$

Here A and B represent the scattering amplitudes corresponding to the spin-independent and spin-dependent parts of the interaction, respectively. The known experimental value for the differential cross section $d\sigma/d\Omega$ may be used. The amplitude, B, for spin-dependent scattering may be estimated by using the Born approximation. Then only the imaginary part of A contributes to P. For small angles this is approximately proportional to the total cross section.

For 300-Mev neutrons incident on carbon, for example, a square-well spin-orbit interaction ($R=1.4A^{\frac{1}{3}}\times10^{-13}$ cm) of 2-Mev depth gives a polarization of 40 percent at five degrees. Though this is probably an overestimate, it suggests that the existence of a small

- $$\boxed{\mathcal{L}_{QCD} = \sum_{n=1}^{N_f} \bar{\psi}_n [i\gamma^\mu (\partial_\mu - ig_s \frac{\lambda^a}{2} G^a_\mu) - m_n]\psi_n - \frac{1}{4}\sum_{a=1}^{8} G^a_{\mu\nu} G^{\mu\nu,a}}$$

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- β calculated to four loops in $\overline{MS}$ scheme: Larin et al (1997), Czakon (2005)

$$\beta_0 = 9/4, \quad \beta_1 = 4, \quad \beta_2 = 10.0599, \quad \beta_3 = 47.228 \qquad (N_f = 3)$$

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- One-loop solution (important for our discussion):

$$a_s(\mu^2) = \frac{a_s(\mu_0^2)}{1 + \beta_0 a_s(\mu_0^2)\ln(\mu^2/\mu_0^2)} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda_{QCD})}, \quad \Lambda_{QCD} \approx 200\,\mathrm{MeV}$$

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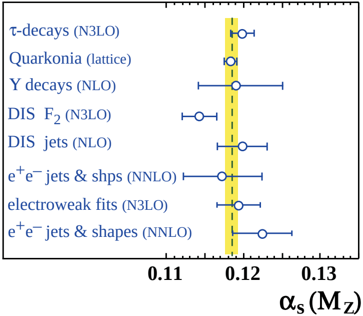

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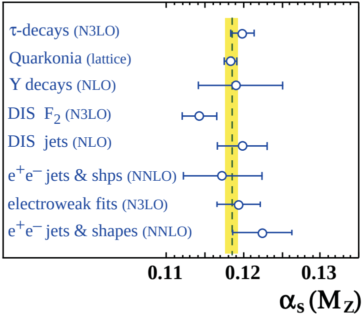

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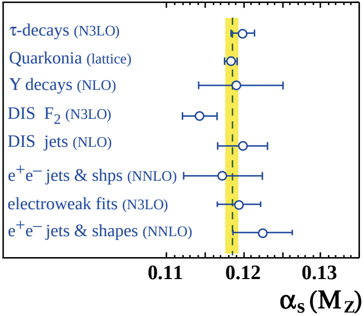

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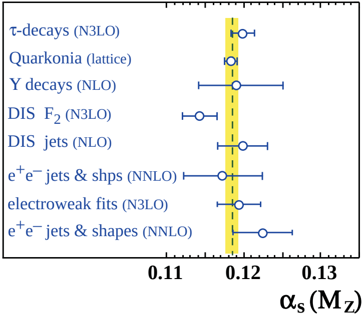

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- Other sources AC Josephson effect and quantum Hall effect

Survey of the α_S determinations

- Measurements from different energies spectacularly confirm RG running of coupling constant as predicted by QCD

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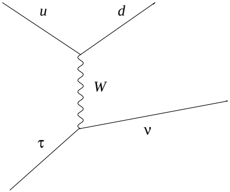

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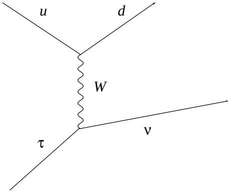

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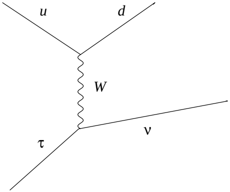

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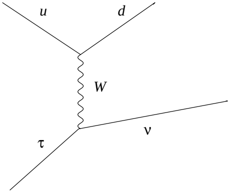

