Super-universality in QCD-String Theories

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Introduction

- String-like defects or solitons occur in a wide variety of physical systems. Some well-known examples are vortices in superfluids, the Nielsen-Olesen vortices of quantum field theories, vortices in Bose-Einstein condensates and QCD-strings.
- Under suitable conditions these objects can behave quantum-mechanically. The challenge then is to find consistent quantum mechanical descriptions in *arbitrary* dimensions.
- A pragmatic approach to would be to treat these objects in some effective manner much as interactions of pions are so succesfully described in terms of chiral effective field theories.
- Two such approaches to effective string theories exist in the literature. One due to Lüscher and collaborators is entirely in terms of the $D-2$ *transverse* physical degrees of freedom.

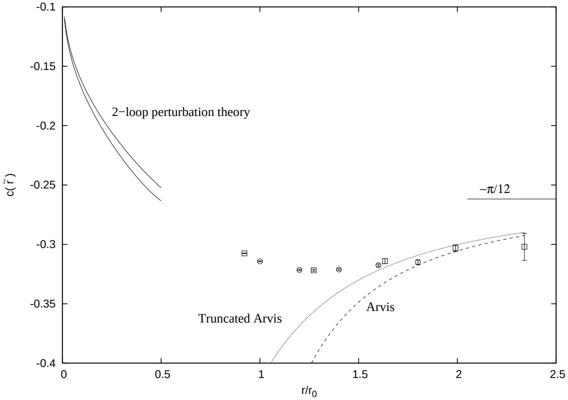
Introduction

- The other approach is the one pioneered by Polchinski and Strominger where the theories are invariant under *conformal transformations* and the physical states are obtained by requiring that the generators of conformal transformations annihilate them.
- Lüscher showed that the leading correction to ground state energy of a (closed)string of length $2\pi R$ takes the form $V(R) = \sigma R - \frac{(D-2)\pi}{24}$.
- Polchinski and Strominger showed, through explicit construction of an approximately conformally invariant action for effective string theories that not only can string-like defects be quantized in arbitrary dimensions but also that the leading correction is the Lüscher term.
- Very recently Lüscher and Weisz, using their path-breaking multilevel algorithm, showed clear numerical evidence for this term in Lattice QCD.

Introduction

- Subsequent large scale numerical simulations by Hari Dass and Majumdar on KABRU pointed to the strong possibility that even the subleading R^{-3} terms in $D = 3$ and $D = 4$ were universal and what is more, coincided with similar terms of Nambu-Goto theory .
- One of the cherished hopes is that non-perturbative QCD calculations can be done in the framework of an effective string description.

Introduction



Does this make sense?

- It is immediately obvious that the string discovered through numerical simulations of QCD can not be the fundamental Bosonic String!
- The fundamental bosonic string lives in 26 dimensions while QCD is a fully consistent theory in 4 dimensions!
- The bosonic string suffers from tachyonic instability, whereas the ground state of the static quark-antiquark sector should be stable and show no such instability..
- Polchinski-Strominger effective string theory solves this by providing a framework valid in all dimensions. It is based on Polyakov's construction of subcritical string theory.

- This rather unexpected result was analytically explained by Drummond and, by Hari Dass and Matlock, using the PS-theory.
- Focus then shifted to finding ways of understanding even higher order corrections, and possibly an analysis to all orders.
- First result we obtained in this direction was a 'proof' that the action that Polchinski and Strominger used, extended to be *exactly* conformally invariant *to all orders* , has the same spectrum as the Nambu-Goto theory to all orders.
- We had called this extended action Polyakov-Liouville Type.
- The main ingredients in this proof was a calculation of the Stress Tensor to all orders and an ansatz for the oscillator algebra that would yield the correct Virasoro Algebra.
- It had also assumed that the string momentum is the same as in the free theory.

Introduction

- Drummond had shown that the next level at which candidate actions could be found was only at R^{-6}.
- However, he had not identified the conformal transformations leaving invariant his actions, four in number, which we have called *Drummond Actions*.
- With the help of a covariant formalism developed by Peter and myself, we had shown that only two of these are linearly independent and we had identified their invariance transformation laws .
- In the next all order result, along the same lines as what we had done for the Polyakov-Liouville action, we had shown that effective string actions with these Drummond terms also do not change the spectrum! .
- I have finally extended the proof, again along the same lines, to the spectrum of *all classes* of PS effective string theories to all orders.

