Hadron Spectrum from Lattice Calculations

David Richards (Jefferson Laboratory)

Hadron Spectrum Collaboration

Strong 2010, Mumbai, 10-12 February, 2010

- Introduction
- Baryon excitation spectrum in quenched and full QCD
- Lattices for spectrum calculations
- Identifying the continuum quantum numbers: meson spectrum
- Electromagnetic properties of excited states
- Conclusions

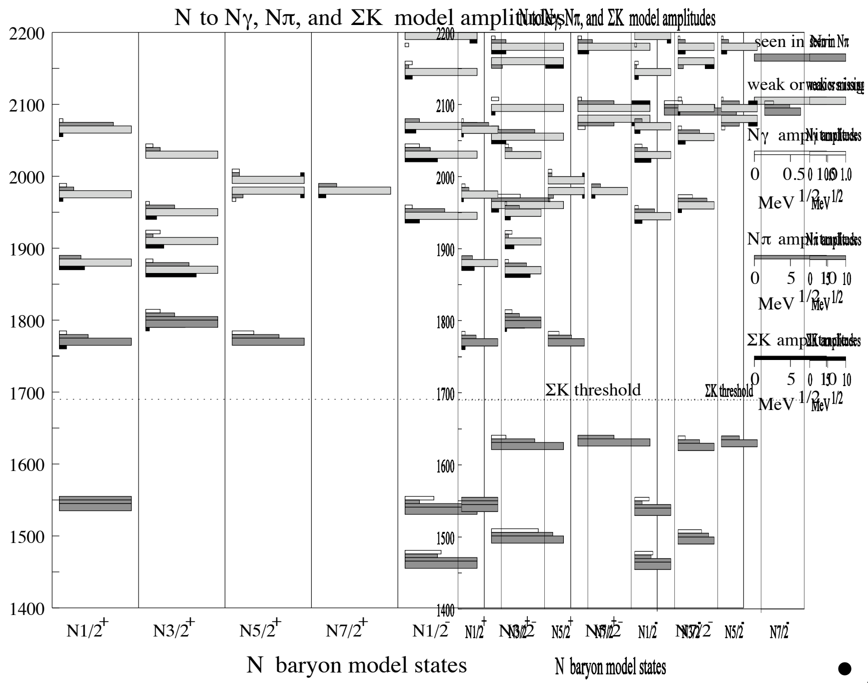
TexPoint fonts used in EMF.

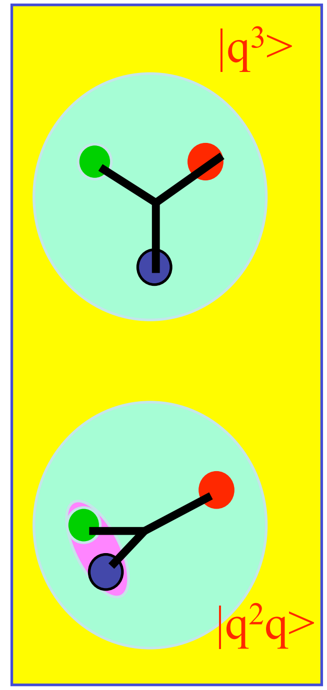
Resonance Spectrum of QCD

- *Why is it important?*
  - *What are the key degrees of freedom describing the bound states?*
  - *What is the role of the gluon in the spectrum – search for exotics?*
  - *What is the origin of confinement, describing 99% of observed matter?*
  - *If QCD is correct and we understand it, expt. data must confront ab initio calculations*
- NSAC Performance Measures
  - *"Complete the combined analysis of available data on single π, η, and K photo-production of nucleon resonances…"* (HP3:2009)
  - *"Measure the electromagnetic excitations of low-lying baryon states (<2 GeV) and their transition form factors…"* (HP12)
  - *"First results on the search for exotic mesons using photon beams will be completed"* (HP15)

Spectroscopy - I

- **Nucleon Spectroscopy: Quark model masses and amplitudes – states classified by isospin, parity and spin.**



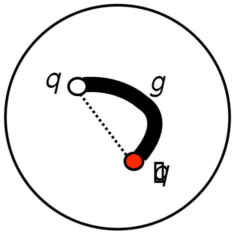


Capstick and Roberts, PRD58 (1998) 074011

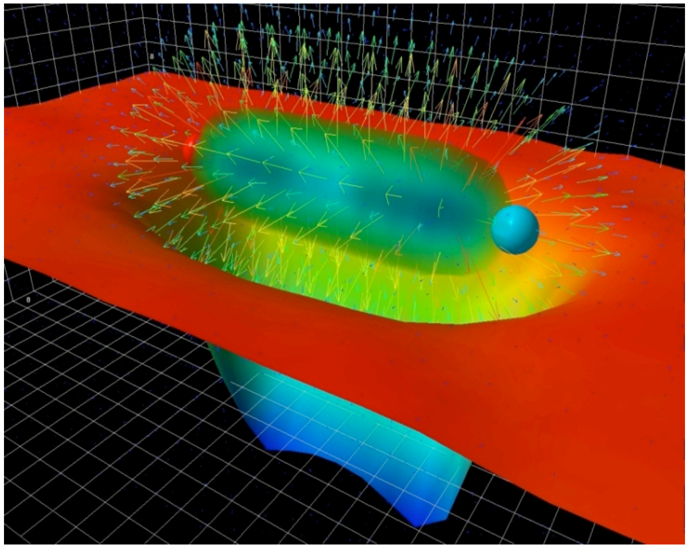
- Are states Missing, because our pictures are not expressed in correct degrees of freedom?
- Do they just not couple to probes?

Exotics – I

- Exotic Mesons are those whose values of J^{PC} are in accessible to quark model
  - Multi-quark states: $q\bar{q}q\bar{q}$
  - Hybrids with *excitations of the flux-tube*

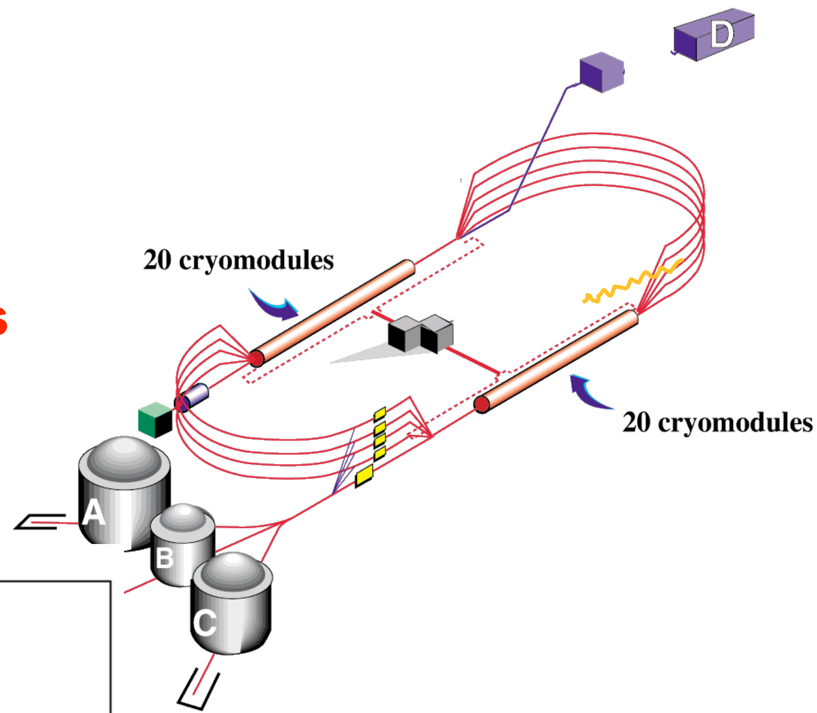


- Study of hybrids: revealing gluonic and flux-tube degrees of freedom of QCD.

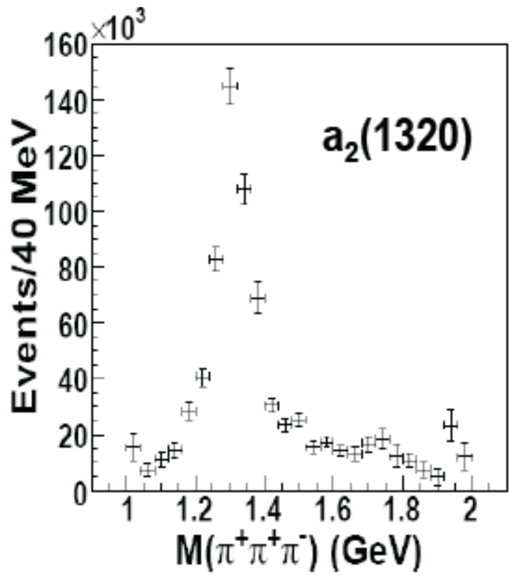


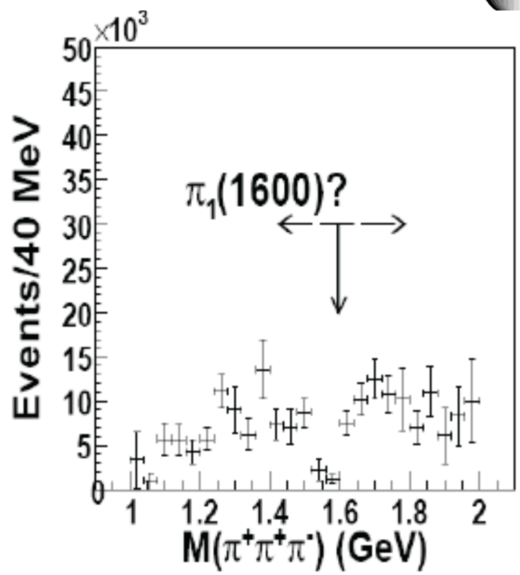
Lattice QCD: Hybrids and GlueX - I

- **GlueX aims to photoproduce hybrid mesons in Hall D at JLab.**
- **Lattice QCD has a crucial role in both predicting the spectrum and in computing the production rates**



π1(1600) in pion production at BNL





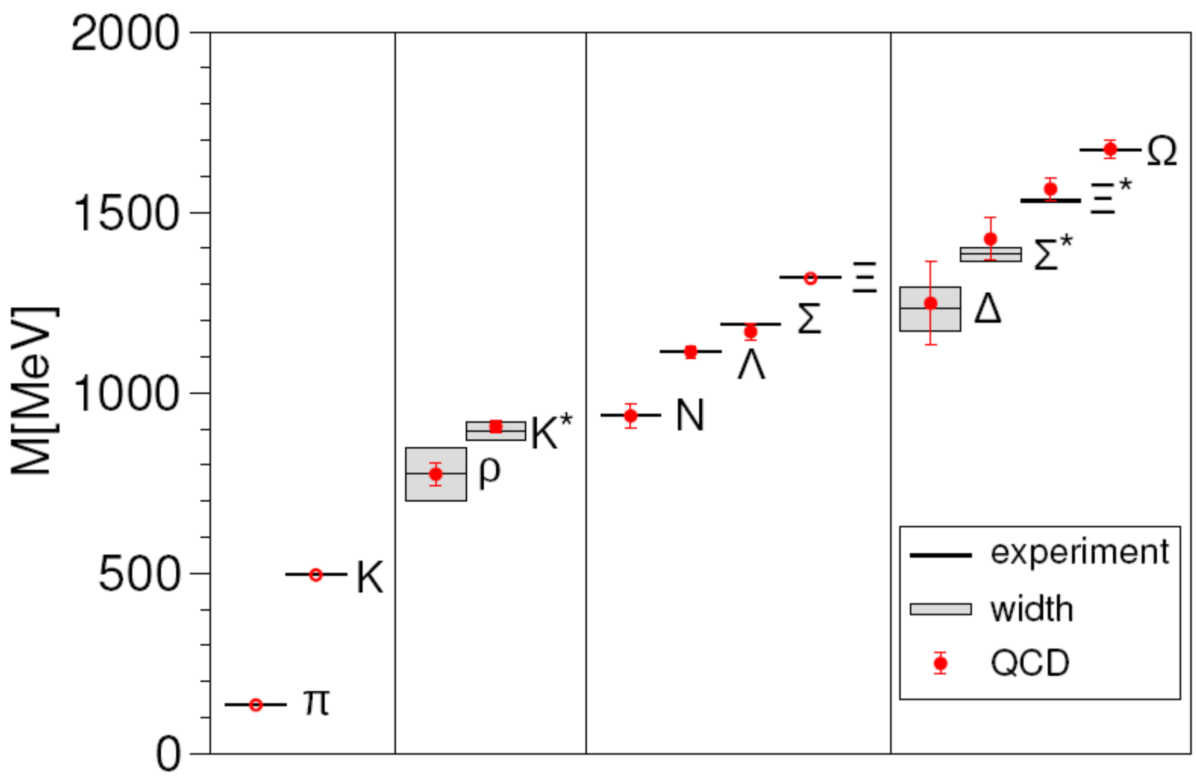
No evidence in photoproduction at CLAS

Important goal for LQCD

Low-lying Hadron Spectrum

$$C(t) = \sum_{\vec{x}} \langle 0 \mid N(\vec{x},t)\bar{N}(0) \mid 0 \rangle \;=\; \sum_{n,\vec{x}} \langle 0 \mid e^{ip\cdot x} N(0) e^{-ip\cdot x} \mid n \rangle \langle n \mid \bar{N}(0) \mid 0 \rangle$$

$$= \mid \langle n \mid N(0) \mid 0 \rangle \mid^2 e^{-E_n t} = \sum_n A_n e^{-E_n t}$$



Durr et al., BMW Collaboration

Science 2008

Control over:

- ***Quark-mass dependence***
- ***Continuum extrapolation***
- ***finite-volume effects (pions, resonances)***

