Anomaly and the QCD Critical Point: A Study in a strongly correlated system

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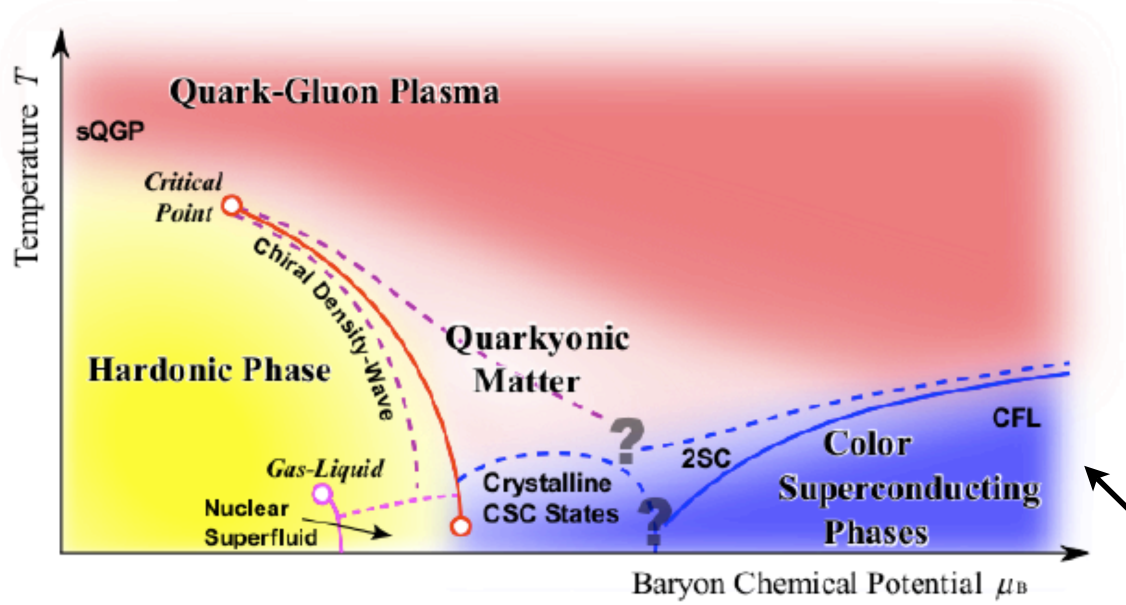
QCD Phase Diagram is Complex!

thanks to Ph. Deforcrand!