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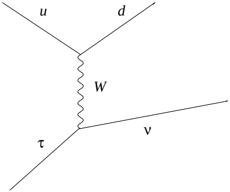

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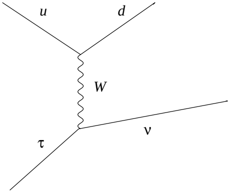

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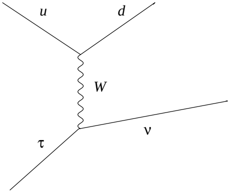

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  $\Rightarrow$ determination of α_s at a low scale $\quad (M_\tau = 1.78 \, GeV)$

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- Stable predictions with reduced errors

- Choice of the renormalization scale

Borel and Renormalization Group Summed Perturbation Theory

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Light quark masses

- The μ dependence of the $\overline{\mathrm{MS}}$ "running mass" $\overline{m}(\mu)$ is given by the renormalization group equation

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- To first order we have the mass relations (for pion mass this has been confirmed spectacularly on the lattice)

$$\begin{aligned}
m_{\pi^0}^2 &= B(m_u + m_d) \\
m_{\pi^+}^2 &= B(m_u + m_d) + \Delta_{EM} \\
m_{K^0}^2 &= B(m_s + m_d) \\
m_{K^+}^2 &= B(m_s + m_u) + \Delta_{EM} \\
m_\eta^2 &= \frac{1}{3} B(4m_s + m_u + m_d)
\end{aligned}$$

Quark mass ratios

- $$\frac{m_u}{m_d} = \frac{2m_{\pi^0}^2 - m_{\pi^+}^2 + m_{K^+}^2 - m_{K^0}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2} = 0.56$$
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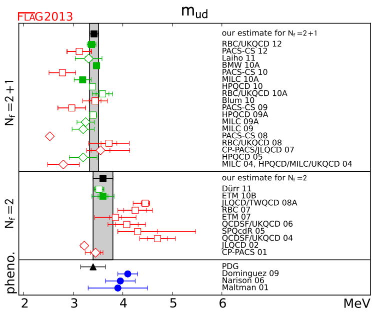
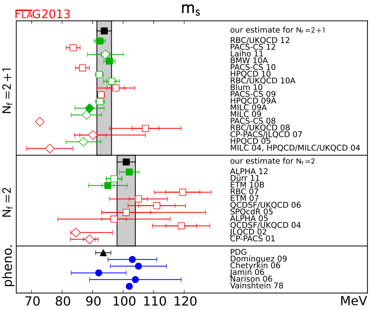


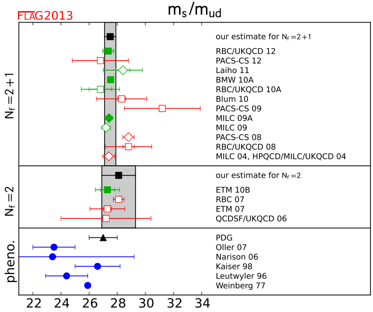
Average mass of the u and d quarks

s quark mass



Mass of the s quark in $\overline{MS}$ scheme at $\mu = 2$ GeV

Quark mass ratio



Ratio of the s quark mass to the average of the u and d quark masses

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- Direct lifetime measurement done by James Cronin and collaborators

Primakoff paper

Walter and Barratt[1] examined and identified the absorption spectra of Li_2, Na_2, K_2, Rb_2, Cs_2, LiK, LiRb, LiCs, NaK, NaRb, NaCs, KRb, RbCs, and KCs.

The identification of a NaLi molecule is complicated by the existence of Na_2 and Li_2 band systems in the regions of the visible, near infrared and ultraviolet. Since the probability of molecular formation is a function of the product of the concentration of the atoms involved, it seemed possible that one component of a sodium-lithium mixture might be held at a low vapor pressure and the other at a high vapor pressure to increase the probability of observing the NaLi molecule.

In our experiment the lithium metal was placed in an absorption cell constructed of nickel and having water-cooled quartz windows. A nickel side tube was connected to the absorption cell to contain the sodium. Heating units were arranged around the absorption cell and side tube to control the temperature of the sodium and lithium metals independently.