Variational Method

- **Extracting excited-state energies described in C. Michael, NPB 259, 58 (1985) and Luscher and Wolff, NPB 339, 222 (1990)**
- **Can be viewed as exploiting the *variational method***
- **Given N x N correlator matrix** $C_{\alpha\beta} = \langle 0 \mid \mathcal{O}_\alpha(t)\mathcal{O}_\beta(0) \mid 0 \rangle$ **, one defines the N *principal correlators* $\lambda_i(t,t_0)$ as the eigenvalues of**

$$C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$$

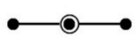
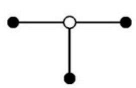
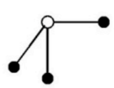
- **Principal effective masses defined from correlators plateau to lowest-lying energies**

$$\lambda_i(t,t_0) \rightarrow e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

Variational Method - II

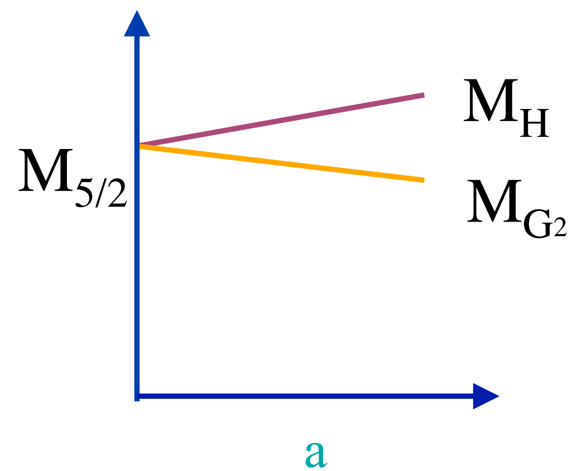
- **Spectrum on lattice looks different – states at rest classified by isospin, parity and representation under cubic group**

Illustration	Name	Explicit form ($\lvert i\rvert \neq \lvert j\rvert \neq \lvert k\rvert$)
	single-site	$\phi^F_{ABC}\;\varepsilon_{abc}\;\tilde{\psi}_{Aa\alpha}\;\tilde{\psi}_{Bb\beta}\;\tilde{\psi}_{Cc\gamma}$
	singly-displaced	$\phi^F_{ABC}\;\varepsilon_{abc}\;\tilde{\psi}_{Aa\alpha}\;\tilde{\psi}_{Bb\beta}\;\left(\tilde{D}^{(p)}_{j}\tilde{\psi}\right)_{Cc\gamma}$
	doubly-displaced-I	$\phi^F_{ABC}\;\varepsilon_{abc}\;\tilde{\psi}_{Aa\alpha}\;\left(\tilde{D}^{(p)}_{-j}\tilde{\psi}\right)_{Bb\beta}\;\left(\tilde{D}^{(p)}_{j}\tilde{\psi}\right)_{Cc\gamma}$
	doubly-displaced-L	$\phi^F_{ABC}\;\varepsilon_{abc}\;\tilde{\psi}_{Aa\alpha}\;\left(\tilde{D}^{(p)}_{j}\tilde{\psi}\right)_{Bb\beta}\;\left(\tilde{D}^{(p)}_{k}\tilde{\psi}\right)_{Cc\gamma}$
	triply-displaced-T	$\phi^F_{ABC}\;\varepsilon_{abc}\;\left(\tilde{D}^{(p)}_{-j}\tilde{\psi}\right)_{Aa\alpha}\left(\tilde{D}^{(p)}_{j}\tilde{\psi}\right)_{Bb\beta}\left(\tilde{D}^{(p)}_{k}\tilde{\psi}\right)_{Cc\gamma}$
	triply-displaced-O	$\phi^F_{ABC}\;\varepsilon_{abc}\;\left(\tilde{D}^{(p)}_{i}\tilde{\psi}\right)_{Aa\alpha}\left(\tilde{D}^{(p)}_{j}\tilde{\psi}\right)_{Bb\beta}\left(\tilde{D}^{(p)}_{k}\tilde{\psi}\right)_{Cc\gamma}$

Extension to $qqq\,\bar{q}q$

Lattice PWA

J	$n^J_{G_1}$	$n^J_{G_2}$	n^J_H
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3



Low-lying Baryon Spectrum

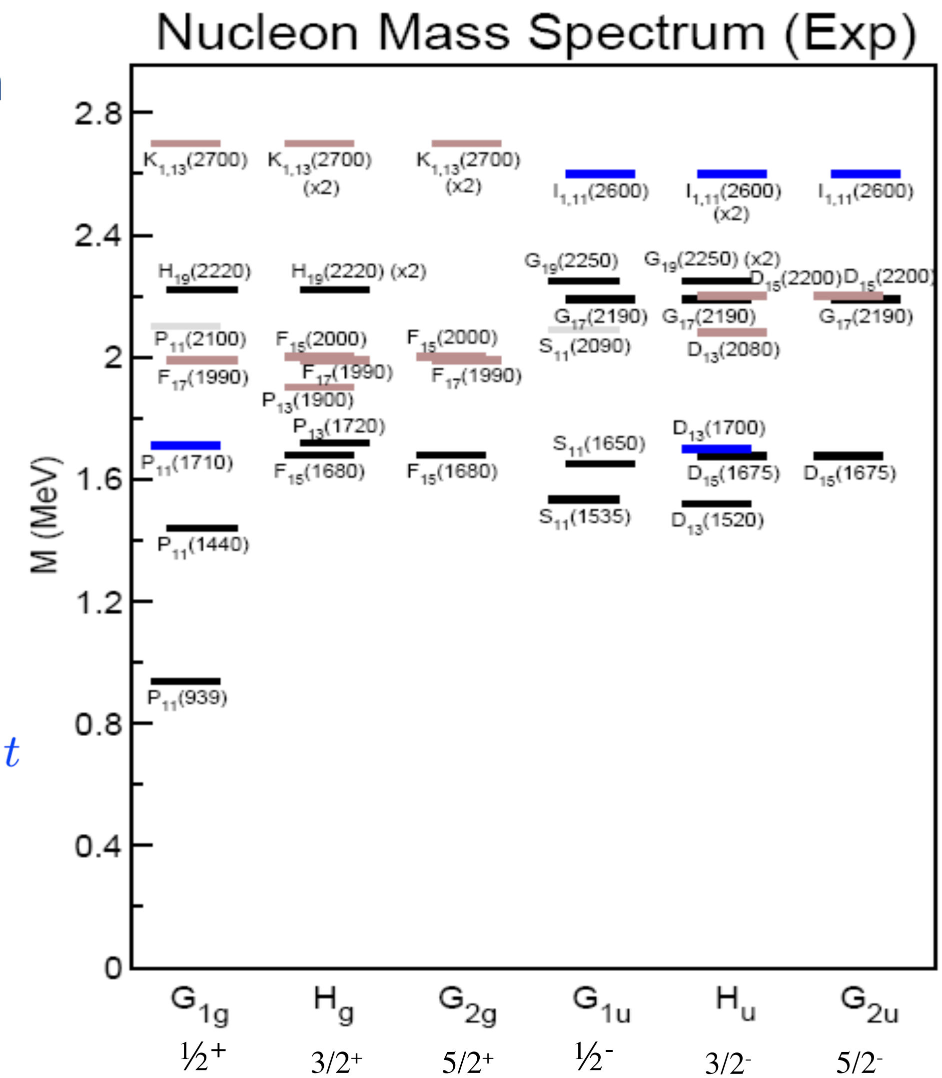
A lattice theorist's view

The nucleon spectrum as seen on the lattice!

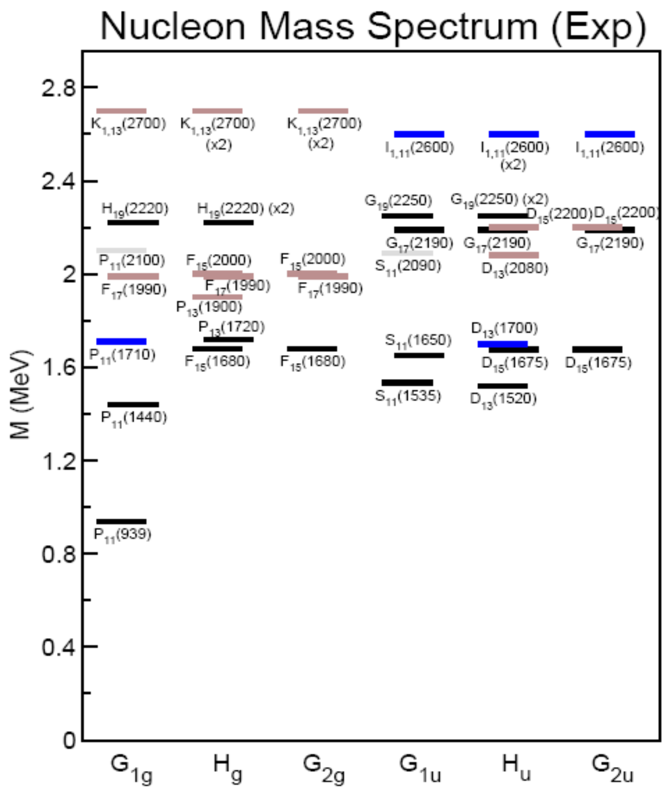
- **Challenges/opportunities:**
  - **Compute excited energies**
  - **Compute decays**

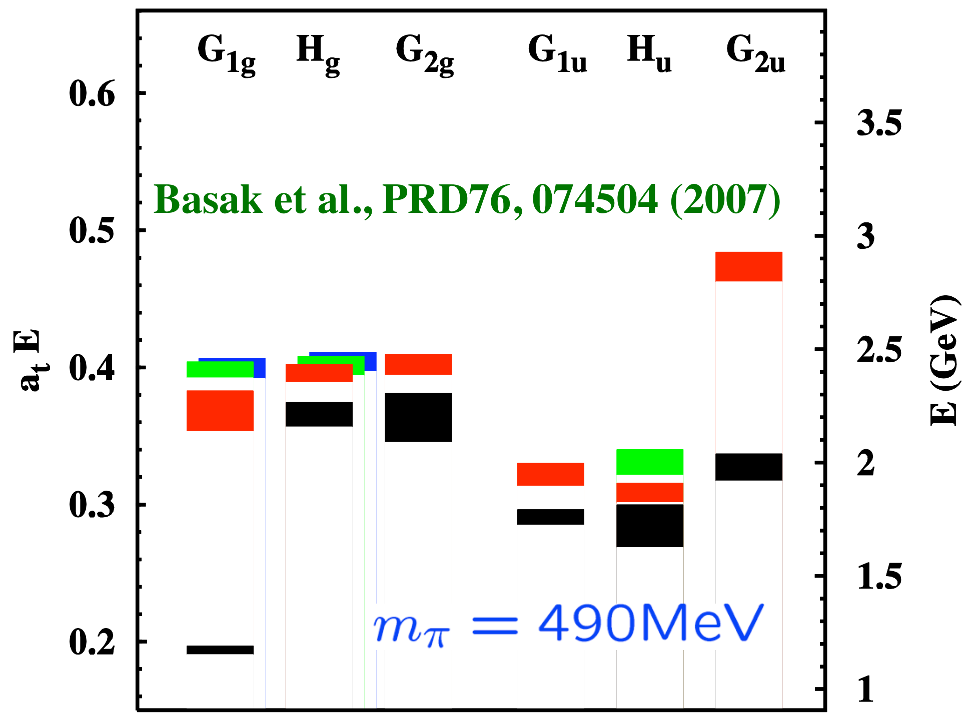
$$\text{signal-to-noise} \simeq e^{-(m_H - \frac{3}{2} m_\pi)t}$$

Anisotropic: $a_t < a_s$: $exp\ (- m\ a_t\ t)$



Resonance Spectrum - Quenched

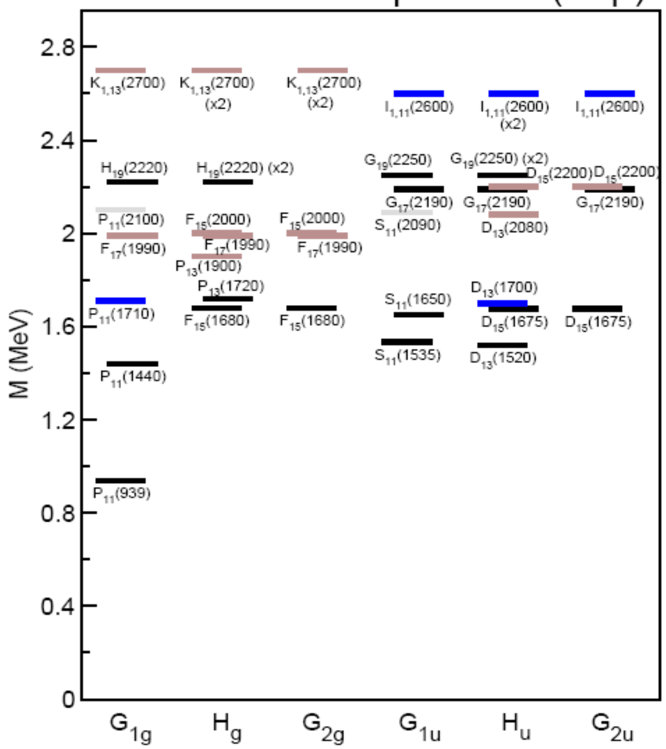


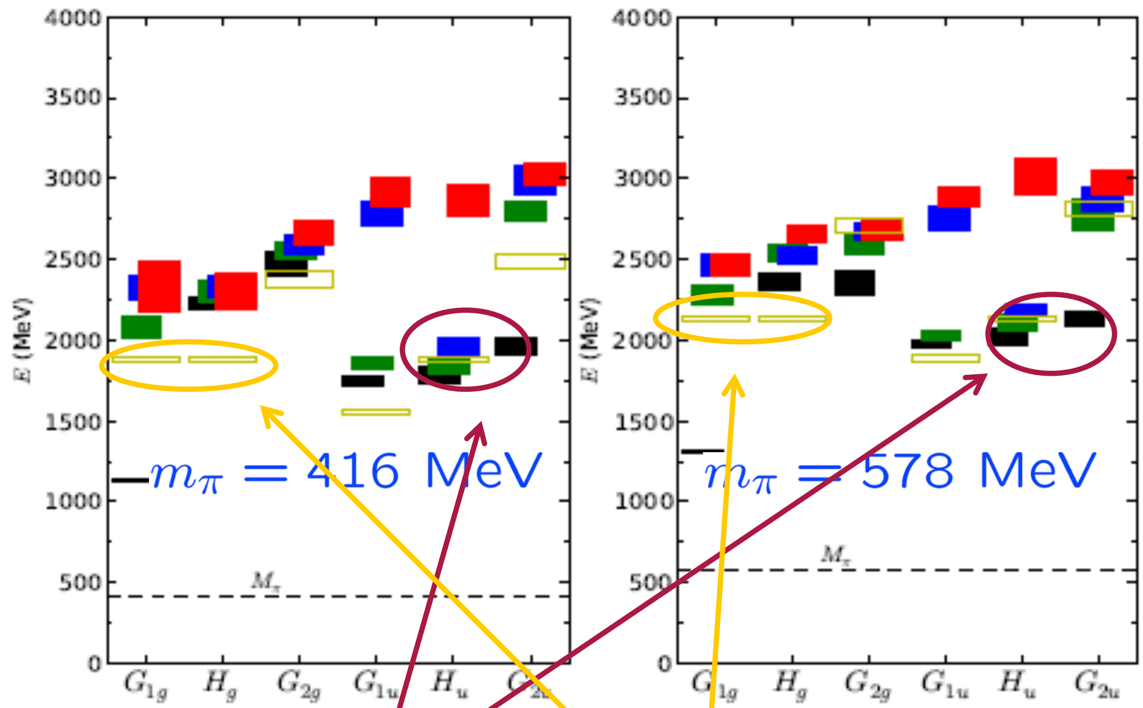


- Demonstration of our ability to extract nucleon resonance spectrum
- Hints of patterns seen in experimental spectrum
- Methodology central to remainder of project
- Do not recover ordering of P_{11} and S_{11}

