Excited string states using the multilevel algorithm.

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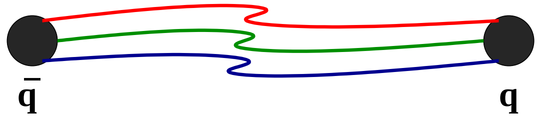
Intodaction

- Mechanism of Confinement : On the lattice there is evidence of flux tube formation.

- Conjecture : Flux tube $\equiv$ bosonic string.



- Effective theories for flux tubes (hadronic strings) .

Energy states $E_n(R) = \sigma\, R \sqrt{1 + \frac{2\pi}{\sigma\, R^2} \left(n - \frac{1}{24}(d-2)\right)}$

σ : string tension , d : # of space time dimensions

- Write down most general (series) action with vanishing conformal anomaly in any dimension. [Polchinski & Strominger]

- Write down action as a series in $1/r$ (r : length of flux tube) and impose open-closed duality. [Lüscher & Weisz]
  Open-closed duality constrains possible string spectra.
  Forbids $1/R^2$ terms in the effective string action.

- Spectrum in PS prescription is universal to $\mathcal{O}(R^{-3})$ and to this order it coincides with the NG spectrum.
  [Drummond, Hari Dass & Matlock, Kuti, Maresca]

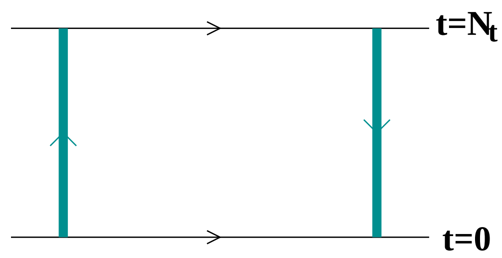
- LW formulation: To $\mathcal{O}(R^{-3})$, spectrum same as NG in 3-d.
  In 4-d one free parameter exists. [Lüscher & Weisz]

- Nambu-Goto partition function respects open-closed duality.

Observables on the lattice

- Polyakov loop correlators:
  Accurate ground state energy.

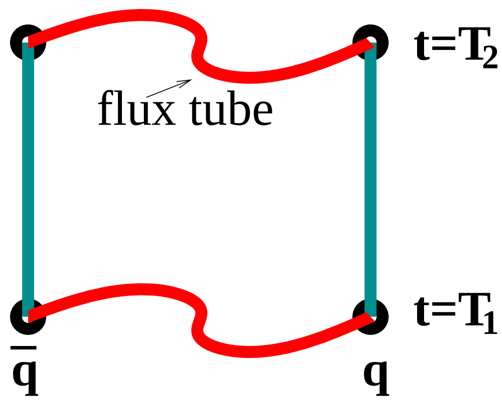
$$\langle P^\dagger P(r)\rangle = \sum_{i=0}^{\infty} b_i \, \mathrm{exp}[-E_i(r)N_t]$$



Polyakov Loop correlator

- Wilson loops :
  Suitable for excited states

$$W(r,\Delta T) = \sum_\alpha \beta_i^\alpha \beta_j^\alpha e^{-E_\alpha(r)\Delta T}$$



Wilson Loop

- The string pictures holds at large $r \Rightarrow$ large loops.

- Note that $W(r, \Delta T) \propto \exp(-r\Delta T)$

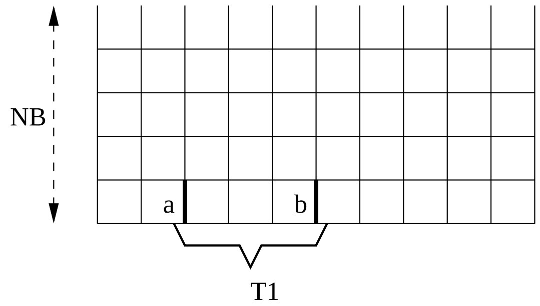
- Since we need large r, we must either work with small ΔT, or have the means to extract exponentially suppressed signals from the noise.

- 1st alternative has been followed by Kuti *et.al.* using asymmetric lattices and a very large number of basis states.

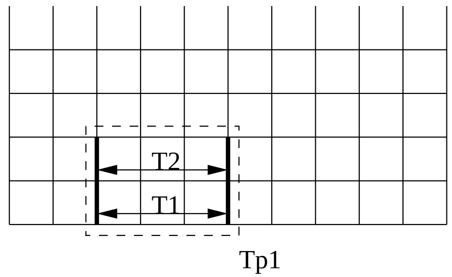
- Advances in algorithms (e.g. multilevel schemes) and computing power now allow for exponential error reduction and reliable extraction of expectation values of large Wilson loops.

Algorithm - Ground State

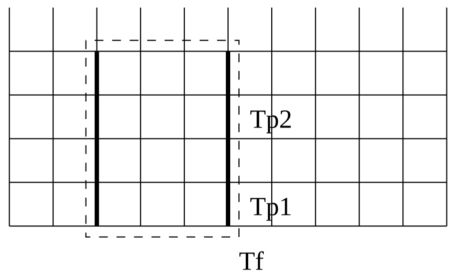
- $a \otimes b =$ T1(2,2,2,2)



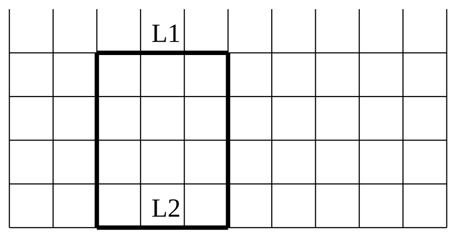
- (T1)$_{ijkl}$(T2)$_{jmln}$=(Tp1)$_{imkn}$ Averaging is carried out for Tp1.



