The critical point of QCD: what measurements can one make?

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ILGTI: TIFR

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Outline

The method

Taylor expansion of the pressure in μ_B

$$P(T,\mu_B) = \sum_n \frac{1}{n!}\chi^{(n)}(T)\mu_B^n$$

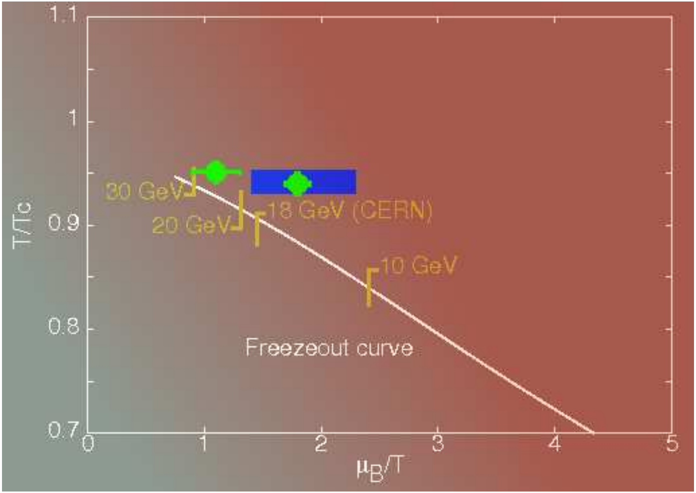
has Taylor coefficients that need to be evaluated only at $\mu_B = 0$ where there is no sign problem. The baryon number susceptibility (second derivative of P) has a related Taylor expansion

$$\chi_B(T,\mu_B) = \sum_n \frac{1}{n!}\chi^{(n+2)}(T)\mu_B^n.$$

χ_B diverges at the critical point. Series expansion can show signs of divergence (Gavai, SG, 2003). If all the coefficients are positive, then the divergence is at real μ_B.

The method is perfectly general and can be applied to any theory.

The phase diagram



▸▸ Skip caveats

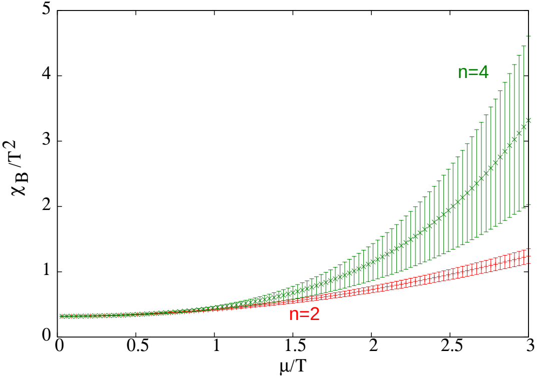
The implementation

- Our implementation is in $N_f = 2$ QCD using staggered quarks.
- Light quark bare masses are tuned to give $m_\pi = 230$ MeV.
- Currently our results from two cutoffs, $\Lambda = 1/a \simeq 800$ MeV ($N_t = 4$) and 1200 MeV ($N_t = 6$).
- Temperature scale setting performed by measuring the renormalized gauge coupling in three different renormalization schemes. At these Λ different schemes give slightly different scales: 1% error estimated from this source.
- Lattice sizes of 4–6 fm per side near T_c: several pion Compton wavelengths, several thermal wavelengths.
- Simulation algorithm is R-algorithm. MD time step has been changed by factor of 10 without any change in results.

Remaining issues

- Series expansion carried out to 8th order. What happens when order is increased? Intimately related to finite volume effects. Finite size scaling tested; works well (Gavai, SG 2004, 2008)
- What happens when strange quark is unquenched (keeping the same action)? Numerical effects on ratios of susceptibility marginal when unquenching light quarks (Gavai, SG, hep-lat/0510044; see also RBRC 2009; de Forcrand, Philipsen, 2007, 2009).
- What happens when m_π is decreased? Estimate of μ_B^E may decrease somewhat: first estimates in Gavai, SG, Ray, nucl-th/0312010; see also Fodor, Katz 2001, 2002.
- What happens in the continuum limit? Estimate of μ_B^E may increase somewhat (Gavai, SG 2008; SG 2009).