Polchinski-Strominger Effective String Theories

- The PS prescription is to write all action terms that are invariant under conformal transformations.
- Drop all terms proportional to the leading order constraints $\partial_\pm X \cdot \partial_\pm X = 0$.
- Drop all terms proportional to the leading order equations of motion $\partial_{+-} X^\mu = 0$.
- Leading order is in the following sense:

$$X^\mu_{\text{cl}} = e^\mu_+ R\tau^+ + e^\mu_- R\tau^-; \tag{1}$$

where $e_-^2 = e_+^2 = 0$ and $e_+ \cdot e_- = -1/2$ satisfies the full EOM.

- Fluctuations around the classical solution are denoted by Y^μ, so that

$$X^\mu = X^\mu_{\text{cl}} + Y^\mu. \tag{2}$$

Polchinski-Strominger Action

- $$S = \frac{1}{4\pi}\int d\tau^+ d\tau^- \left\{ \frac{1}{a^2}\partial_+ X^\mu \partial_- X_\mu + \beta \frac{\partial_+^2 X \cdot \partial_- X \partial_+ X \cdot \partial_-^2 X}{(\partial_+ X \cdot \partial_- X)^2} + \mathcal{O}(R^{-3}) \right\}. \quad (3)$$

- This action is invariant, i.e $\delta S < \mathcal{O}(R^{-2})$, under the modified conformal transformations

$$\delta_- X^\mu = \epsilon^-(\tau^-)\partial_- X^\mu - \frac{\beta a^2}{2}\partial_-^2 \epsilon^-(\tau^-)\frac{\partial_+ X^\mu}{\partial_+ X \cdot \partial_- X}, \quad (4)$$

- (and another; $\delta_+ X$ with $+$ and $-$ interchanged).

Consistency in all dimensions.

- The energy-momentum tensor in terms of the fluctuation field is

$$T_{--} = -\frac{R}{a^2} e.\partial_- Y - \frac{1}{2a^2} \partial_- Y \cdot \partial_- Y - \frac{\beta}{R} e_+ \cdot \partial_-^3 Y + \ldots \quad (5)$$

- The Operator Product Expansion(OPE) of $T_{--}(\tau^-)T_{--}(0)$ is given by

$$\frac{D + 12\beta}{2(\tau^-)^4} + \frac{2}{(\tau^-)^2} T_{--} + \frac{1}{\tau^-} \partial_- T_{--} + \mathcal{O}(R^{-1}). \quad (6)$$

Consistency in all dimensions.

- In the absence of the PS action the OPE would have given D as the matter central charge.
- In order to cancel the central charge -26 from gauge fixing D would have to be 26.
- But now the special value $\beta = \beta_c$ with $\beta_c = \frac{26-D}{12}$ cancels the gauge fixing central charge for all values of D!

Spectrum of PS Theory.

- Making the mode expansion $\partial_- Y^\mu = a \sum_{m=-\infty}^{\infty} \alpha_m^\mu e^{-im\tau^-}$
- The Virasoro generators are given by

$$\begin{aligned} L_n &= \frac{R}{a} e_- \cdot \alpha_n + \frac{1}{2} \sum_{m=-\infty}^{\infty} : \alpha_{n-m} \cdot \alpha_m : \\ &+ \frac{\beta_c}{2} \delta_n - \frac{a \beta_c n^2}{R} e_+ \cdot \alpha_n + \mathcal{O}(R^{-2}). \end{aligned} \tag{7}$$

- The oscillator algebra that yields Virasoro algebra is

$$[\alpha_m^\mu, \alpha_n^\nu] = m \eta^{\mu\nu} \delta_{m,-n}$$

- The quantum ground state is $|k, k; 0\rangle$ which is also an eigenstate of α_0^μ and $\tilde{\alpha}_0^\mu$ with common eigenvalue ak^μ. This state is annihilated by all α_n^μ for positive-definite n.

Spectrum of PS Theory.