Resonance Spectrum – Nf=2

N_f=2: Hadron Spectrum Collab., Phys.Rev.D79:034505 (2009)



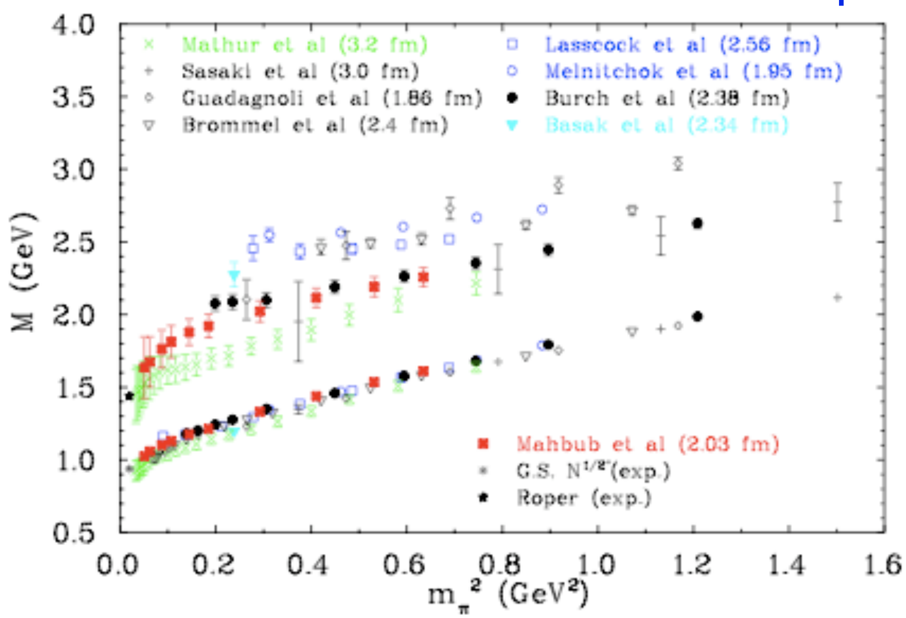


- ***First identification of spin-5/2 state in LQCD***

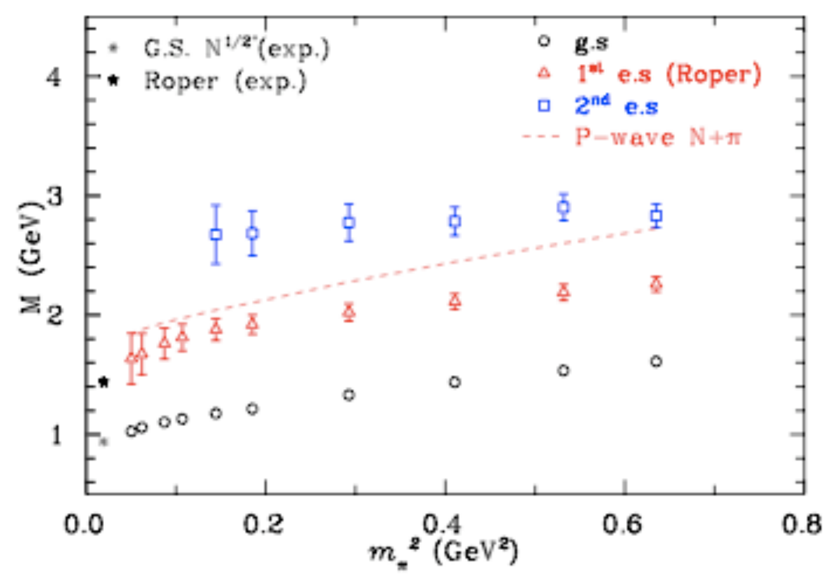
Little evidence for multi-particle states

Roper Resonance - I

Roper (1440): lightest positive parity excitation of the nucleon – lighter than the N(1535) negative-parity excitation. Hard to reconcile with constituent quark model.



Two quenched calculations observe light Roper



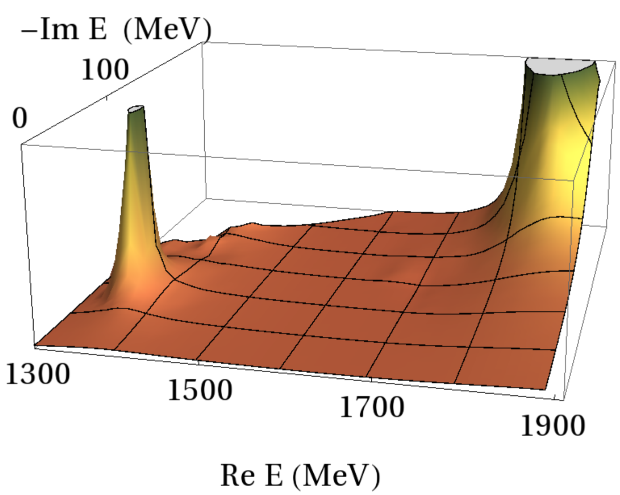
Mahbub et al., arXiv:0910:2789

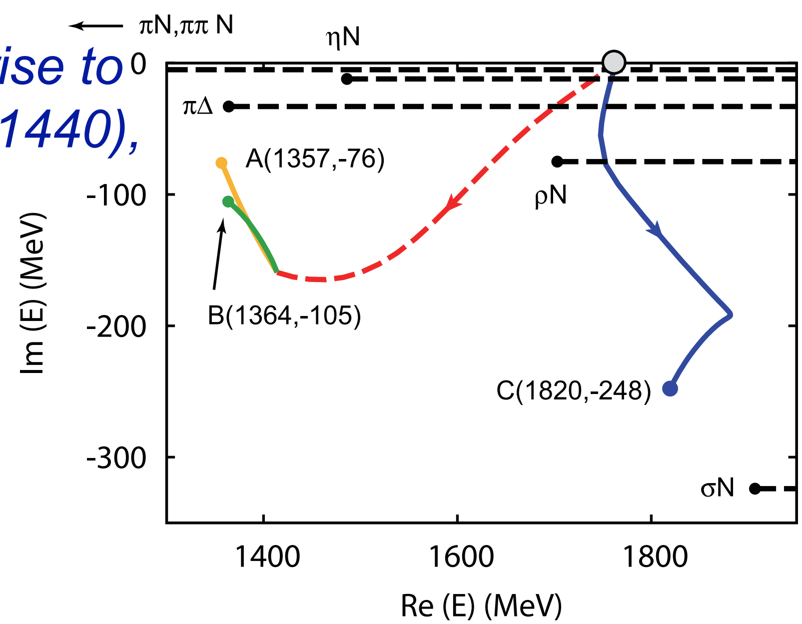
Roper from Amplitude analysis

Reaction model developed to analyse pion-nucleon reaction data to W = 2 GeV, and pion production data from Jlab.

Analytic continuation method to extract parameters of nucleon resonances within EBAC dynamical coupled-channel model.

Single bare state in P_{11} channel gives rise to three poles: two around the Roper N(1440), and the other around the N*(1710).*





Juelich-DCC; Roper generated dynamically

Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama, Sato, PRL (2010) to appear

Challenges

- Lattices with two light and strange quark
- Identification of spin
- Seeking two-particle states in spectrum of energies – *region where states unstable.*

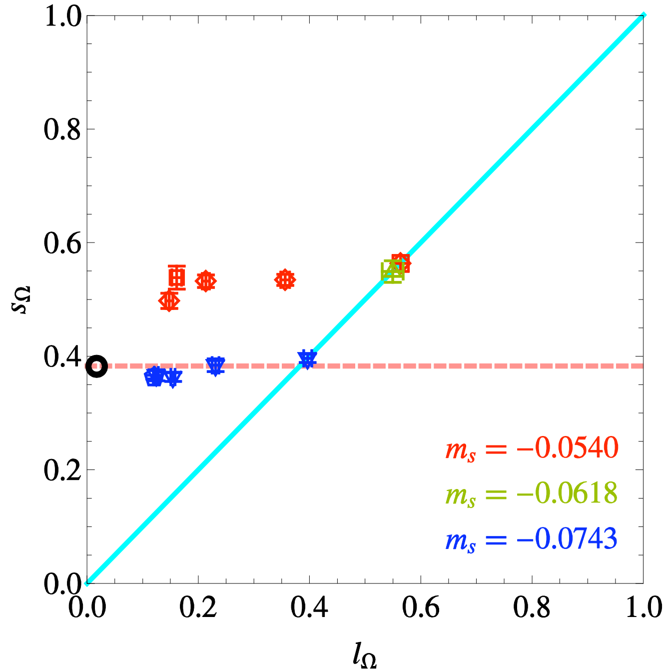
Anisotropic Clover Generation - I

- "Clover" Anisotropic lattices $a_t < a_s$: *major gauge generation program under INCITE and discretionary time at ORNL designed for spectroscopy*

Challenge: setting scale and strange-quark mass

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega

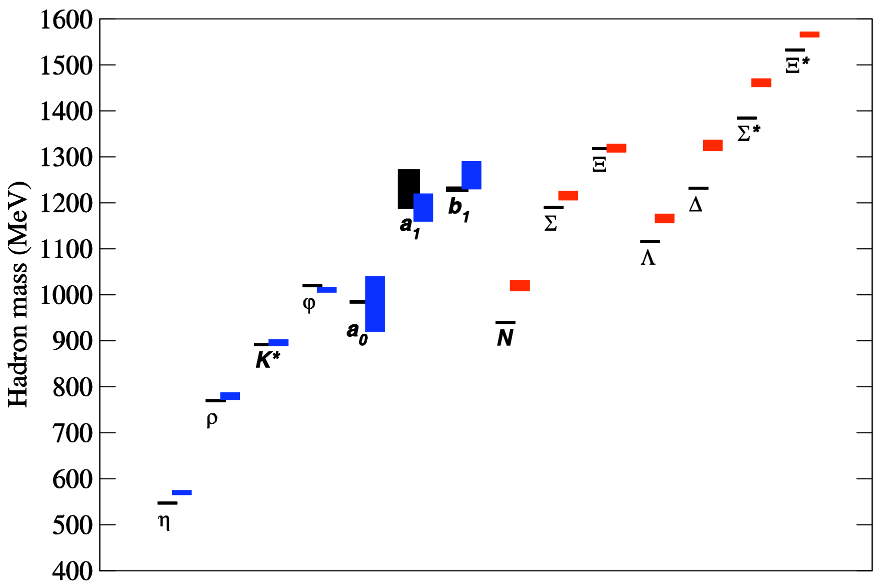


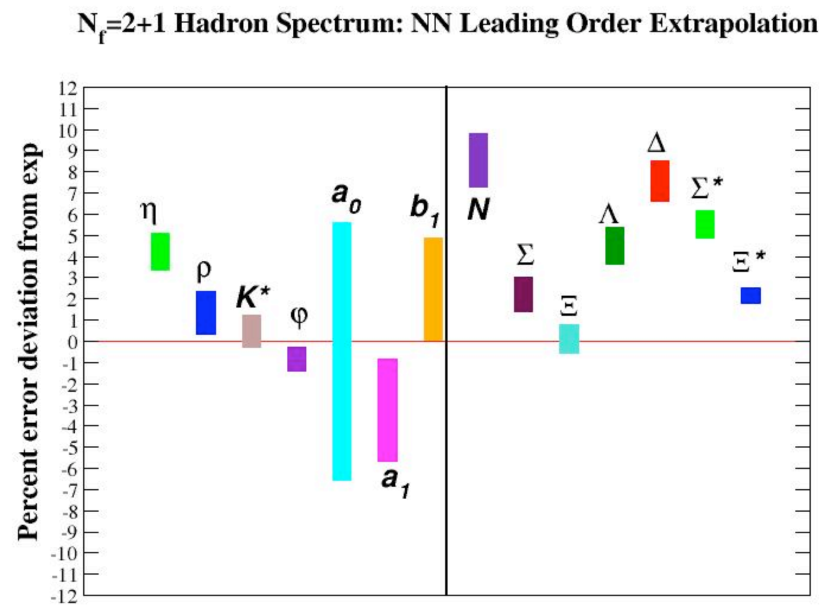
$$l_X = (9/4)m_\pi^2/m_X^2$$

Express physics in (dimensionless) (l,s) coordinates

H-W Lin et al (Hadron Spectrum Collaboration), PRD79, 034502 (2009)

Anisotropic Clover – II





L_s(fm)	2.45fm	2.95fm	3.93fm	4.91fm
m_π(MeV)	$20^3 \times 128$	$24^3 \times 128$	$32^3 \times 256$	$40^3 \times 256$
833	6k, TACC[1.0M](10)			
560	7k, TACC[1.5M](6.7)			
448	8k, TACC[2.1M](5.4)			
383	13k, done	13k, done	11k, Tenn[22M](7.4)	
230		6k, PSC[6M](3.2)	11k, ORNL[70M](4.2)	
140				11k, INCITE[390M](3.4)

Two volumes

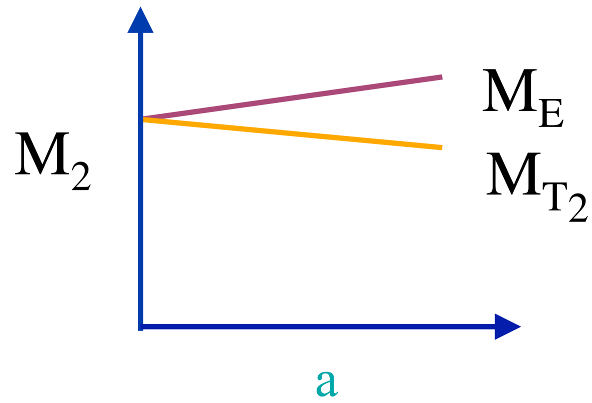
Discovering the continuum quantum numbers: low-lying meson spectrum

Identification of Spin - I

- **We have seen lattice does not respect symmetries of continuum: *cubic symmetry for states at rest***

Problem: requires data at several Lattice spacings – density of states in each irrep large.