lattice QCD



model studies

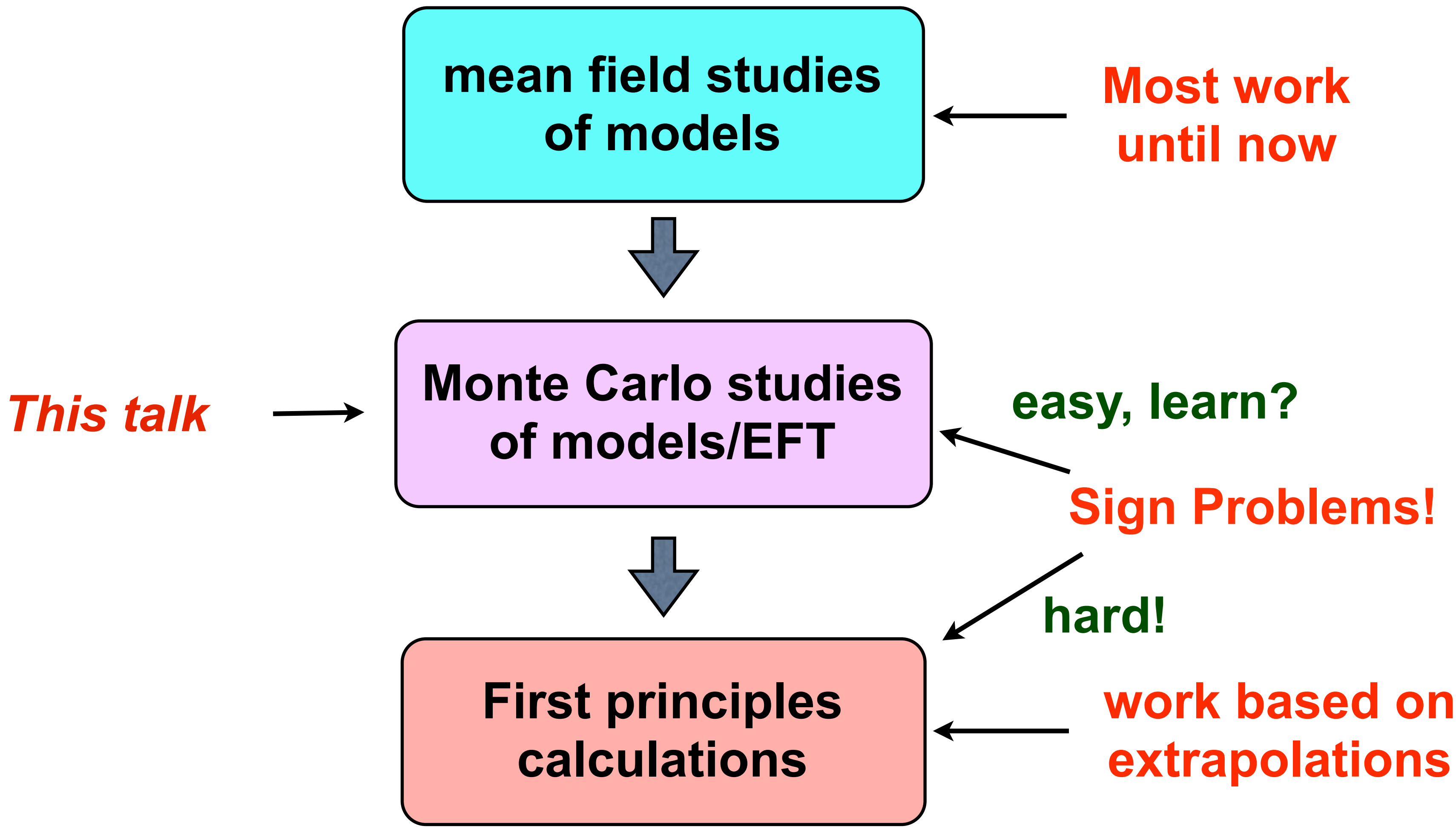
By Fukushima

perturbative QCD

Sign problem hinders progress!

A Fundamental problem for strong interaction physics in the 21st Century!

Need fresh new ideas!



Facts about the sign problem

- Sign problems depend on the “variables” used to write the partition function.
- By a solvable sign problem we mean we can find a new set of variables where the “Boltzmann weights” are all positive. (Not sufficient ??)
- In the conventional variables both Bosons and Fermions both suffer from a sign problem in the presence of a chemical potential
- The presence of non-abelian gauge fields with a chemical potential is notoriously difficult. (QCD!)

Progress over the past decade

- In the absence of gauge fields many bosonic systems "solvable" in the presence of a chemical potential. **poster by Debasish (TIFR)**

- Indeed strong coupling lattice QCD with staggered fermions have a solvable sign problem for even number of colors. (Bosons are baryons!)

  Karsch & Mutter (extension)

- In the presence of Abelian gauge fields and chemical potential many sign problem are solvable even at weak coupling in bosonic systems.

  contrast with fermions.....

- New ideas are emerging: fermion-bag approach (meron-clusters) etc. **Wenger 2008, SC 2009**

Model for a QCD-like phase diagram?

Begin with ignoring the fermionic nature of baryons

Famous example: Two color QCD
but baryon mass = pion mass (symmetry)

break the symmetry by hand!

Avoid gauge interactions
Try strong couplings!

Preserve chiral symmetries of QCD?

Study two flavor strongly coupled Z_2 lattice gauge theory!