- The ground state momentum is

$$p^{\mu}_{\text{gnd}} = \frac{R}{2a^2}(e^{\mu}_{+} + e^{\mu}_{-}) + k^{\mu}$$

- The total rest energy is

$$(-p^2)^{1/2} = \sqrt{\left(\frac{R}{2a^2}\right)^2 - k^2 - \frac{R}{a^2}(e_+ + e_-)\cdot k}. \qquad (8)$$

- The physical state conditions $L_0 = \tilde{L}_0 = 1$ fix k, so that

$$k^1 = 0, \qquad k^2 + \frac{R}{a^2}(e_+ + e_-)\cdot k = \frac{(2-\beta_c)}{a^2}. \qquad (9)$$

Spectrum of PS Theory.

$$(-p^2)^{1/2} = \frac{R}{2a^2}\sqrt{1 - \frac{D-2}{12}\left(\frac{2a}{R}\right)^2}, \qquad (10)$$

- This is the precise analog of the result obtained by Arvis for open strings.
- Expanding this and keeping only the first correction, one obtains for the static potential

$$V(r) = \frac{R}{2a^2} - \frac{D-2}{12}\frac{1}{R} + \cdots . \qquad (11)$$

Absence of R^{-3} corrections to the Arvis spectrum.

- It turns out that both the action and transformation law given by PS hold to order R^{-3}.
- Peter Matlock and I, and Drummond, have shown that there is no correction to the Nambu-Goto spectrum at the R^{-3} level also!
- This requires showing first that there are no candidate action terms whose leading behaviour is R^{-3}.
- Hence to investigate corrections to the spectrum at R^{-3} it suffices to work with the PS action.
- The result is that
$$\Delta L_m = m^2 X$$
- The oscillator algebra that yields the Virasoro algebra is also unchanged from what was used by PS.

Investigating Higher Order Terms in $V(R)$

- Numerical data shows clear departure from the Arvis potential.
- This raises the question of going beyond the R^{-3} corrections.
- The original PS prescription amounted to specifying an action and determining the appropriate transformation laws.
- This, as evidenced in the early days of Supergravity theories, is clumsy and unwieldy.
- A better approach is desirable. To see one way of achieving this let us return to our earlier comment about the Liouville theory.

Hints from PolyakovTheory

- The starting point is the *Liouville Action*

$$S_{Liou} = \frac{26-D}{48\pi} \int d\tau^+ \, d\tau^- \;\; \partial_+\phi\partial_-\phi \qquad (12)$$

- Accrding to PS, we are to replace the conformal factor e^ϕ by the component $\partial_+ X \cdot \partial_- X$ of the induced metric on the world sheet, and replace $(26-D)/12$ by a parameter β.
- A straightforward transcription of this idea would have suggested the total action

$$\begin{aligned} S_{(2)} &= \frac{1}{4\pi}\int \mathrm{d}\tau^+\mathrm{d}\tau^- \left\{ \frac{1}{a^2}\partial_+X^\mu\partial_-X_\mu \right. \\ &\left. + \beta\frac{\partial_+(\partial_+X\cdot\partial_-X)\partial_-(\partial_+X\cdot\partial_-X)}{(\partial_+X\cdot\partial_-X)^2} \right\}. \end{aligned} \qquad (13)$$

- Dropping leading order EOM lead to the PS action.

All order analysis of $V(R)$

- Now $S_{(2)}$ to *all orders* is conformally invariant. Thus we can use it to study all order corrections to $V(R)$.
- Such an all order result may not necessarily suffice to explain the numerical data but it can give one explicit model for higher order corrections.
- Actually Drummond had shown that there are no action terms whose leading order behaviour is R^{-4}, R^{-5}.
- Thus even for comparison with data, the R^{-4}, R^{-5} terms from the Liouville action will be *exact* as far as Effective String Theories are concerned.

All Order Analysis...

- The main result is the on-shell T_{--}:

$$T^{\beta}_{--}|_{hol} = \frac{\beta}{2}\{2\frac{\partial_{--}L_h}{L_h} - 3\frac{(\partial_{-}L_h)^2}{L_h^2}\}$$

where

$$L_h = -\frac{R^2}{2} + Re_{+} \cdot \partial_{-}G$$

- It should be noted that the T_{--} derived only involves e_{+} and various $-$ derivatives of G_{-}.