Solution: exploit known continuum behavior of overlaps



- Construct interpolating operators of *definite* (continuum) JM: O^{JM}

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

- Use projection formula to find subduction under irrep. of cubic group

$$O^{[J]}_{\Lambda\lambda}(t, \vec{x}) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R \in O_h^D} D^{(\Lambda)*}_{\lambda\lambda}(R) U_R O^{J,M}(t, \vec{x}) U_R^\dagger$$
$$= \sum_M S^{J,M}_{\Lambda,\lambda} O^{J,M}$$

Identification of Meson Spins

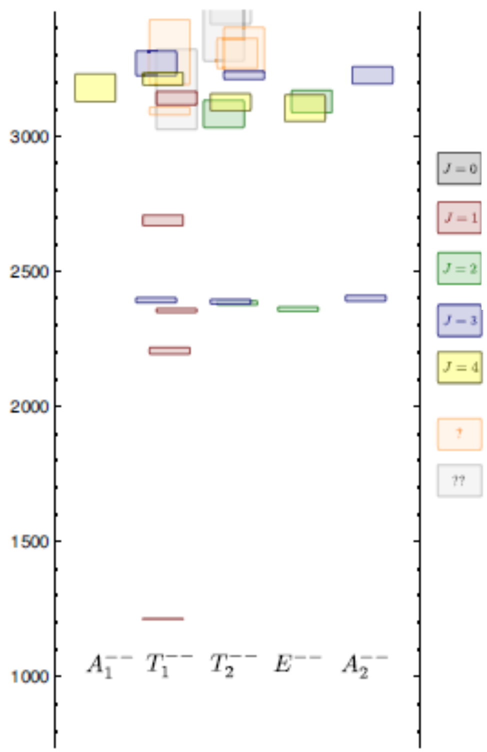
Hadspec collab. (dudek et al), 0909.0200, PRL

Overlap of state onto subduced operators

$$\langle 0 \mid O^{J,M} \mid J', M' > \;=\; Z_J \delta_{J,J'} \delta_{M,M'}$$

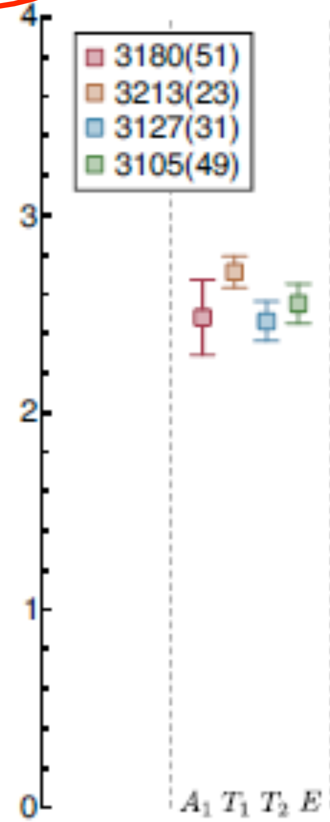
$$\langle 0 \mid O^{J}_{\Lambda,\lambda} \mid J', M' \rangle \;=\; S^{J,M'}_{\Lambda,\lambda} Z_J \delta_{J,J'}$$

Common across irreps., up to O(a)