Two-Flavor Massless QCD

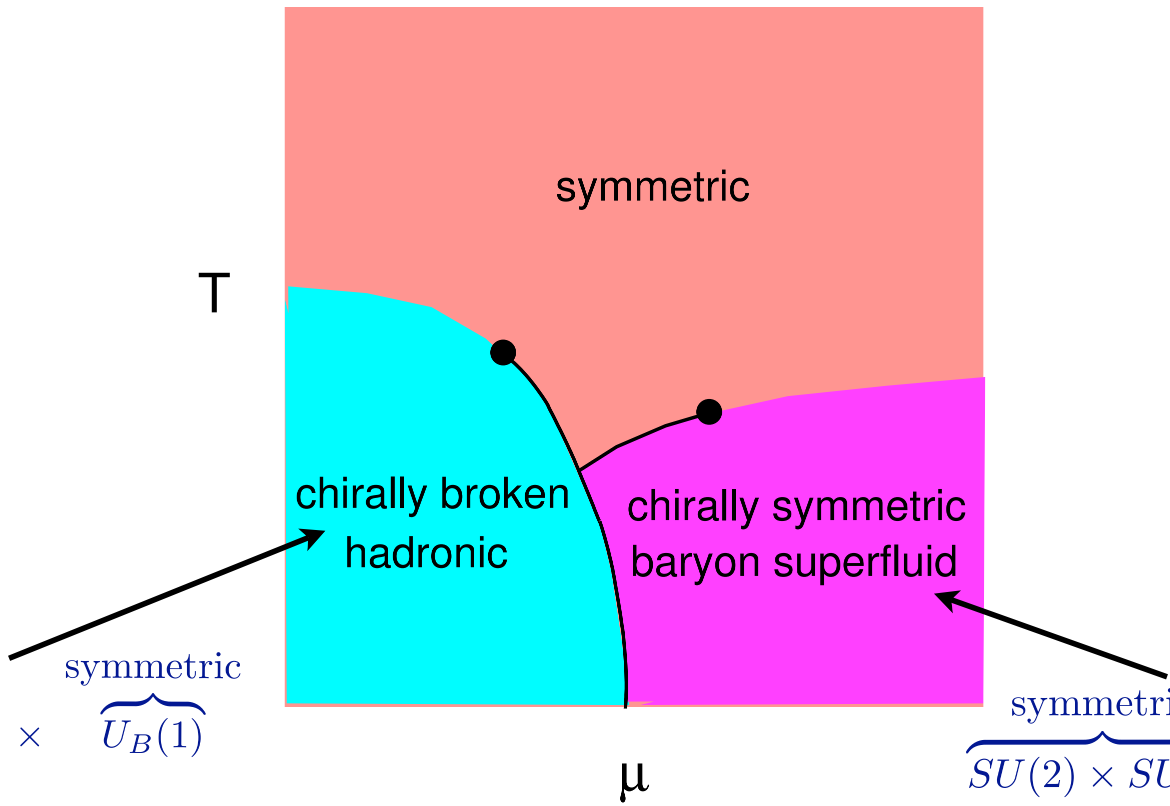
Global symmetries

$$\underbrace{SU(2) \times SU(2)}_{\text{chiral symmetry}} \quad \times \quad \underbrace{U_B(1)}_{\text{baryon number}} \quad \times \quad \underbrace{U_A(1)}_{\text{broken by anomaly}}$$

A two flavor Z_2 gauge theory with staggered fermions contains these symmetries and can be solved efficiently using the worm algorithm.

What can we expect?

The simplest (non-trivial) phase diagram



$$\underbrace{SU(2)\times SU(2)}_{\text{broken}} \times \overbrace{U_B(1)}^{\text{symmetric}}$$

$$\overbrace{SU(2)\times SU(2)}^{\text{symmetric}} \times \underbrace{U_B(1)}_{\text{broken}}$$

The location of the tri-critical points will be sensitive to the anomaly!

A strongly correlated model

Action: On a hyper cubic lattice

$$S = -\sum_{x,\alpha} t_\alpha \left[(\overline{\psi}_x \psi_{x+\alpha})(\overline{\psi}_{x+\alpha}\psi_x) + \frac{\mathrm{e}^{-m_B}}{2}\left\{ \mathrm{e}^{\mu_B \delta_{\alpha,t}} (\overline{\psi}_x \psi_{x+\alpha})^2 + \mathrm{e}^{-\mu_B \delta_{\alpha,t}} (\overline{\psi}_{x+\alpha}\psi_x)^2 \right\} \right]$$

$$-\delta \sum_{x,\alpha} \frac{(t_\alpha)^2}{2} \left\{ (\overline{\psi}_x \psi_{x+\alpha})(\overline{\psi}_{x+\alpha}\psi_x) \right\}^2 - \frac{c}{2} \sum_x (\overline{\psi}_x \psi_x)^2$$

$$\psi_x = \begin{pmatrix} u_x \\ d_x \end{pmatrix}, \quad \overline{\psi}_x = \begin{pmatrix} \overline{u}_x & \overline{d}_x \end{pmatrix}$$

$$\delta = \frac{\mathrm{e}^{-m_B}}{2}$$

$$t_\alpha = 1, \; \alpha \in \text{spatial}$$

$$t_\alpha = \gamma, \; \alpha \in \text{temporal}$$

Control Parameters:

- γ **Temperature (temporal hopping)**
- m_B **Baryon mass**
- μ **Baryon chemical potential**
- c **Anomaly**

Quark Fields do not carry “color” or “Dirac” index

Symmetries of the action

When c = 0 the action is invariant under:

$$SU(2) \times SU(2) \times U_B(1) \times U_A(B)$$

$$\psi_{x_e} \to \mathrm{e}^{i\theta_A + i\theta_B} L \psi_{x_e}, \qquad \psi_{x_o} \to \mathrm{e}^{-i\theta_A + i\theta_B} R \psi_{x_o},$$

$$\overline{\psi}_{x_o} \to \overline{\psi}_{x_o} L^\dagger \mathrm{e}^{-i\theta_A - i\theta_B}, \qquad \overline{\psi}_{x_e} \to \overline{\psi}_{x_e} R^\dagger \mathrm{e}^{i\theta_A - i\theta_B},$$

when c ≠ 0 the action is only invariant under

$$SU(2) \times SU(2) \times U_B(1)$$

Thus, the parameter c controls the anomaly strength

Differences with QCD

- Model is a Z_2 NOT SU(3) gauge theory
- No continuum limit only a lattice field theory model.
- Baryons are hard-core bosons NOT fermions
- Baryon mass is NOT connected with chiral symmetry breaking
- Only one type of baryon: (u + d)
- Tunable anomalous symmetry

Still interesting.....