Summation bad; resummation good



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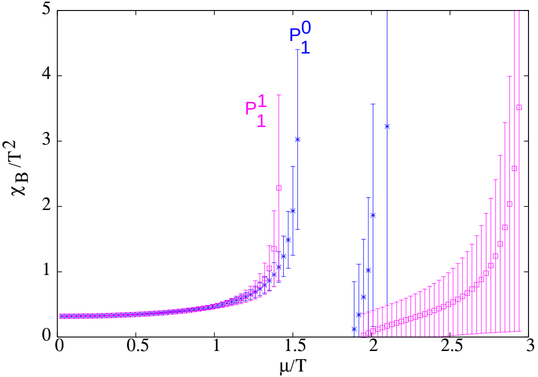

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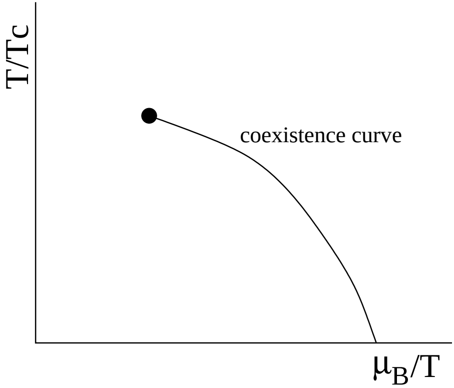

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If $\chi_B^{(2)} \simeq 1/|\mu - \mu_B|^\phi$ then $\chi_B^{(n)} \simeq 1/|\mu - \mu_B|^{(\phi+n-2)}$. Critical index δ currently unknown.

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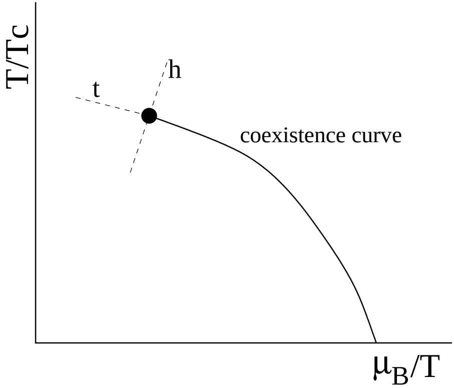

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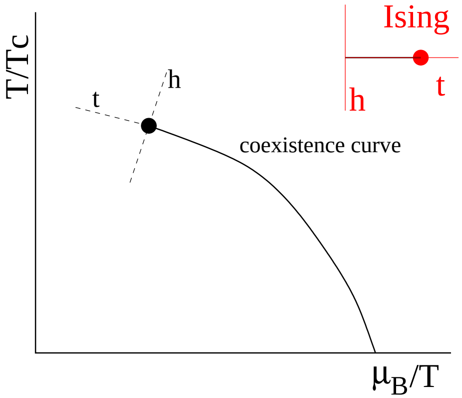

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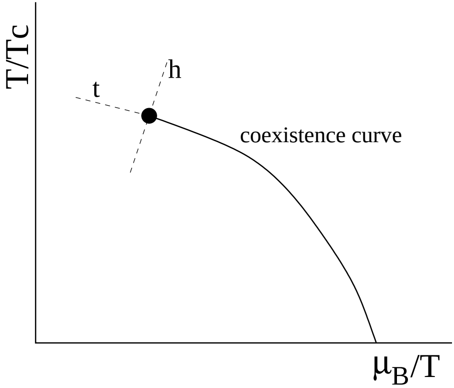