J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

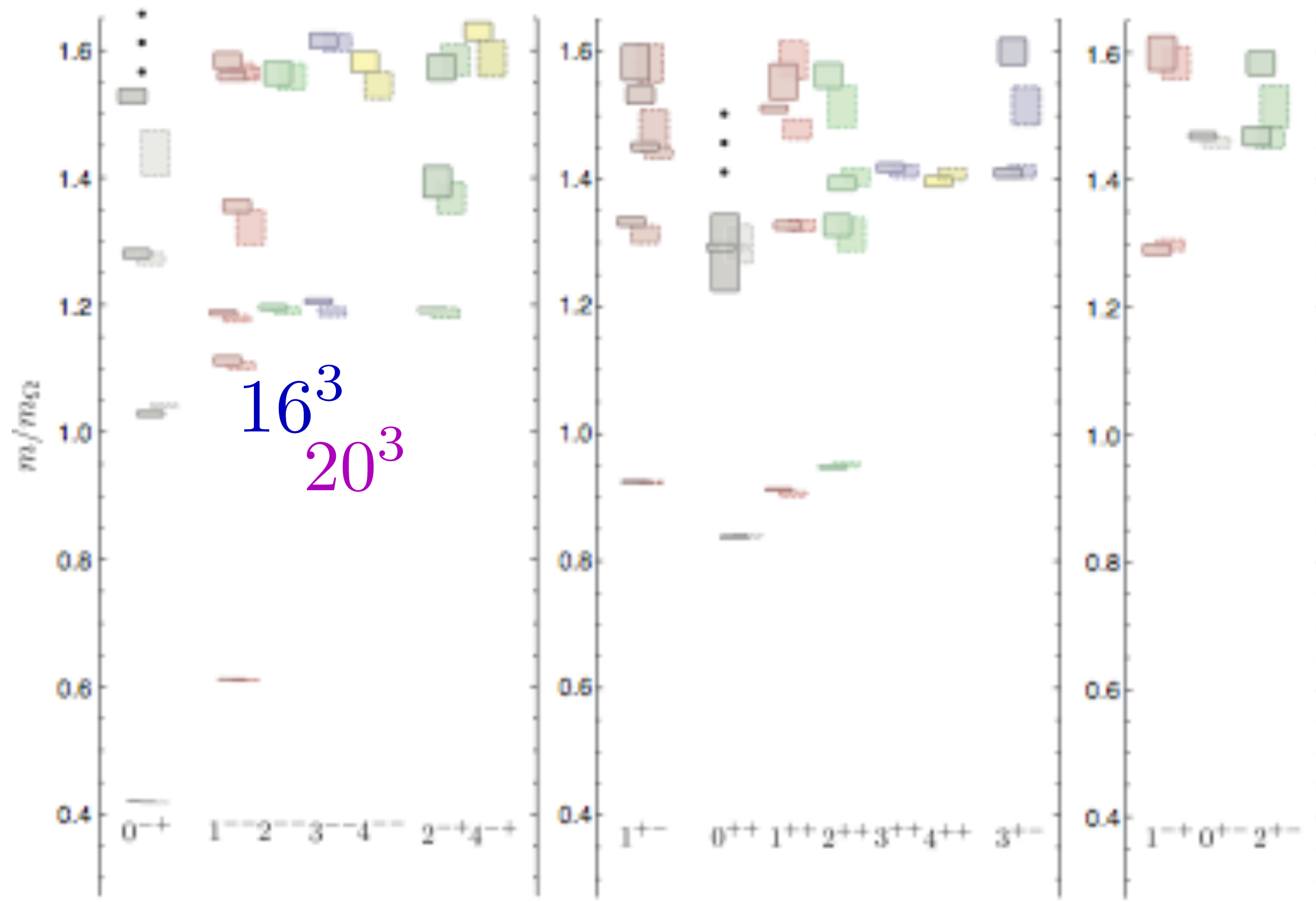
$$N_f = 3$$



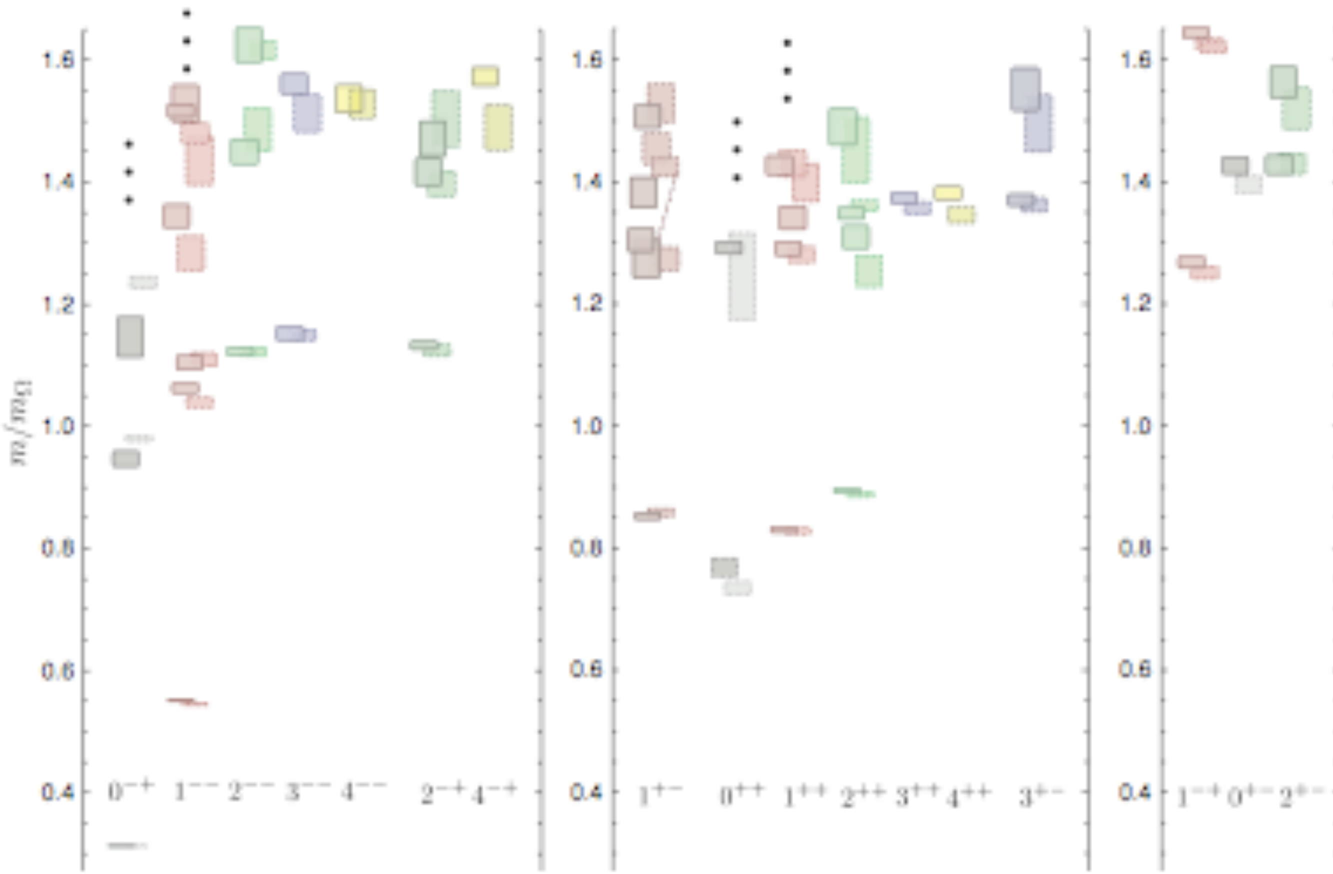
Nf = 3 Spectrum

Dudek et al. (HadSpec Collab), in preparation

Exotic quantum numbers

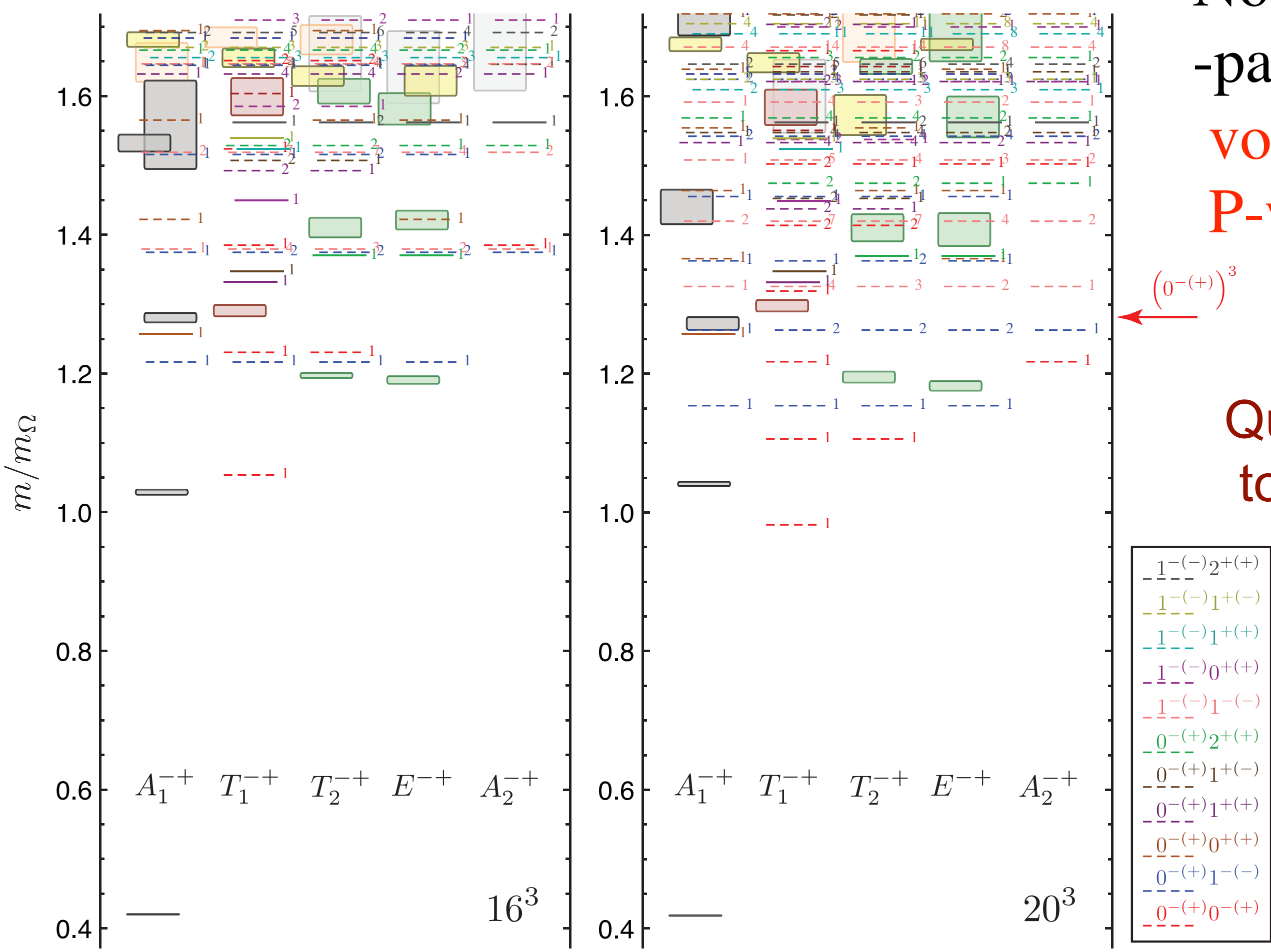


Nf = 2 + 1 Spectrum



Spectrum of light isovector mesons: m_{π}=520 MeV

Whence the multi-hadrons?

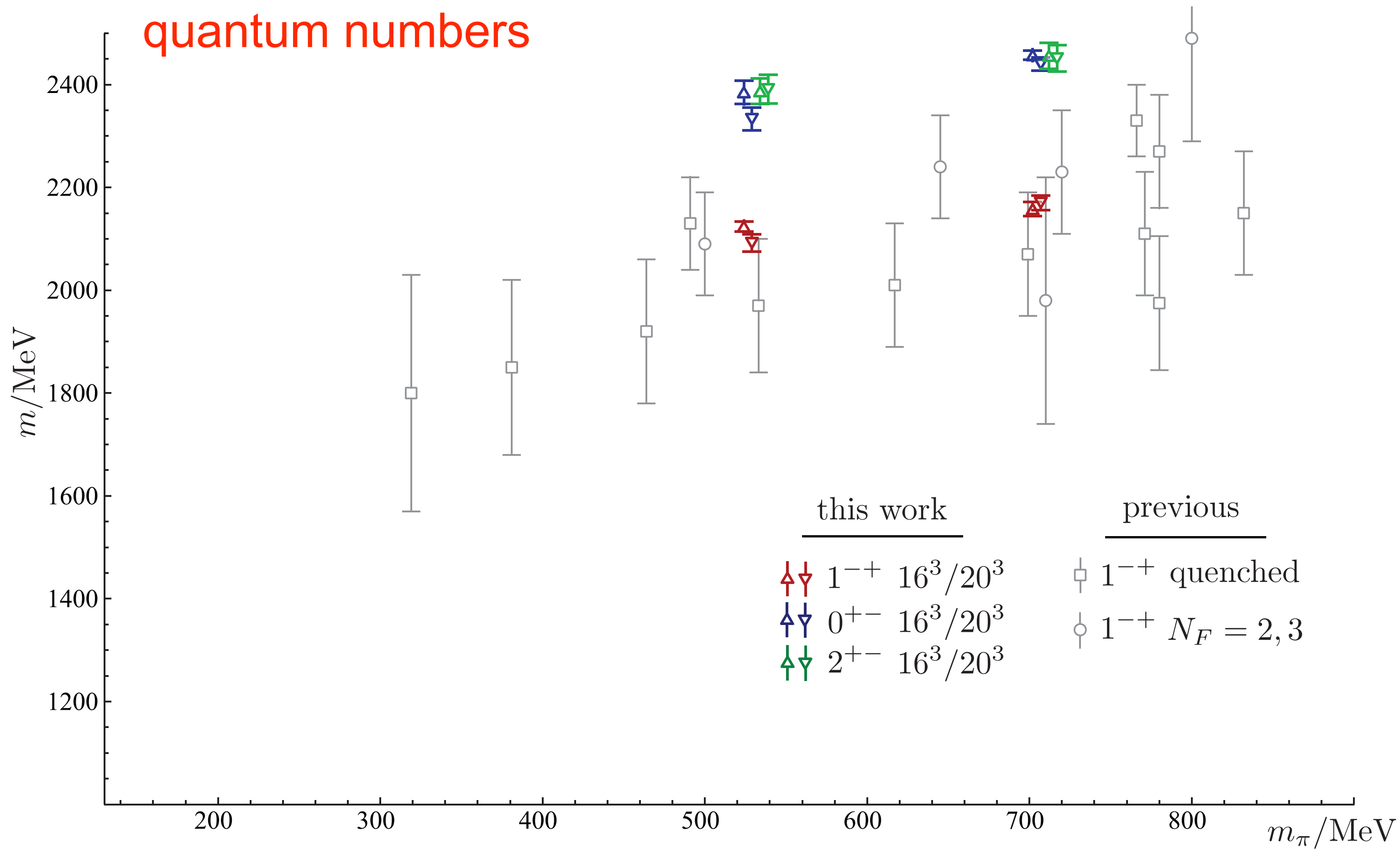


Non-interacting two -particle energies: volume-dependent for P-wave

Quark bilinears insensitive to mult-hadron states

Low-lying Exotic Spectrum

High-precision calculation of mesons spectrum, and those with exotic quantum numbers



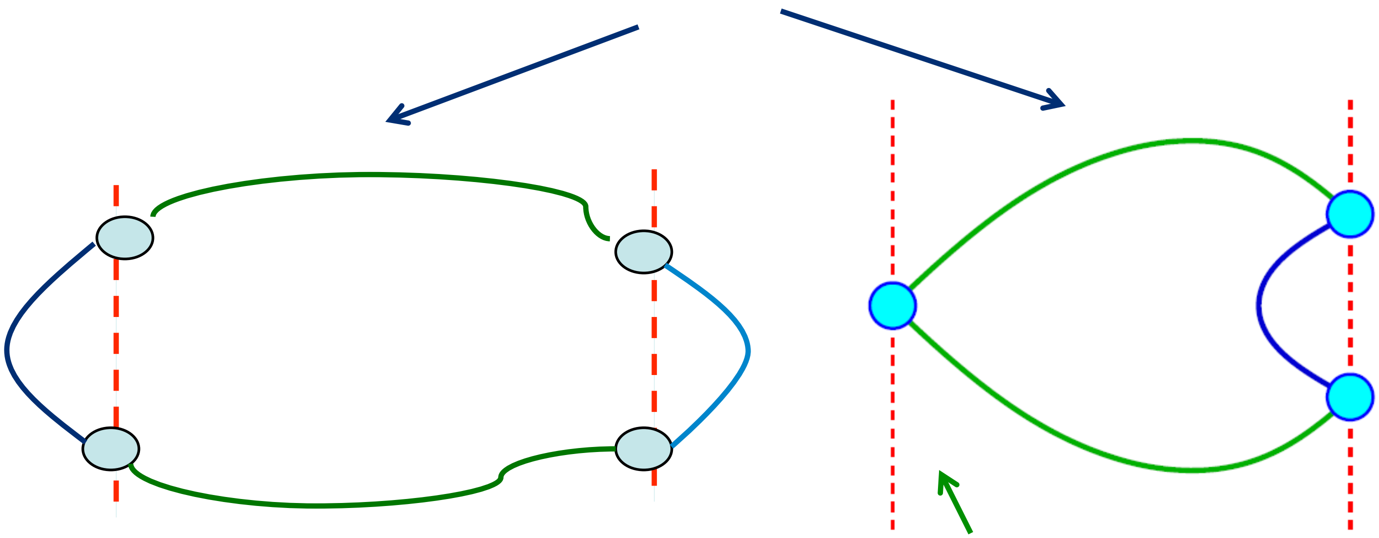
HadSpec Collaboration (J. Dudek et al.), preliminary

Multi-hadron States and Strong Decays

See also talk of Nilmani

Multi-hadron Operators

Need "all-to-all"

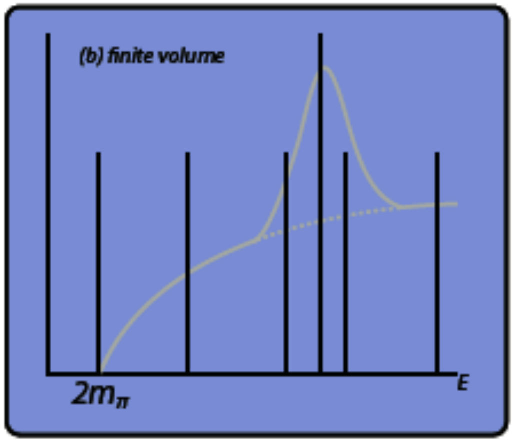


Usual methods give "point-to-all"

Strong Decays

Thanks to Jo Dudek

- In QCD, even ρ is unstable under strong interactions – *resonance in π-π scattering (quenched QCD not a theory – won't discuss).*
- Spectral function continuous; finite volume yields discrete set of energy eigenvalues



Momenta quantised: known set of free-energy eigenvalues

$$E_n = 2\sqrt{m_\pi^2 + (\frac{2n\pi}{L})^2}$$

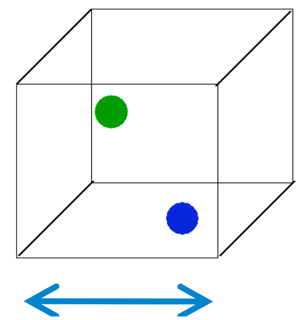
Strong Decays - II

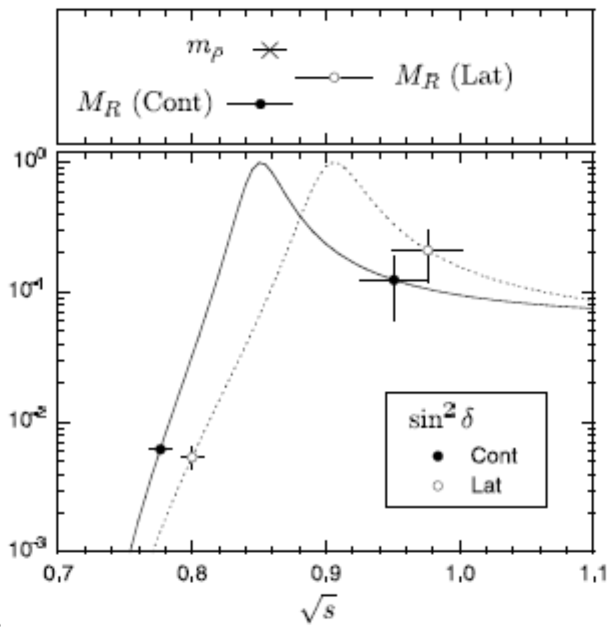
- For interacting particles, energies are shifted from their free-particle values, by an amount that depends on the energy.
- Luscher: relates shift in the free-particle energy levels to the phase shift at the corresponding E.

Breit-Wigner fit

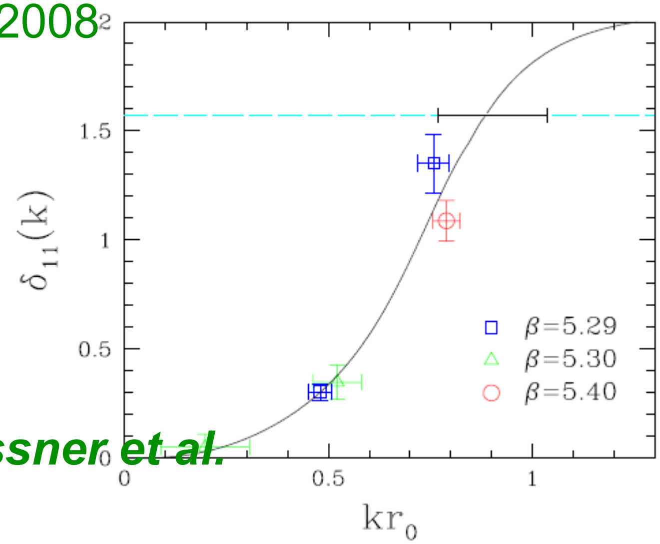
CP-PACS, arXiv:0708.3705

$$\delta E(L) \leftrightarrow \delta(E)$$





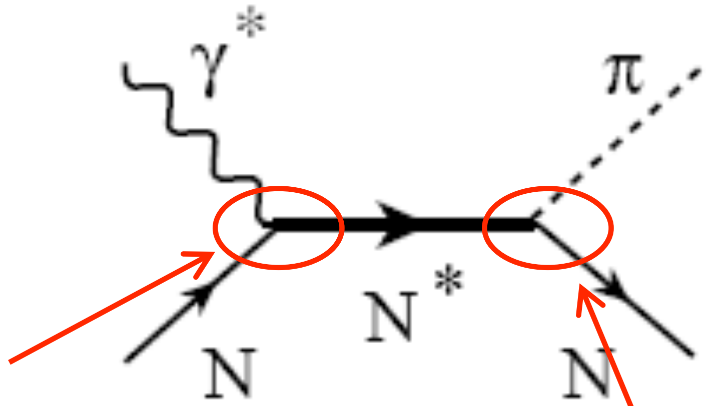
QCDSF, 2008



Ulf Meissner et al.