Symmetry Breaking

Four types of conserved currents:

1. **flavored vector current (Y_V)**
2. **flavored chiral currents (Y_C)**
3. **axial current (Y_A)**
4. **baryon current (Y_B)**

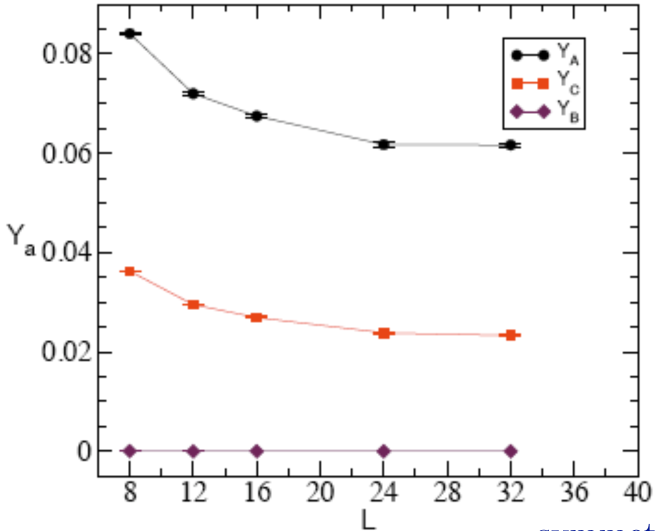
SSB studied through conserved-current correlations

$$Y_a = \frac{1}{L^3} \sum_{x,y} \left\langle J^a_\alpha(x) \;\; J^a_\alpha(y) \right\rangle$$

$$\lim_{L\to\infty} Y_a = \begin{cases} \rho_a \neq 0 & \text{Broken Phase} \\ A \exp(-aL) & \text{Symmetric Phase} \end{cases}$$

Results: L^3 x 4, m_B = 0.1, c = 0, μ = 0

$$\gamma < \gamma_c$$



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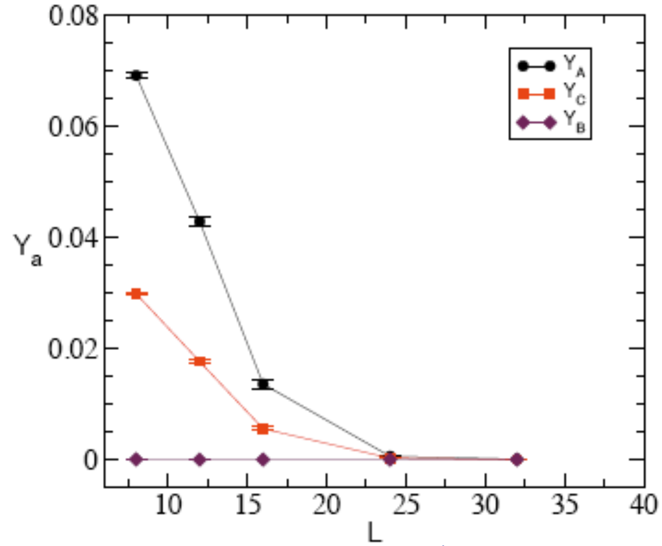
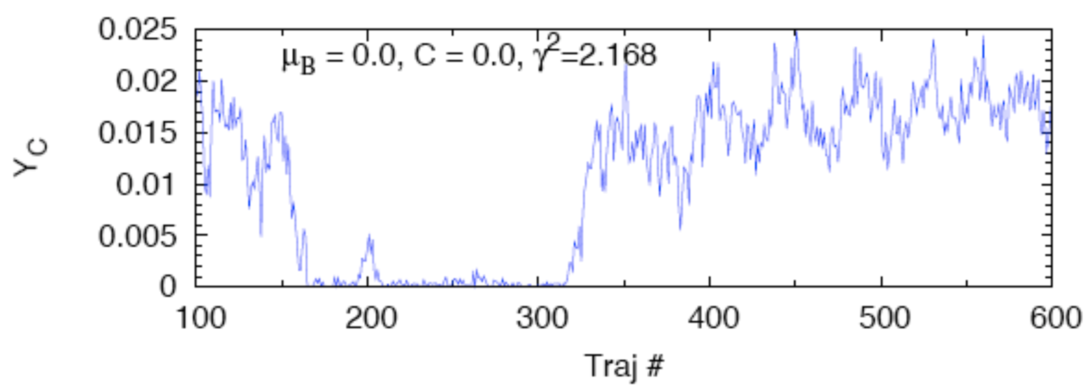


$$\overbrace{SU(2) \times SU(2) \times U_A(1) \times U_B(1)}^{\text{symmetric}}$$