The lithium metal was maintained at 850°C. A series of absorption spectrograms was then taken with the sodium at temperatures of 435, 460, 485, and 510°C, respectively. A similar procedure was used for maintaining constant high sodium with increasing lithium vapor pressures.

The results of this experiment confirm the previous work of Walter and Barratt. No bands attributable to a NaLi molecule were observed in the region 3000 to 8000A. No explanation is available, particularly as it is the only member not observed of the complete set of binary molecular systems obtainable with the alkali metals.

* Contribution No. 10, Department of Physics, Kansas State College, Manhattan, Kansas.
† Now at Airport Station, Weather Bureau, Memphis, Tennessee.
‡ Now at South Dakota State College, Brookings, South Dakota.
[1] J. M. Walter and S. Barratt, Proc. Roy. Soc. (London) **A119**, 257 (1928).

Photo-Production of Neutral Mesons in Nuclear Electric Fields and the Mean Life of the Neutral Meson*

H. Primakoff†
Laboratory for Nuclear Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts

January 2, 1951

It has now been well established experimentally that neutral π-mesons (π^0) decay into two photons.[1] Theoretically, this two-photon type of decay implies zero π^0 spin;[2] in addition, the decay has been interpreted as proceeding through the mechanism of the creation and subsequent radiative recombination of a virtual proton anti-proton pair.[3] Whatever the actual mechanism of the (two-photon) decay, its mere existence implies an effective interaction between the π^0 wave field, φ, and the electromagnetic wave field, **E**, **H**, representable in the form:

$$\text{Interaction Energy Density} = \eta(\hbar/\mu c)(\hbar c)^{-\frac{1}{2}}\varphi\mathbf{E}\cdot\mathbf{H}. \quad (1)$$

Here φ has been assumed pseudoscalar, the factors $\hbar/\mu c$ and $(\hbar c)^{-\frac{1}{2}}$ are introduced for dimensional reasons ($\mu \equiv$ rest mass of π^0), and η is a dimensionless constant determined by the decay mechanism.[4]

One can obtain η immediately (by a first-order perturbation calculation) in terms of the mean life, τ, of a neutral π-meson at rest, *viz.*:[5]

$$\tau^{-1} = \pi^2\eta^2\mu c^2/2\hbar. \quad (2)$$

The effective interaction of Eq. (1) can now be used for a calculation of the probability of the inverse process: π^0 production in photon-photon collisions, or, for the calculation of the probability of the more interesting process: π^0 production in the collision of a photon with an external, approximately static electric field; e.g., the Coulomb field of a (slowly recoiling) nucleus. The total cross section σ for this last process is, from a first-order perturbation treatment of Eq. (1), proportional to η^2; i.e., to τ^{-1}; one obtains[6]

$$\sigma \approx 32\pi^2\frac{\hbar/\mu c}{c\tau}Z\left(\frac{e^2}{\hbar c}\right)\left(\frac{\hbar}{\mu c}\right)^2\frac{4}{3}\left(\frac{\hbar\kappa}{\mu c}\right)^3, \quad \text{for} \quad \hbar\kappa \ll \hbar k \approx \mu c \quad (3)$$

$$\sigma \approx 32\pi^2\frac{\hbar/\mu c}{c\tau}Z^2\left(\frac{e^2}{\hbar c}\right)\left(\frac{\hbar}{\mu c}\right)^2, \quad \text{for} \quad R(k-\kappa) \approx \frac{(2Z)^{\frac{1}{3}}}{2}\frac{\mu c}{\hbar k} \ll 1. \quad (4)$$

In Eqs. (3) and (4), $\hbar k$, $\hbar\kappa = \hbar k[1-(\mu c/\hbar k)^2]^{\frac{1}{2}}$ are, respectively, the momenta of the incident photon and produced neutral π-meson; the angular distribution of the mesons is strongly collimated about the direction of the incident photon if $\hbar k \gg \mu c$. In deducing Eq. (3), it has been supposed that the nuclear protons remain approximately at rest during time intervals of the order of several periods of the incident electromagnetic wave [since $v_{\text{proton}} \approx \frac{1}{5}c$ and $(ck)^{-1} \lesssim \hbar/\mu c^2$], and that the probability of finding any pair of protons a distance r apart is proportional to $\exp(-r/R)$, where $R \approx \hbar(2Z)^{\frac{1}{3}}/\mu c$ is the nuclear radius. It is seen from Eqs. (3) and (4) that the electric fields of the Z protons contribute "coherently" to the π^0 production, once the photon energy exceeds $\frac{1}{2}(2Z)^{\frac{1}{3}}\mu c^2$.