EM Properties of Excited States

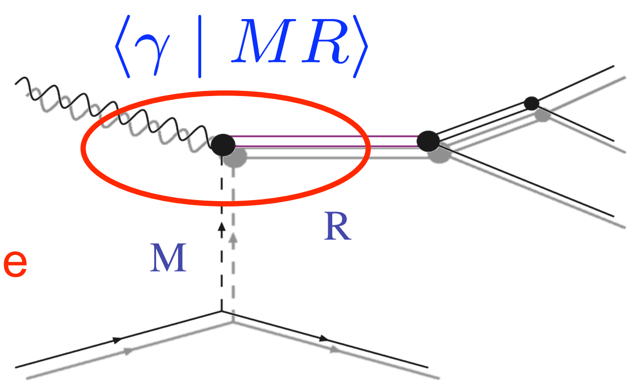
Example: Single-pion photoproduction



Radiative transition amplitudes

Example: Photoproduction at GlueX

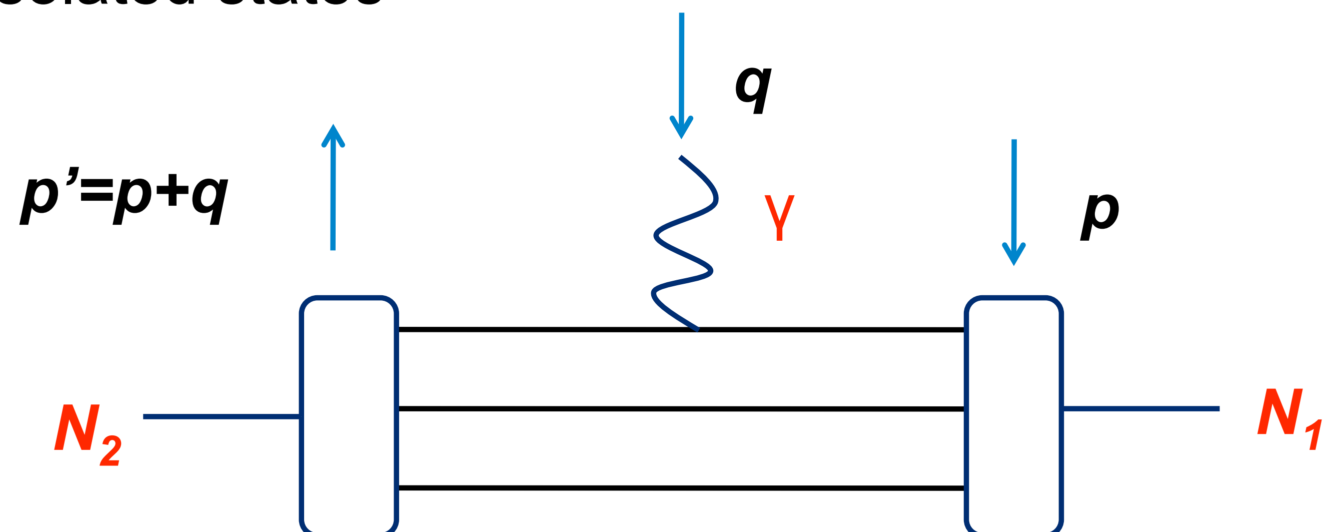
Axial-vector Couplings?



Radiative transition amplitude

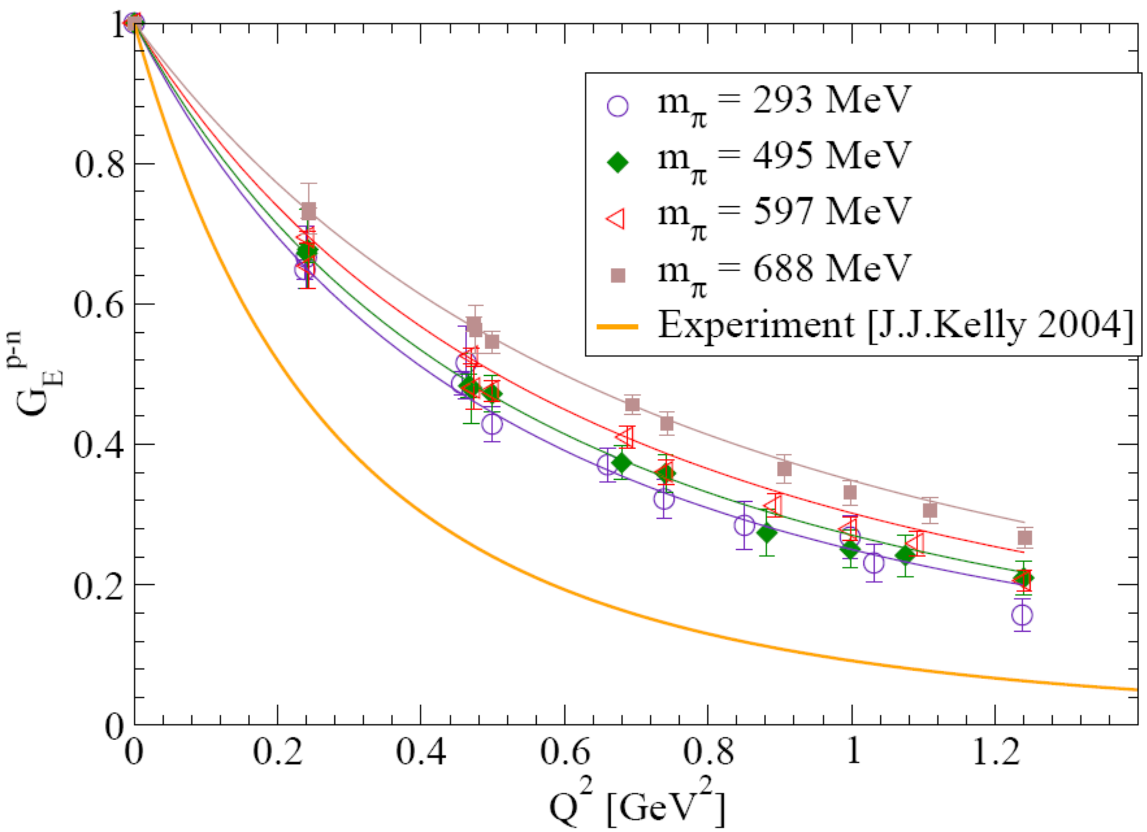
Anatomy of a Calculation - I

- Lattice QCD computes the transition between isolated states



$$\langle N_2 \left| V_\mu \right| N_1 \rangle_\mu(q) \;=\; \overline{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_\mu - \frac{q_\mu}{q^2} - (M_{N_2} - M_{N_1}) \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq\cdot x},$$

Isovector Form Factor

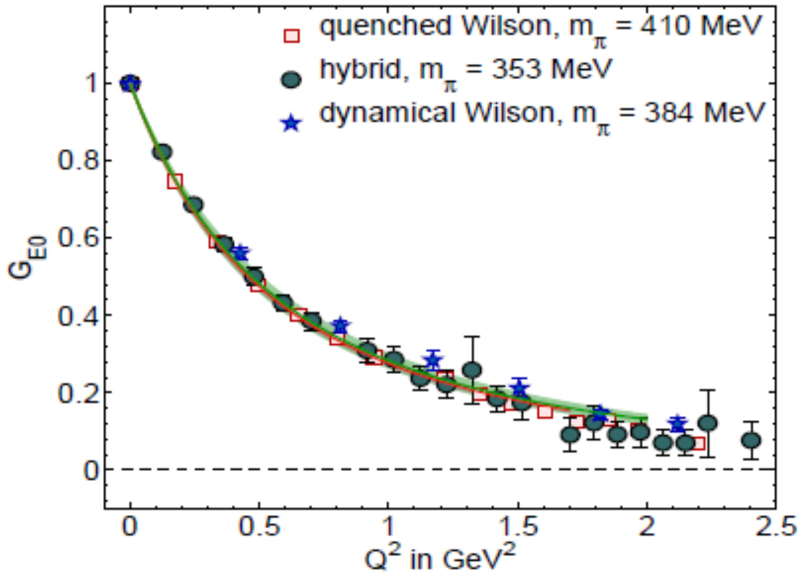


J.D.Bratt et al (LHPC), arXiv:0810.1933

Euclidean lattice: *form factors in space-like region*

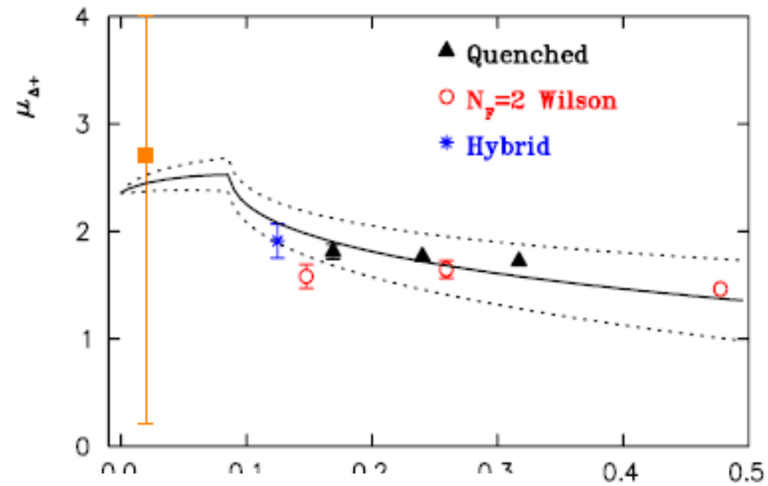
Extension to higher Q^2

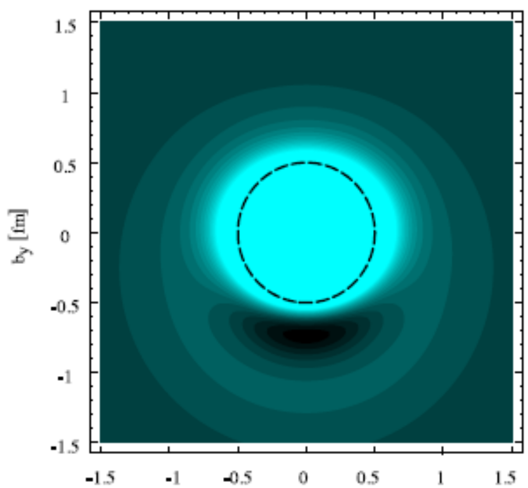
EM Properties of Delta



Alexandrou et al., PRD79, 014509 (2009)

Electric form factor





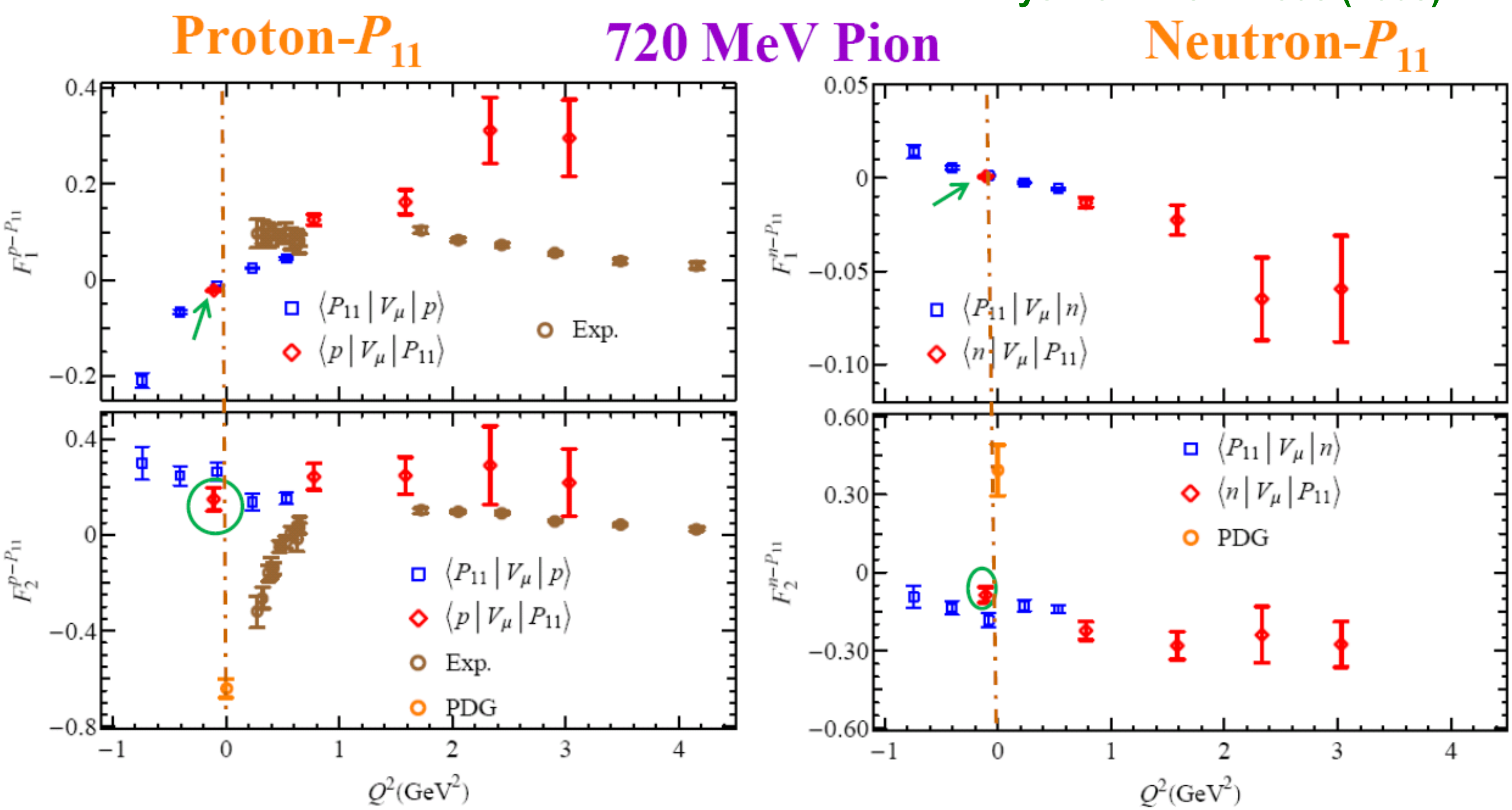
$$\rho^{\Delta}_{T\, s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\,\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \times \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp \rangle,$$

$$Q^{\Delta}_{s_\perp} \equiv e \int d^2\vec{b}\, (b_x^2 - b_y^2)\, \rho^{\Delta}_{T\, s_\perp}(\vec{b}).$$