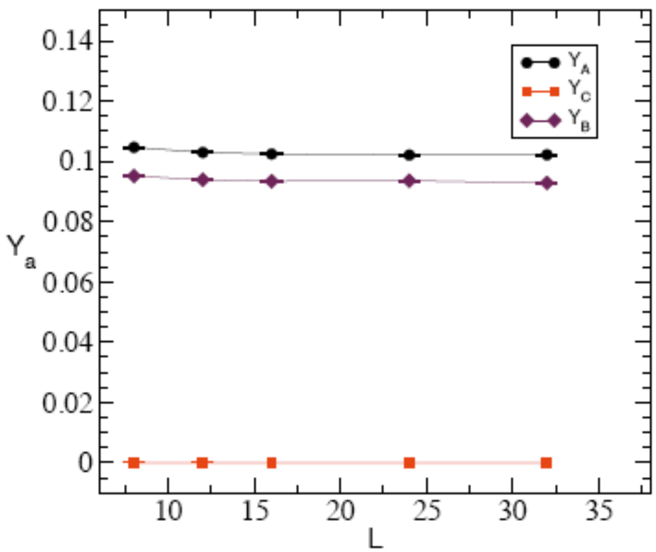
two state signal

$$\gamma = \gamma_c$$



Results: L^3 x 4, m_B = 0.1, c = 0, μ = 0.8

$\gamma < \gamma_c$



$$\overbrace{SU(2) \times SU(2)}^{\text{symmetric}} \times \underbrace{U_A(1) \times U_B(1)}_{\text{broken}}$$

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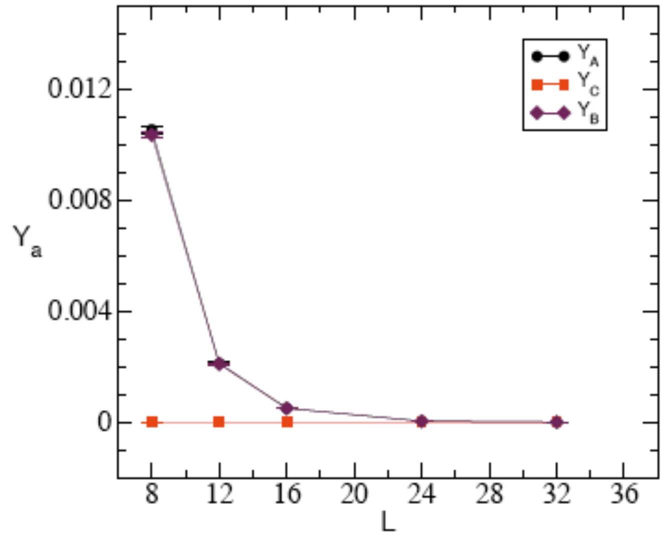