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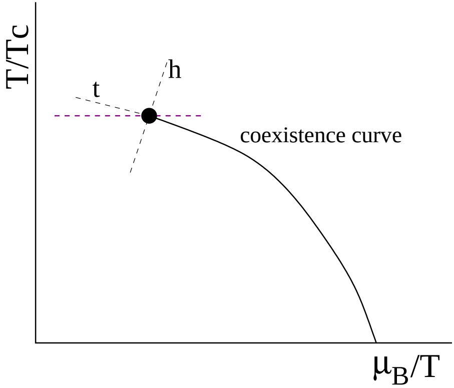

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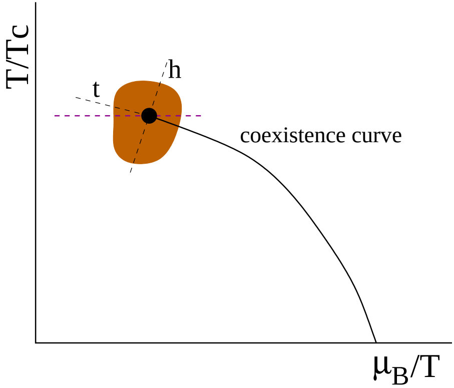

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Outline

Lattice measurements
- The critical point
- NLS at finite μ_B

Experimental measurements
- The method
- Lattice predictions

Gaussian Fluctuations

Normal fluctuations are Gaussian

Suggestion by Stephanov, Rajagopal, Shuryak: measure the width of momentum distributions. Better idea, use conserved charges, because at any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B \rangle.$$

Bias-free measurement possible: Asakawa, Heinz, Muller; Jeon, Koch.

Why Gaussian?

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution: **central limit theorem** (CLT).

Is the current RHIC point non-critical?

Answer

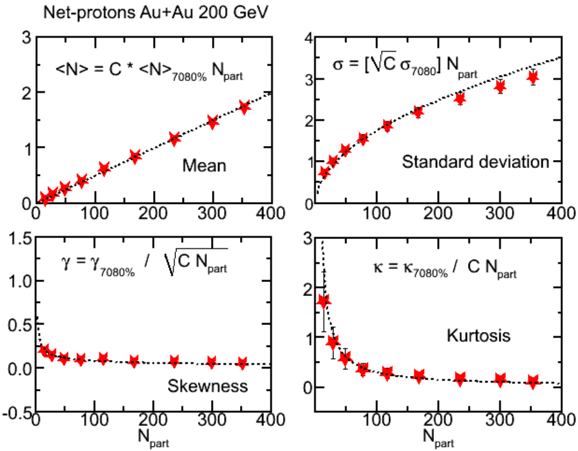
Check whether CLT holds.
Recall the scalings of extensive quantity such as B and its variance σ^2, skewness, $\mathcal{S}$, and Kurtosis, $\mathcal{K}$, given by

$$B(V) \propto V, \quad \sigma^2(V) \propto V, \quad \mathcal{S}(V) \propto \frac{1}{\sqrt{V}}, \quad \mathcal{K}(V) \propto \frac{1}{V}.$$

Caveat

Make sure that the nature of the physical system does not change while changing the volume. Perhaps best accomplished by changing rapidity acceptance while keeping centrality fixed. Alternative tried by STAR is to change the number of participants.

STAR measurements



STAR Collaboration: QM 2009, Knoxville.

What to compare with QCD

The cumulants of the distribution are related to Taylor coefficients—

$$[B^2] = T^3 V \left(\frac{\chi^{(2)}}{T^2} \right), \quad [B^3] = T^3 V \left(\frac{\chi^{(3)}}{T} \right), \quad [B^4] = T^3 V \chi^{(4)}.$$

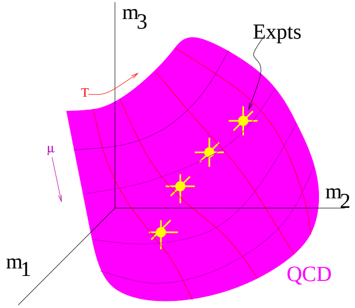
T and V are unknown, so direct measurement of QNS not possible (yet). Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Construct the ratios

$$m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with QCD (Gavai, SG, 2010).