Nucleon Radiative Transition - I

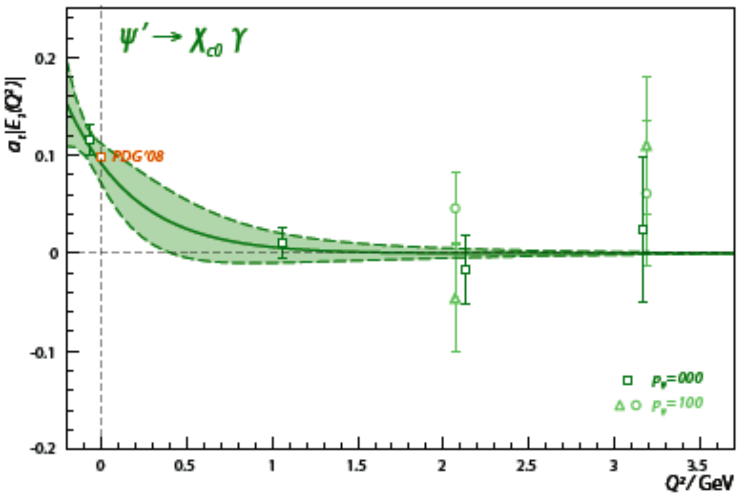
N_f=0 exploratory: P11->Nucleon transition

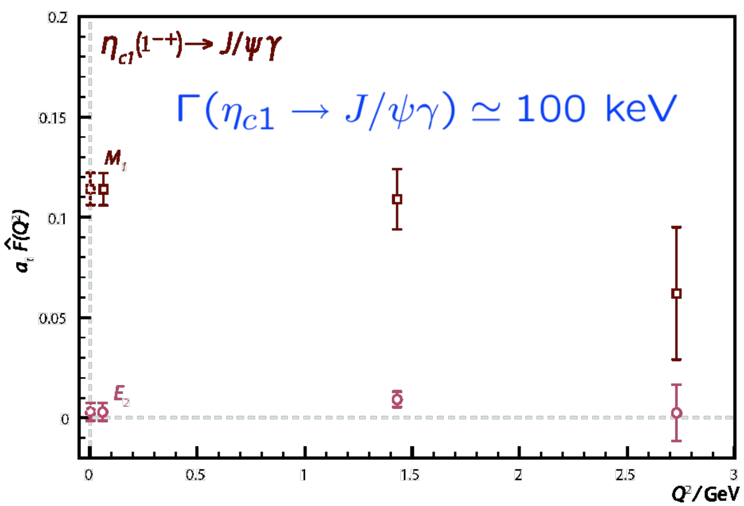
H-W Lin et al.,
Phys.Rev.D78:114508 (2008).



Spectrum and Properties of Mesons in LQCD

Initial studies in charmonium





J Dudek, R Edwards, C Thomas, Phys. Rev. D79:094504 (2009).

Use of variational method, and the optimized meson operators, to compute radiative transitions between excited states and exotics.

considerable phenomenology developed from the results - supports non-relativistic models and limits possibilities for form of excited glue

Radiative width of hybrid comparable to conventional meson – important for GlueX

HP15

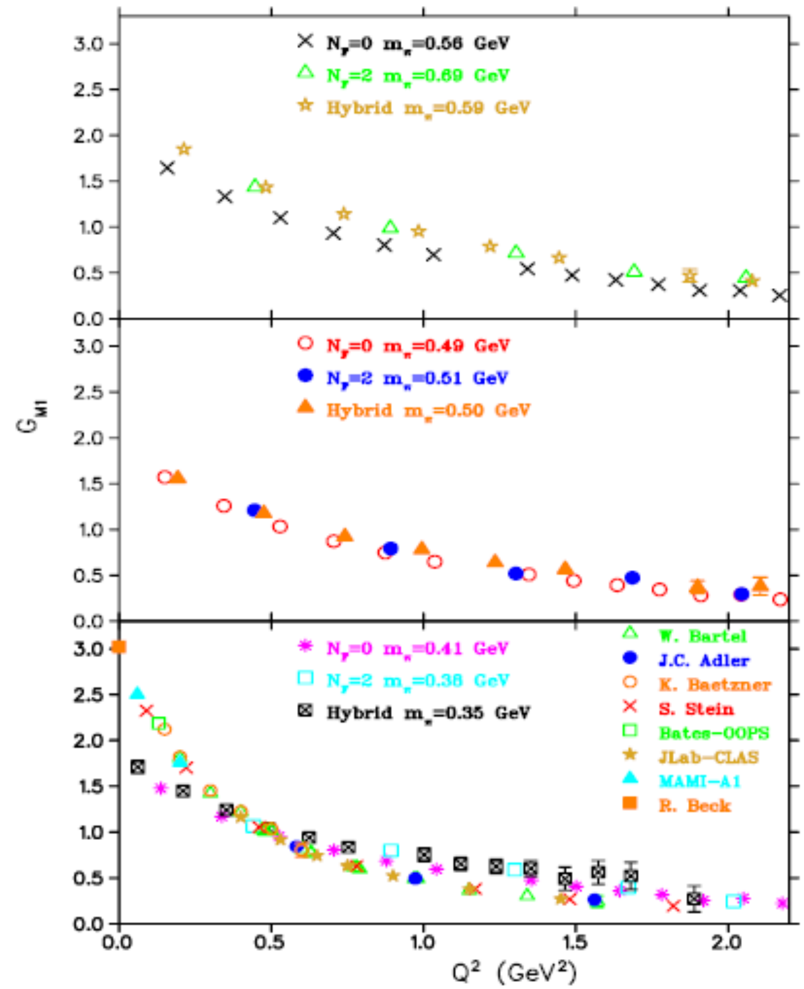
Conclusions

- Lattice calculations evolving from studies of properties of ground-state hadrons to those of resonances
  - Lattices with correct spectrum of flavors
  - Variational method to precisely determine energies
  - Identification of spin both for mesons and for baryons
  - New correlator construction methods: many operators, high precision
- Properties of lowest-lying resonances studied
  - Delta form factor and charge distribution
  - "Roper" transition form factor
  - Radiative transitions between mesons
- Challenges:
  - Identifying the multiparticle states
  - Entering regime of strong decays
  - Transition Form Factors at higher Q^2
  - Mapping to Chiral Perturbation Theory

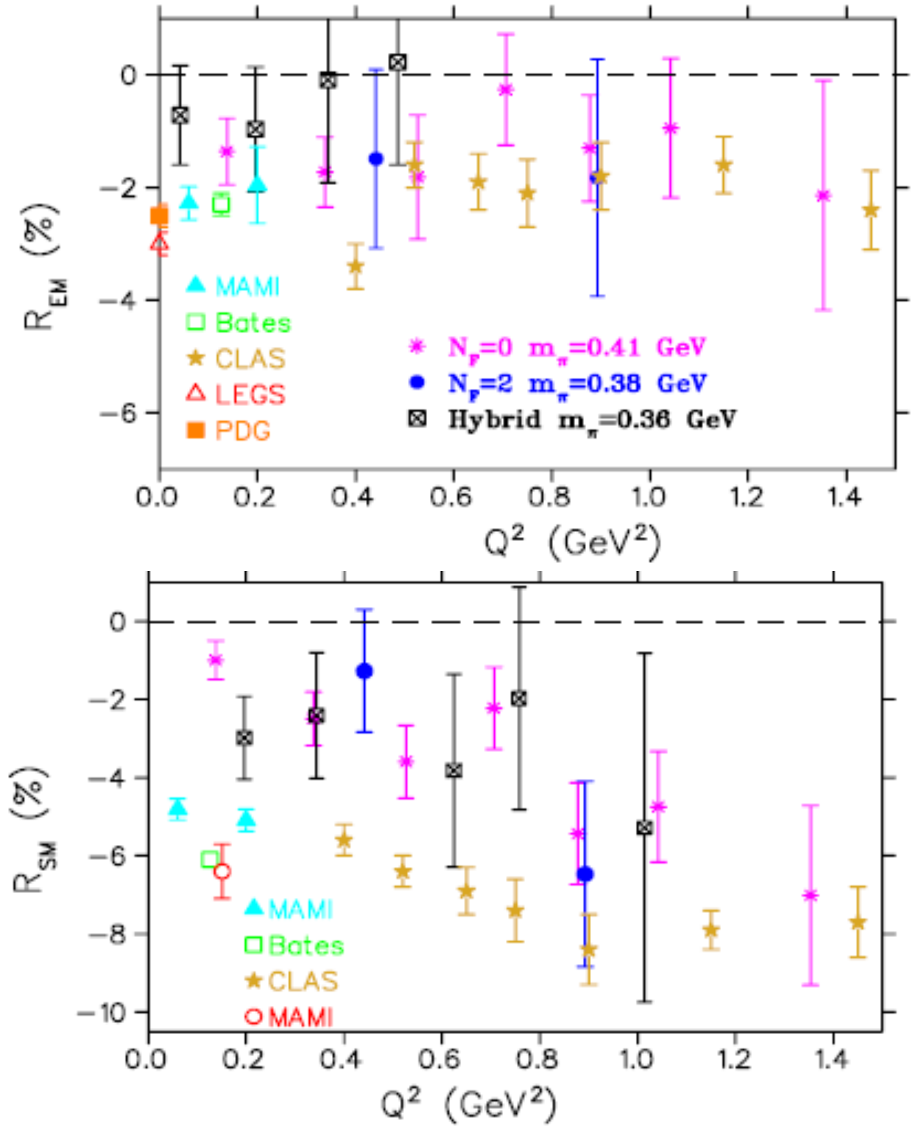
N-Δ Transition Form Factor - I

- **Transition between lowest lying I=3/2, J=3/2 (Δ), and I=1/2, J=1/2 (N)**
- **Comparison between different lattice calculations and expt.**
  - **Milder Q^2 dependence than experiment but**
  - ***Quark masses corresponding to pion masses around 350 MeV***
  - ***Q^2 range up to around 2 GeV^2***

Alexandrou et al, arXiv:0710.4621



N-Δ Transition Form Factor - II

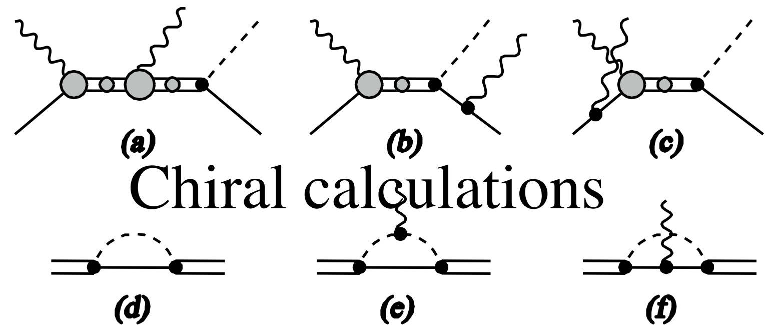


$R_{EM} \to +1$

Alexandrou et al, arXiv:0710.4621

Deformation in nucleon or delta

Delta Form Factors

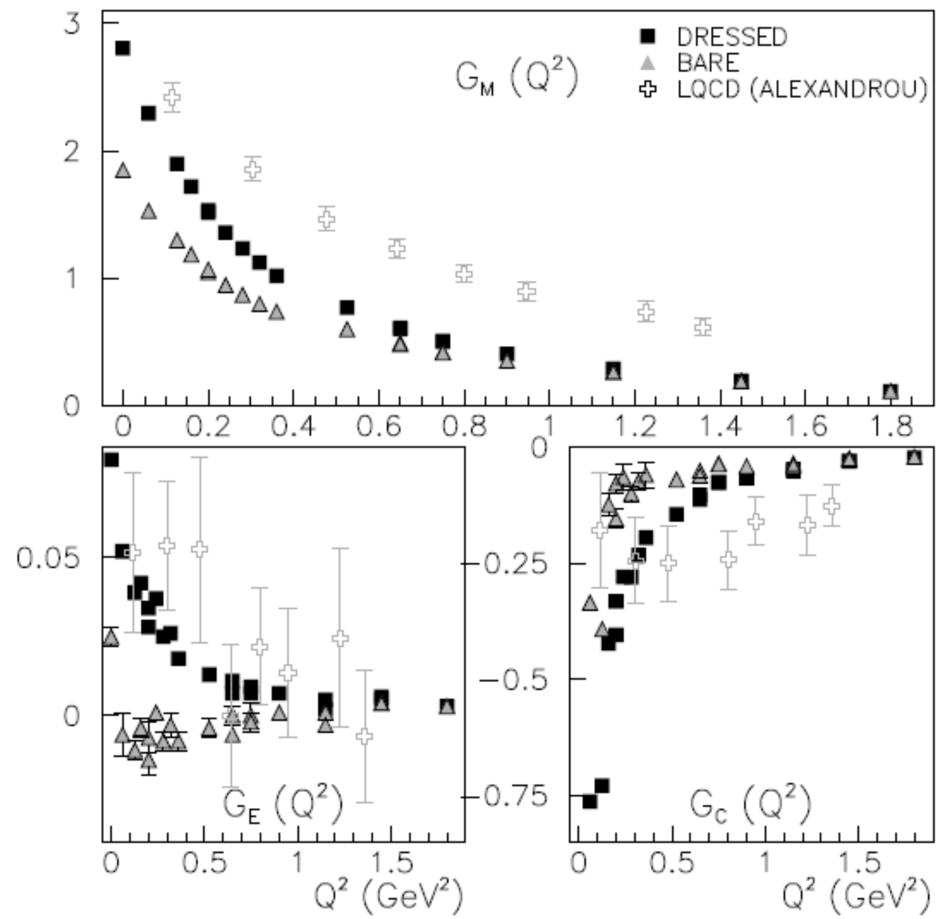


Pascalutsa, Vanderhaeghen (2004)Thomas, Young (…)