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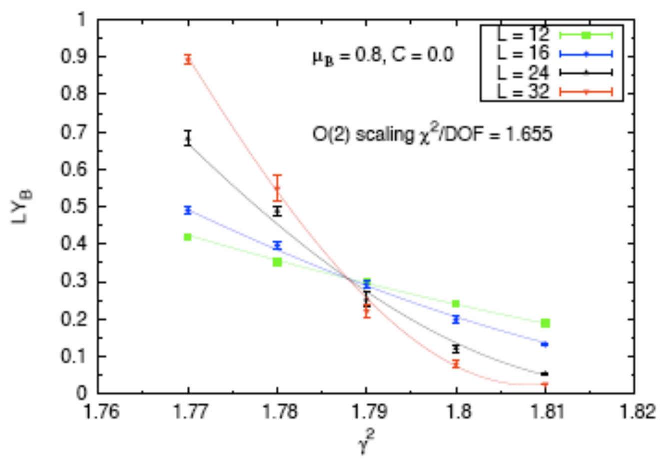
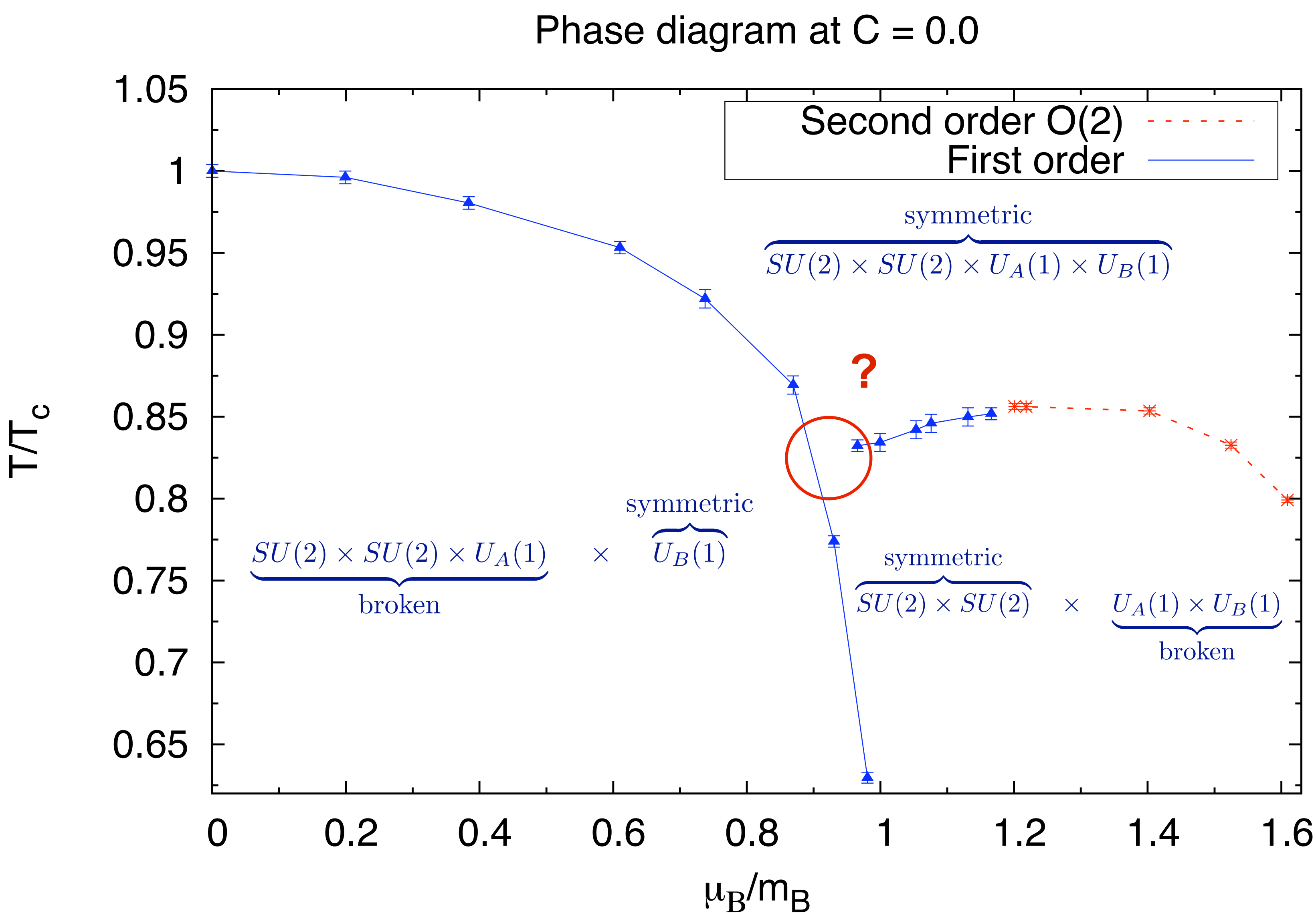
O(2) scaling