Is there an internally consistent check that all backgrounds and systematic effects are removed and comparison with lattice QCD possible?

How to compare with QCD



Possible measurements lie on a surface. By a comparison with QCD, a measurement of T/T_c and μ_B/T is immediate. Similarly for Q and S. SG, 2009

Out of equilibrium near CP: finite lifetime (Berdnikov, Rajagopal) and finite size (Stephanov). One more ratio of moments sufficient to check equilibrium. Also check freezeout conditions.

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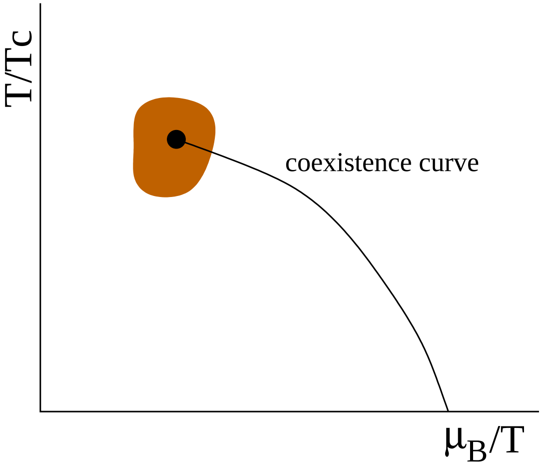

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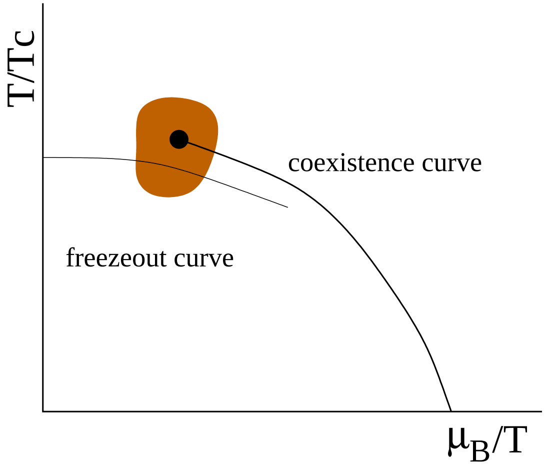

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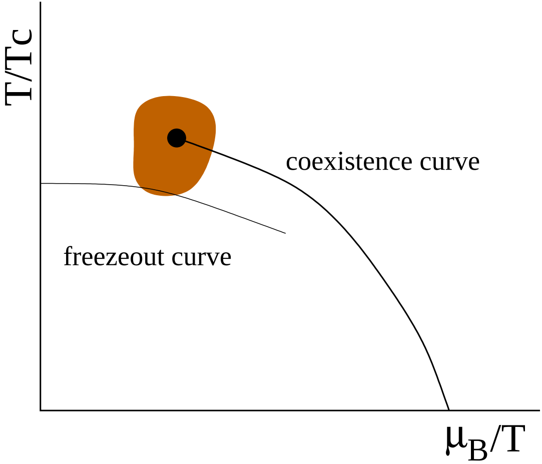

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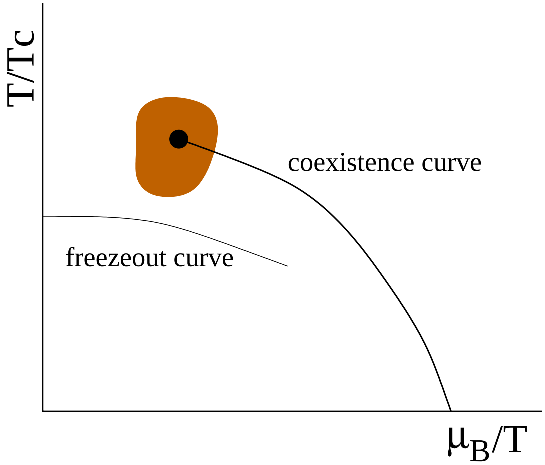


Setting the scale

One uncertainty

Different lattice computations give $T_c \simeq 190$ MeV (RBRC) or 175 MeV (BW). Maybe lower?