All Order Analysis: A field redefinition

- This T_{--}, by the following holomorphic field redefinition

$$G_-{}' = G_- + \frac{\beta a^2}{2} R e_+ \partial_{--} L_h^{-1}$$

can be transformed to

$$\bar{T}_{--} = const.\partial_{--} \log\{1 - \frac{2}{R} e_+ \cdot G_-\}$$

- This field redefinition does not have any factor ordering problems.
- Hence the classical proof of equivalence of theories under field redefinitions can be carried over.
- But the second form has vanishing L_0 and thus the spectrum to all orders is the same as that of Nambu-Goto theory.

- Thus the all order candidate conformal effective string theory does not give any corrections to bring about agreement with numerical simulations.
- This is the super-universality alluded to in the title.
- To make a complete analysis we need to have a systematic procedure to construct all effective actions invariant under

$$\delta X^\mu = -\epsilon^- \partial_- X^\mu$$

- Towards this end we have constructed two types of Covariant Calculi, one of which is based on the induced metric, and the other on a generalized Weyl-Coordinate covariance applicable to higher-derivative, nonpolynomial actions.
- Either of them can be used to construct conformally invariant effective string actions.

Analysis of all S^j_{cov}

- The only way to get a T^h_{--} is for the Lagrangean to be made up of two factors one of which, called $\mathcal{L}^h$ has holomorphic terms in it and another, called $\mathcal{L}^{hvar}$ whose *Nöether variation* has holomorphic parts in it.
- Further, their product must be such that there are equal number of + and - indices.
- A detailed analysis shows that this is not possible.
- Hence, every S^j_{cov} has a vanishing on shell T_{--}.

The issue of String Momentum

- Our contention that the spectrum of all conformally invariant effective string theories are the same as that of Nambu-Goto theory is based on taking the space-time momentum of string to be

$$p^\mu = \frac{R}{2a^2}(e_+^\mu + e_-^\mu) + k^\mu$$

- This is certainly true of Nambu-Goto theory.
- Due to the higher derivative actions the string momentum density is certainly modified.
- A field theory analysis, which unfortunately is not as powerful as CFT, has shown that at least to order R^{-3} the total momentum is indeed uncorrected.
- Extension to higher orders, which is very tedious, is under way.

The issue of oscillator algebra

- In our work we had found an oscillator algebra that consistently reproduces the correct Virasoro Algebra at all orders.
- A more systematic way is to derive these starting from the basic canonical commutation relations of the underlying field theory.
- Due to the higher-derivative and non-polynomial nature of these field theories, this analysis is quite involved.
- This work is also under way.

Conclusions

- Modulo these two caveats, we have shown that the entire class of conformally invariant Polchinski-Strominger effective string theories has the same spectrum as Nambu-Goto theory to all orders in R^{-1}.
- An immediate comparison can be made with the recent results of Aharony and Karzbrun who, following the Lüscher-Weisz approach, showed similar results to order R^{-5} in $D = 3$ but claimed that for $D \geq 4$ there could be corrections at the R^{-5} level.
- This discrepancy needs to be understood.
- It may mean that the *symmetry content* of effective string theories needs a fresh look. First principle derivation of the effective actions as done by Akhmedov et al could throw valuable light on these issues .

Conclusions

- Numerical data clearly shows a deviation from Nambu-Goto theory at intermediate distances. This is so with simulations of percolation models as well .
- One possibility of reconciling this is if at these intermediate scales the string-like object has not formed at all.
- If not, one will have to conclude that conformally invariant effective string theories do not work.
- Our analysis does not seem to provide any room for extrinsic curvature string effects. But the Polyakov action does not have the conformal invariance used here.
- What additional physics is coded by these highly non-trivial actions considered here? Studies of scattering amplitudes and Partition functions may be useful.
- What is the deeper physical understanding of our results? It is clear that we are still a long way from understanding QCD-Strings.

Thick Strings

- The flux tubes of QCD have thickness. Hence it is very important to incorporate thickness into the effective description.
- At this stage it is not clear how to provide an ab initio description of thick strings.
- An interesting idea in this regard is that of Polchinski and Susskind, who have argued that the four dimensional projection of certain thin AdS_5 strings behave like thick strings. I am investigating this line of thought with Vikram Vyas.
- Even in this approach, integrating out the radial coordinate would result in an effective description in four dimensions. It would appear that our results should be applicable here too.

A short bibliography

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