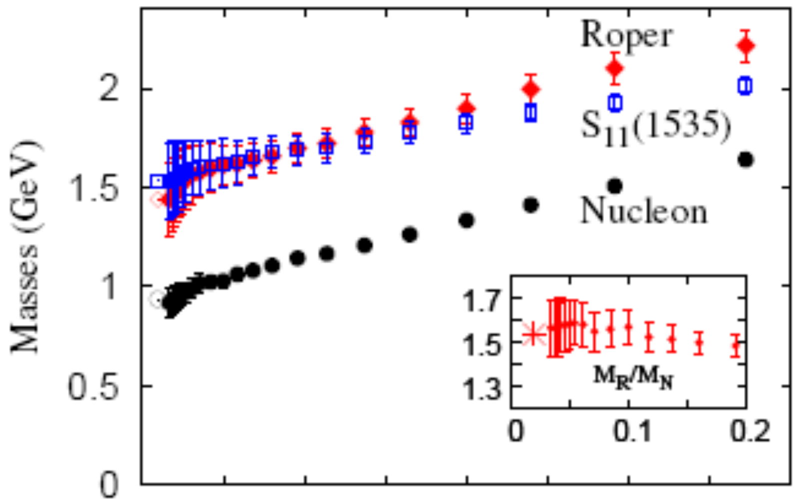
Interpretation of Parameters

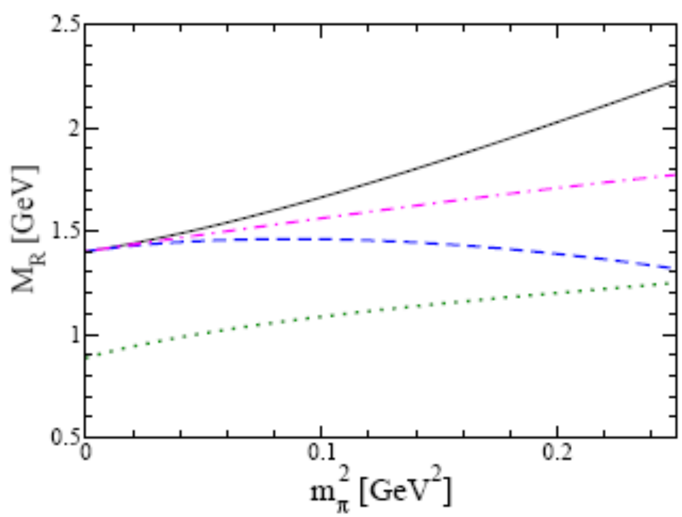
Julia-Diaz et al., Phys.Rev. C75 (2007) 015205

Comparison of LQCD, EFT + expt: lattice QCD can vary quark masses



Roper Resonance





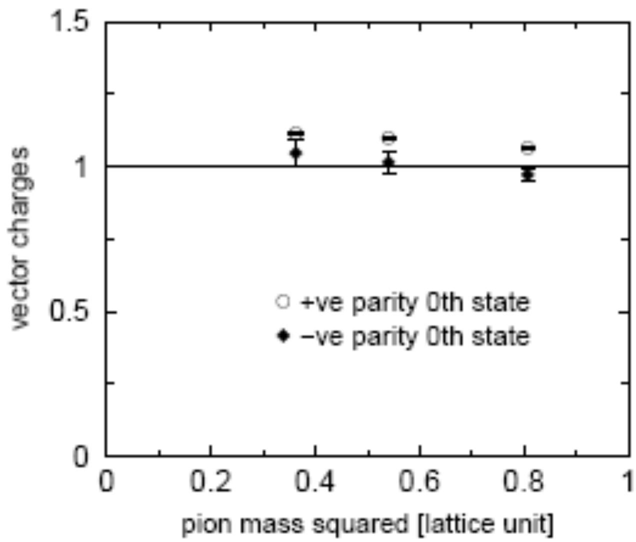
- **Bayesian statistics and constrained curve fitting**
- **Used simple three-quark operator**

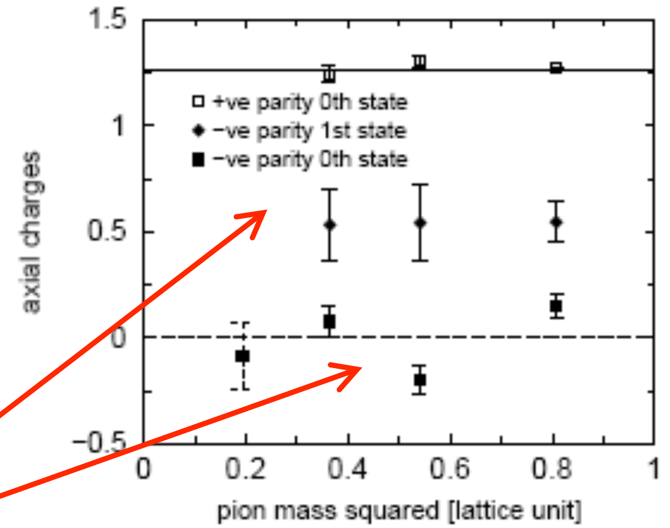
Dong et al., PLB605, 137 (2005)

Borasoy et al., Phys.Lett. B641 (2006) 294-300

Axial-vector Charges

- The axial-vector charges $g_A^{N1\ N2}$ can provide additional insight into hadron structure
- Recent calculation of axial-vector charges of two lowest-lying ½- states, associated with N(1535) and N(1650).

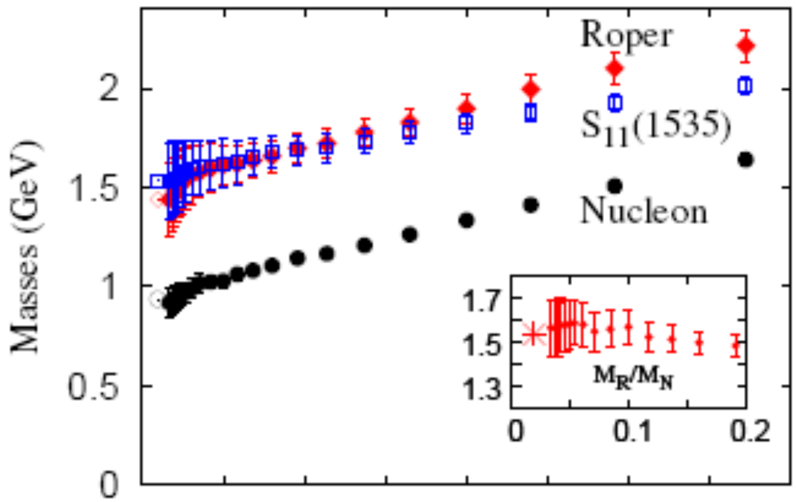


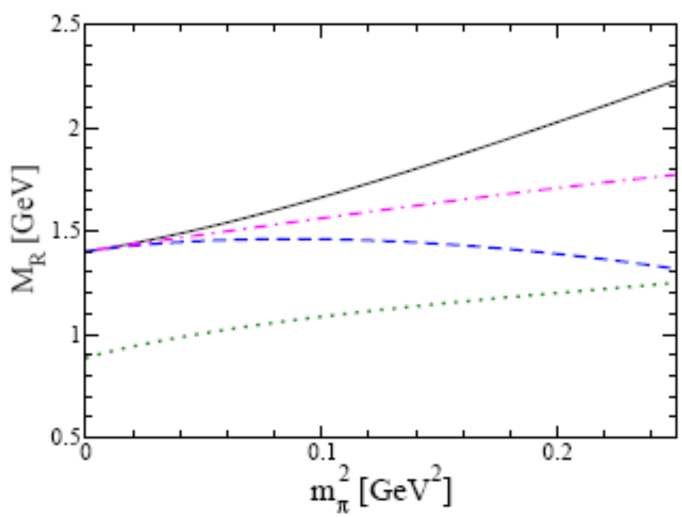


Takahashi, Kunihiro, arXiv:0801.4707

Consistent with NR quark model

Roper Resonance





- Bayesian statistics and constrained curve fitting
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Borasoy et al., Phys.Lett. B641 (2006) 294-300

Correlation functions: Distillation

- Use the new "distillation" method.
- Observe

Eigenvectors of Laplacian

$$L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$

- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- Meson correlation function

Includes displacements

$$C_M(t,t') = \langle 0 \mid \bar{d}(t')\Gamma^B(t')u(t')\bar{u}(t)\Gamma^A(t)d(t)|0\rangle$$

- Decompose using "distillation" operator as

M. Peardon *et al.*, arXiv :0905.2160

$$C_M(t,t') = \mathrm{Tr}\langle \phi^A(t')\tau(t',t)\Phi^B(t)\tau^\dagger(t',t), \rangle$$

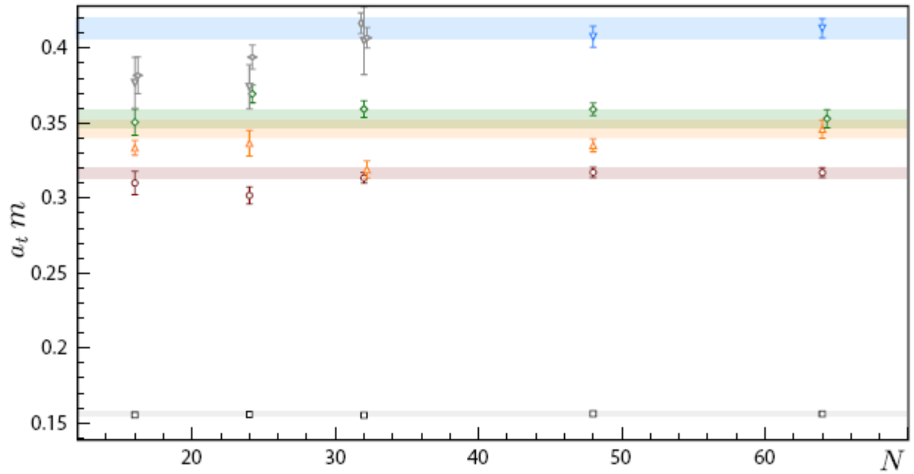
where

$$\Phi^{A,ij}_{\alpha\beta} = v^{*(i)}(t)[\Gamma^A(t)\gamma_5]_{\alpha\beta} v^{(j)}(t')$$

Perambulators →

$$\tau^{ij}_{\alpha\beta}(t,t') = v^{*(i)}(t') M^{-1}_{\alpha\beta}(t',t) v^{(j)}(t).$$

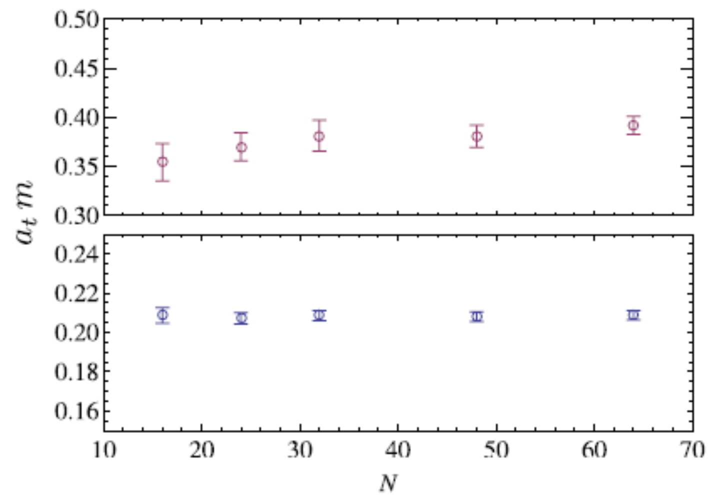
Distillation Results

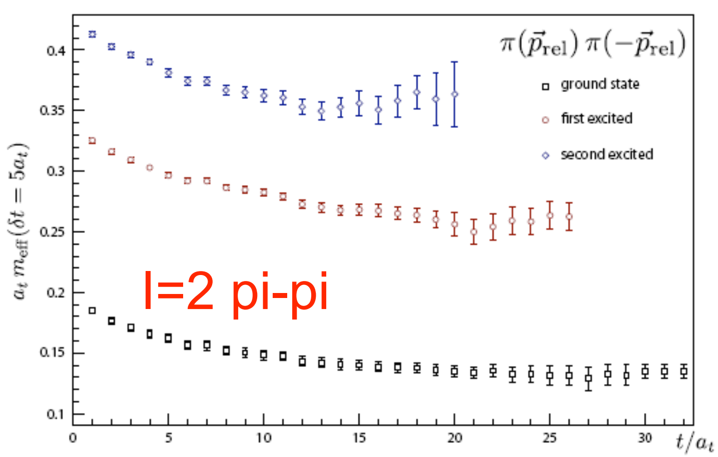


Nucleon Variational Analysis

ρ Variational Analysis

Errors < 3%





I=2 pi-pi

Overall momentum 0
Basis: pairs of back-to-back operators at momentum ***p***