Thus, if τ is less than, say, 10^{-17} sec, Eq. (4) indicates that a Z^2 term should be observable in the total cross section for production of neutral π-mesons in photon-nucleus collisions. Since no such term has so far been experimentally detected,[7] one can set a very rough lower limit on τ: $\tau > 5\times10^{-16}$ sec. An approximate upper limit of 5×10^{-14} sec seems to be indicated by cosmic-ray data.[8]

* Assisted by the joint program of the ONR and AEC.
† On leave from Washington University, St. Louis, Missouri.
[1] Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950); Panofsky, Aamodt, Hadley, and Phillips, Phys. Rev. **80**, 94 (1950).
[2] C. N. Yang, Phys. Rev. **77**, 243 (1950); D. C. Peaslee, Helv. Phys. Acta **23**, 845 (1950); we exclude the possibility of the π^0 spin being >1.
[3] J. Steinberger, Phys. Rev. **76**, 1180 (1949), and other references quoted there.
[4] Marshak, Tamor, and Wightman, Phys. Rev. **80**, 765, 766 (1950); K. Brueckner, Phys. Rev. **79**, 641, 187 (1950).
[5] The mechanism of π^0 decay via interaction with virtual proton anti-proton pairs gives, if for example γ_5 coupling is used between the meson and nucleon wave fields, $\tau^{-1} = (\mu c^2/16\pi^2\hbar)(\mu e^2/M\hbar c)^2(g^2/\hbar c)$ (reference 3), so that in this case, $\eta = (\mu/8^{\frac{1}{2}}\pi^2 M)(g^2/\hbar c)^{\frac{1}{2}}(e^2/\hbar c)$.
[6] Another possible process predicted from Eq. (1) involves the one-photon decay of a π^0 in an external (nuclear) electric field. If τ' is the mean life of this decay, one obtains (with N as the number of nuclei per unit volume, and using Eq. (4))

$$\tau/\tau' = \tau c N\sigma 2k^2[\kappa^4+(\mu c\kappa/\hbar)^2]^{-\frac{1}{2}} \approx 64\pi^2 Z^2(e^2/\hbar c)(\hbar/\mu c)^3 N \ll 1.$$

[7] Observations of Steinberger, Panofsky, and Steller quoted by R. F. Mozley, Phys. Rev. **80**, 493 (1950).
[8] Carlson, Hooper, and King, Phil. Mag. **41**, 701 (1950).

Neutral pion width with chiral corrections

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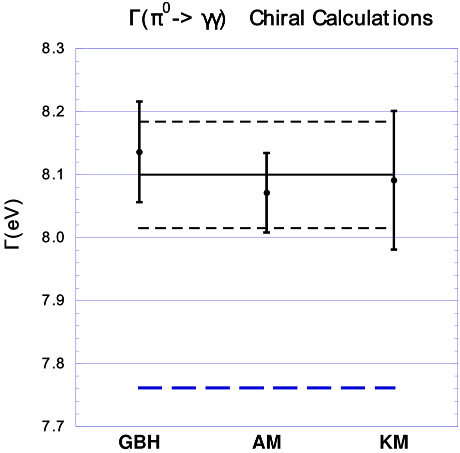
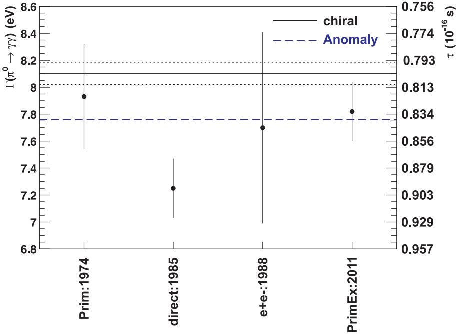


