In Search of the Perfect Fluid

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The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases \((10^{-6} \text{K})\) and the quark gluon plasma \((10^{12} \text{K})\) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.
Measures of Perfection

Viscosity determines shear stress ("friction") in fluid flow

\[ F = A \eta \frac{\partial v_x}{\partial y} \]

Dimensionless measure of shear stress: Reynolds number

\[ Re = \frac{n}{\eta} \times mvr \]

- \([\eta/n] = \hbar\]

- Relativistic systems \( Re = \frac{s}{\eta} \times \tau T \)
Other sources of dissipation (thermal conductivity, bulk viscosity, ...) vanish for certain fluids, but shear viscosity is always non-zero.

There are reasons to believe that $\eta$ is bounded from below by a constant times $\hbar s/k_B$. In a large class of theories $\eta/s \geq \hbar/(4\pi k_B)$.

A fluid that saturates the bound is a “perfect fluid”.
Fluids: Gases, Liquids, Plasmas, …

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.

Historically: Water

$(\rho, \varepsilon, \vec{\pi})$
Simple fluid: Conservation laws for mass, energy, momentum

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \vec{\nabla} (\rho \vec{v}) &= 0 \\
\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \epsilon &= 0 \\
\frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} &= 0
\end{align*}
\]

Constitutive relations: Energy momentum tensor

\[
\Pi_{ij} = P \delta_{ij} + \rho v_i v_j + \eta \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)
\]

reactive  
dissipative  
2nd order

Measure of perfection:

\[
\frac{\delta \Pi_{ij}}{\Pi_{ij}} \sim \frac{\eta (\partial \cdot v)}{P} \sim \frac{\eta}{n} \frac{1}{\tau T}
\]
Kinetic Theory

Kinetic theory: conserved quantities carried by quasi-particles

\[
\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]
\]

\[
\eta \sim \frac{1}{3} n \bar{p} l_{mfp}
\]

Normalize to density. Uncertainty relation suggests

\[
\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar
\]

Also: \( s \sim k_B n \) and \( \eta/s \geq \hbar/k_B \)

Validity of kinetic theory as \( \bar{p} l_{mfp} \sim \hbar \)?
Effective Theories for Fluids (Here: Weak Coupling QCD)

\[ \mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a \]

\[ \frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \quad (\omega < T) \]

\[ \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < g^4 T) \]
Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

- CFT temperature $\iff$ Hawking temperature
- CFT entropy $\iff$ Hawking-Bekenstein entropy $\sim$ area of event horizon
- Shear viscosity $\iff$ Graviton absorption cross section $\sim$ area of event horizon

\[
T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \\
g_{\mu\nu} = g_{\mu\nu}^0 + \gamma_{\mu\nu}
\]
Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy $\leftrightarrow$ Hawking-Bekenstein entropy

$\sim$ area of event horizon

shear viscosity $\leftrightarrow$ Graviton absorption cross section

$\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)

Strong coupling limit universal? Provides lower bound for all theories?
Effective Theories (Strong coupling)

\[ \mathcal{L} = \bar{\lambda} (i \sigma \cdot D) \lambda - \frac{1}{4} G_{\mu \nu}^a G_{\mu \nu}^a + \ldots \ \Leftrightarrow \ S = \frac{1}{2 \kappa_5^2} \int d^5 x \sqrt{-g} \mathcal{R} + \ldots \]

\[ \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < T) \]
Kinetics vs No-Kinetics

AdS/CFT low viscosity goo

pQCD kinetic plasma
Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \text{Im} G_R(\omega, 0)$ associated with $T_{xy}$

weak coupling QCD

strong coupling AdS/CFT

transport peak vs no transport peak
Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

(Almost) scale invariant systems
Perfect Fluids: The contenders

QGP (T=180 MeV)

Trapped Atoms
(T=0.1 neV)

Liquid Helium
(T=0.1 meV)
Perfect Fluids: The contenders

QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms
$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

Liquid Helium
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios $\eta/s$
## Kinetic Theory: Quasiparticles

<table>
<thead>
<tr>
<th></th>
<th>Low Temperature</th>
<th>High Temperature</th>
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</thead>
<tbody>
<tr>
<td><strong>Unitary Gas</strong></td>
<td>Phonons</td>
<td>Atoms</td>
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<tr>
<td><strong>Helium</strong></td>
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<td><strong>QCD</strong></td>
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Theory Summary

- Unitary gas
- $^4$He
- QCD
I. Experiment (Liquid Helium)

Kapitza (1938)
viscosity vanishes below $T_c$
capillary flow viscometer

Hollis-Hallett (1955)
roton minimum, phonon rise
rotation viscometer

$\eta/s \simeq 0.8 \hbar/k_B$
II. Hydrodynamics (Cold atoms)

Radial breathing mode

Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla P \over mn - \nabla V \over m$$

Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}} \omega_\perp$$

Damping small, depends on $T/T_F$.

experiment: Kinast et al. (2005)
Viscous Hydrodynamics

Energy dissipation ($\eta, \zeta, \kappa$: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2} \int d^3 x \eta(x) \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2$$

$$- \int d^3 x \zeta(x) (\partial_i v_i)^2 - \frac{1}{T} \int d^3 x \kappa(x) (\partial_i T)^2$$

Shear viscosity to entropy ratio
(assuming $\zeta = \kappa = 0$)

$$\frac{\eta}{s} = \left(3\lambda N\right)^{\frac{1}{3}} \frac{\Gamma}{\omega_\perp} \frac{E_0}{E_F} \frac{N}{S}$$

Schaefer (2007), see also Bruun, Smith
Dissipation

\[ R_i \quad [\mu m] \]

\[ t [ms] \]

\[ R_\perp \]

\[ R_z \]

\[ \theta [^\circ] \]

\[ E/E_F = 0.56 \]

\[ E/E_F = 2.1 \]

Dissipation

\[
\begin{align*}
(\delta t_0)/t_0 & = \begin{cases} 
0.008 \\
0.024 
\end{cases} \\
(\delta a)/a & = \begin{cases} 
0.008 \\
0.024 
\end{cases} \left( \frac{\langle \alpha_s \rangle}{1/(4\pi)} \right) \left( \frac{2 \cdot 10^5}{N} \right)^{1/3} \left( \frac{S/N}{2.3} \right) \left( \frac{0.85}{E_0/E_F} \right)
\end{align*}
\]

t_0: “Crossing time” \(b_\perp = b_z, \theta = 45^\circ\)
a: amplitude
III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

Hydro model

\[ v_2 \]

\[
\begin{array}{c}
\text{Hydro model} \\
\text{PHENIX Data} \\
\text{STAR Data}
\end{array}
\]

\[
\begin{array}{cccc}
\pi & K & K^+ + K^- & p + \bar{p} \\
\text{Hydro model} & \text{PHENIX Data} & \text{STAR Data}
\end{array}
\]

Viscosity and Elliptic Flow

Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$
(applicability of Navier-Stokes)

$$\eta + \frac{4}{3} \zeta \ll \frac{3}{4} (\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm

Many questions: Dependence on initial conditions, freeze out, etc.
Outlook

Too early to declare a winner.

\[ \eta/s \simeq 0.8 \text{ (He)}, \quad \eta/s \leq 0.5 \text{ (CA)}, \quad \eta/s \leq 0.5 \text{ (QGP)} \]

Other experimental constraints, more analysis needed.

Kinetic theory: o.k. in He (all \( T \)), o.k. close to \( T_c \) in CA, QGP?

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large N, epsilon expansions, \ldots)