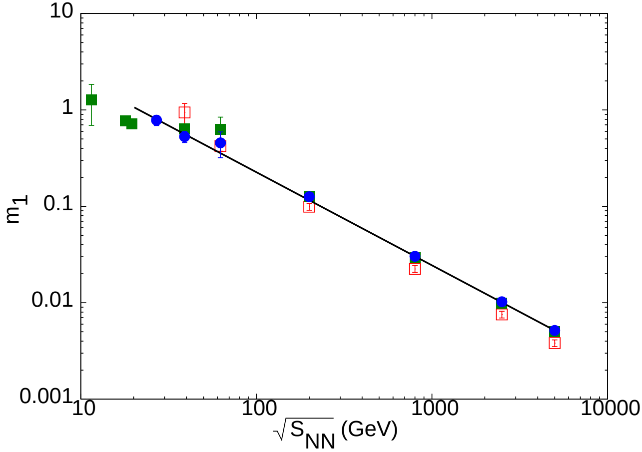
Deal with it

We present results with T and μ_B from resonance gas model for freezeout and two different scenarios for T_c. CP closer to freezeout curve when T_c lower: signals enhanced. Opens possibility of experimental measurement of T_c through experiments.

Result

Fairly robust predictions away from end point (RHIC top energy, LHC 2010 and top energies). Near end point continuum and thermodynamic limits yet to be taken. Feasibility clear (Gavai, SG 2010)

Lattice results along the freezeout curve



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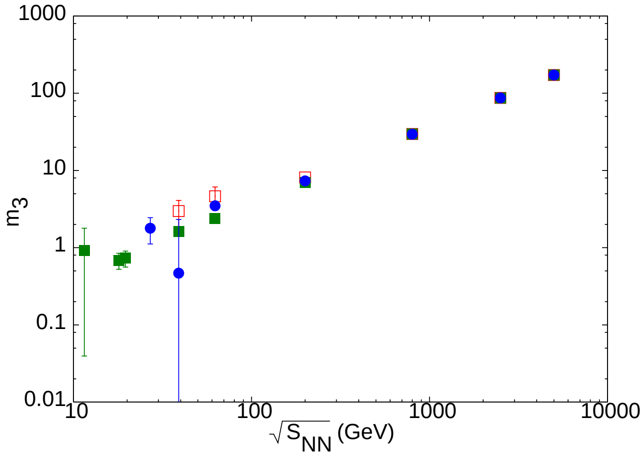

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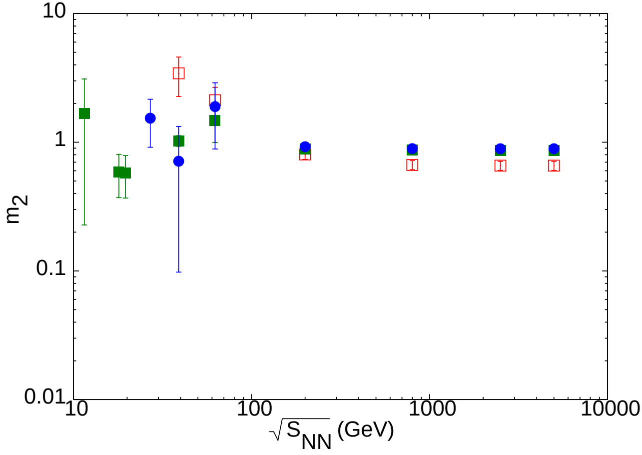

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Go to beginning