Predictions of the width with chiral corrections due to $\pi^0 - \eta$ mixing. Results to Goity, Bernstein and Holstein; Ananthanarayan and Moussallam; Kampf and Moussallam

Summary of the width



Summary of the neutral pion width measurements

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- "Blind spots" where new physics scenarios evade experimental bounds are where the iso-spin violating couplings appear to be important
- For the MSSM studied in detail in Andreas Crivellin, Martin Hoferichter, Massimiliano Procura, Lewis C. Tunstall, JHEP 1507 (2015) 129

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- For reviews on V_{us} from τ-decay, see A. Lusiani, arXiv:1411.4526; from lattice, see V. Lubicz, arXiv:1309.2530; fom kaon decays, see C. Bloise, PoS GQL2010 (2011) 016

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- $f_+(0) = 1$ in the limit of $m_d = m_u = m_s$ ($SU(3)$ limit). Corrections to the relation due to $SU(3)$ breaking $\sim 20\%$. Even smaller due to Ademollo-Gatto theorem (symmetry breaking effects are 2nd order in the breaking term)

More kaon decays

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- Crucial for knowledge of Cabibbo-Kobayashi-Maskawa matrix as the combination $f_+(0)V_{us}$ appears in the expression for rates and Dalitz plot densities.
- Recent determinations from the lattice, e.g., RBC+UKQCD collaboration [P. A. Boyle et al., Physical Review Letters 100 (2008) 141601] gives $f_+(0) = 0.964(5)$. They use 2+1 flavour of dynamical wall quarks.
- Crucial work by H. Leutwyler and M. Roos, Zeitschrift für Physik, C25 (1984) 91.
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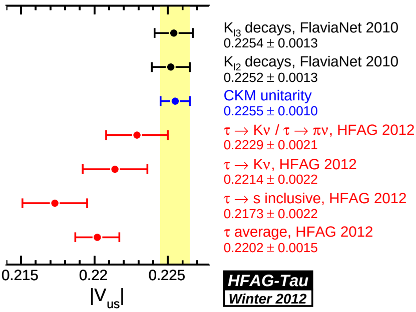
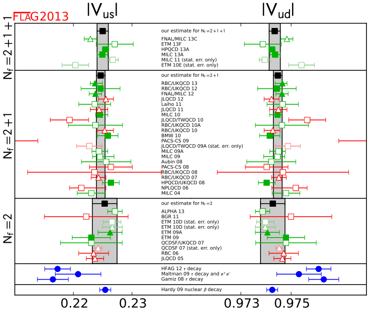
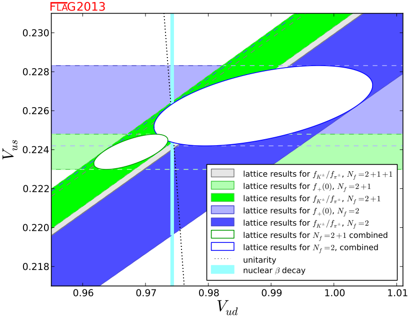


Summary of CKM matrix elements



Summary of CKM matrix elements from FLAG report



Allowed regions in the CKM matrix element plane

CP violation in the light quark sector

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  - improvements of knowledge of symmetry breaking including, e.g., the pion-nucleon sigma term with implications to dark matter searches

Summary

- We have described the state of the art of the knowledge of the
  - The strong coupling constant α_S
  - The three light quark masses
  - The magnitudes of the elements of the CKM matrix $|V_{us}|$ and $|V_{ud}|$
  - The evidence of the complex nature of the CKM matrix elements from ϵ'/ϵ measurements
- The methods include those coming from standard methods of field theory, such as
  - renormalization group methods
  - dispersion relations and sum rules
  - lattice theory
  - large variety of experiments at high precision
  - Tests of anomaly sector such as $\pi^0 \to 2\gamma$ width
- Challenges remain for the standard model such as in the setting of
  - the $g-2$ of the muon, both theory and experiment
  - tests of CP conservation in, e.g, the neutron dipole moment
  - improvements of knowledge of symmetry breaking including, e.g., the pion-nucleon sigma term with implications to dark matter searches
- This is indeed a precision science, unprecedented in history of particle physics.