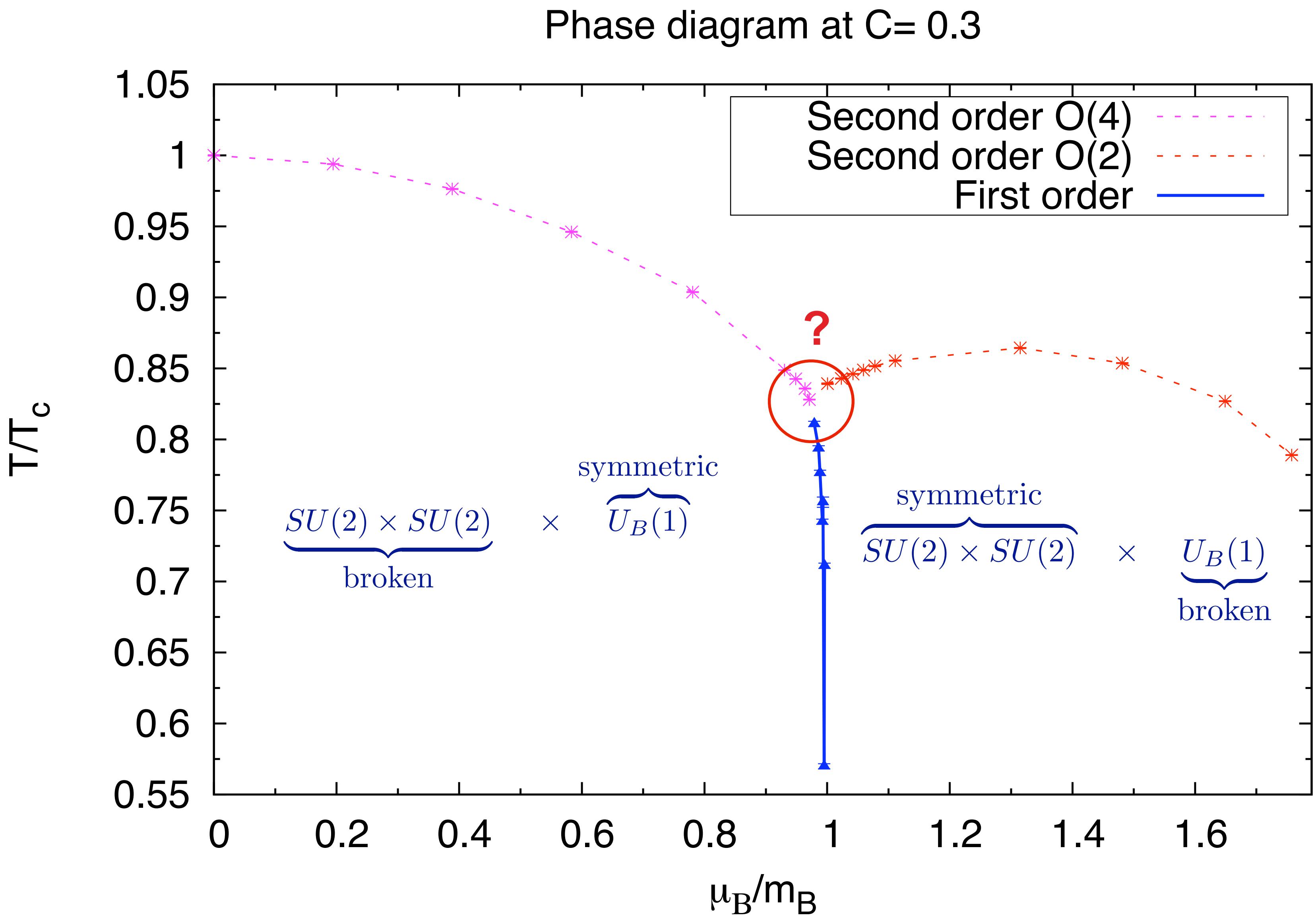
$\gamma = \gamma_c$



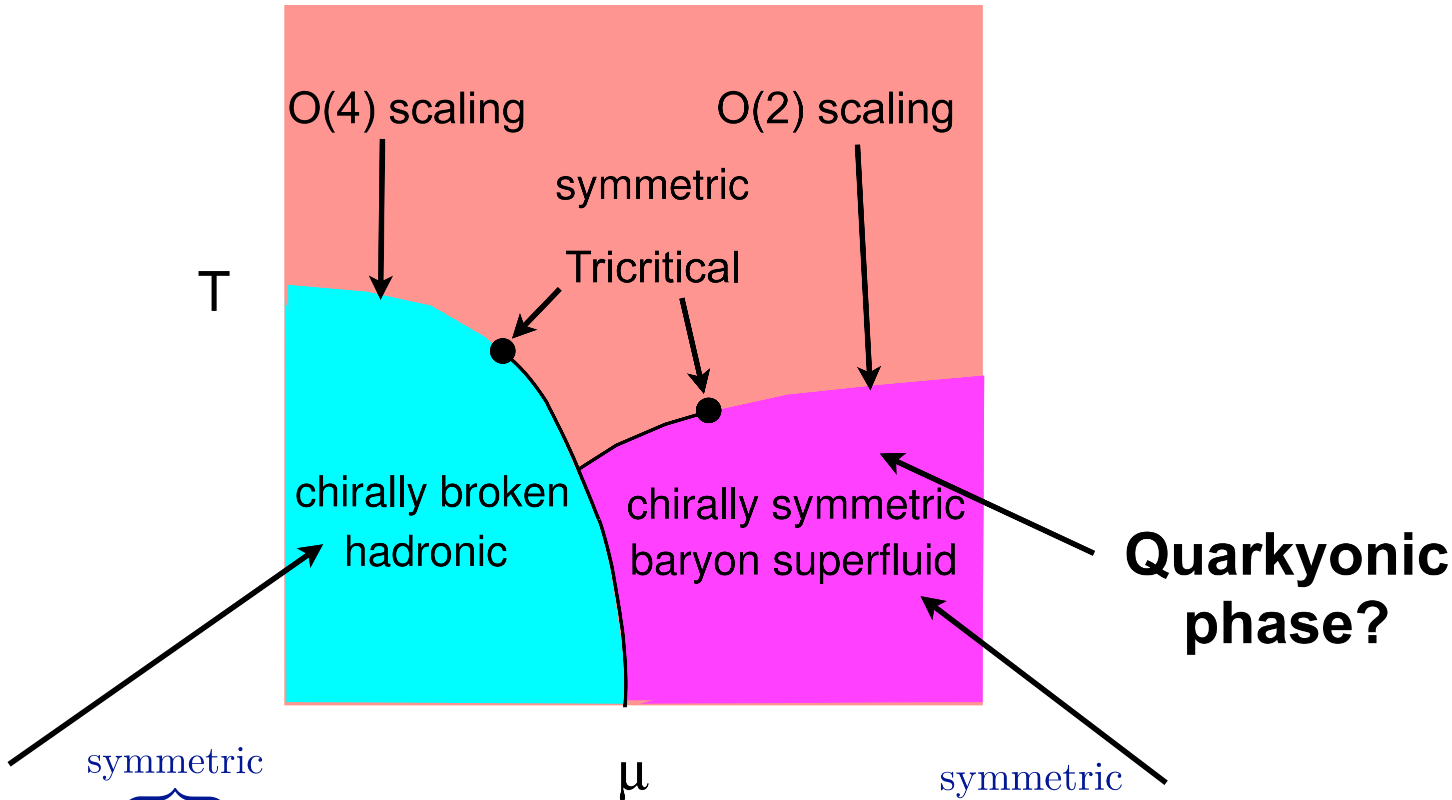
Anomaly switched off



Anomaly Switched on



Phase Diagram for "small" anomaly



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Anomaly moves the "tri-critical point" according to the conventional wisdom

Conclusions

- Possible to study T-μ phase diagrams in models with global symmetries similar to QCD
- A “Chirally symmetric”, “Baryon Superfluid” phase can exist.
  - Quarkyonic phase?
- The strength of the anomaly moves the critical point according to “conventional” wisdom.
- The axial symmetry and chiral symmetry can be disentangled at high baryon density.
  - interesting consequences for large N_c?

Two-color and Two-flavor QCD could be more interesting!

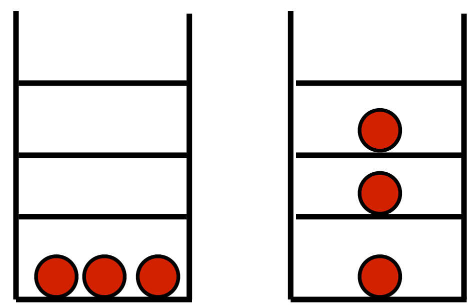
Origin of the sign problem in QCD

Sign problems arise because we sample the "wrong" partition function

other names:
overlap problem
signal to noise ratio

$$\left\langle \text{Sign} \right\rangle = \frac{Z}{Z_b} = e^{-(F-F_b)}$$

Example:
Sign problem due to "fermionic" nature of particles



bosons fermions

$E_f > E_b$

In QCD both "gauge fields" and "fermionic baryons" are responsible for the sign problem!

Fermionic Z_3 gauge theory (unsolvable!)

$$S = -\sum_{x,\alpha} \eta_{x,\alpha} \left[\overline{\psi}_{x+\alpha} z_{x,\alpha} \mathrm{e}^{\mu\delta_{\alpha,t}} \psi_x - \overline{\psi}_x z^*_{x,\alpha} \mathrm{e}^{-\mu\delta_{\alpha,t}} \psi_{x+\alpha} \right] - S_g([z])$$

Bosonic Z_3 gauge theory (solvable!)

$$S = -\beta \sum_{x,\alpha} \left[\overline{\Phi}^*_{x+\alpha} z_{x,\alpha} \mathrm{e}^{\mu\delta_{\alpha,t}} \Phi_x + \Phi^*_x z^*_{x,\alpha} \mathrm{e}^{-\mu\delta_{\alpha,t}} \Phi_{x+\alpha} \right] - S_g([z])$$

Generic phase diagram with a small non-zero anomaly