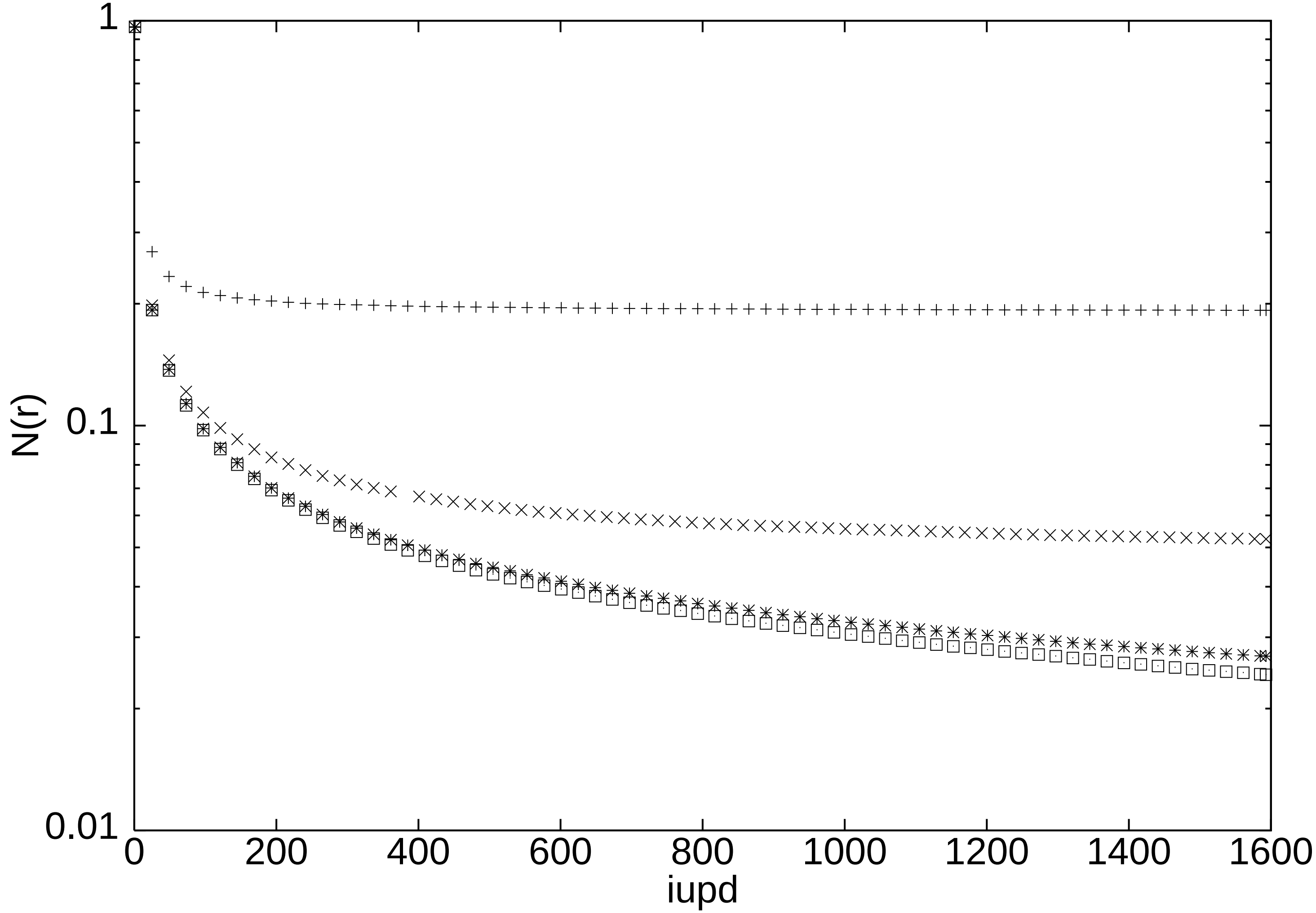
- The averaged Tp's are multiplied together to form the averaged propagator Tf.



- L1, L2 & Tf are multiplied together to produce the Wilson loop.



Important parameters of the algorithm : time slice thickness - Tp1 & the number of sublattice updates $iupd$.



2-link norm vs $iupd$ for r=2,4,6 and 8 at $\beta = 3$

Some of the applications :

1. Ground state of the flux tube.

2. Excited states of the flux tube.

3. Profile of the flux tube.

4. Breaking of the flux tube.

5. 3-quark potential.

6. Glueball spectrum in SU(3) & U(1).

7. Energy momentum tensor of the gluonic field.

Ground state of the flux tube

Potential between static $q\bar{q}$ pair: (series in r^{-n})

$$V(r) \sim \sigma r + \widehat{V} - c/r + \cdots$$

String predictions (d=3)

$$\text{L.O.}\; f(r) = \sigma + \left(\frac{\pi}{24}\right)\frac{1}{r^2} \qquad c(r) = -\frac{\pi}{24}$$

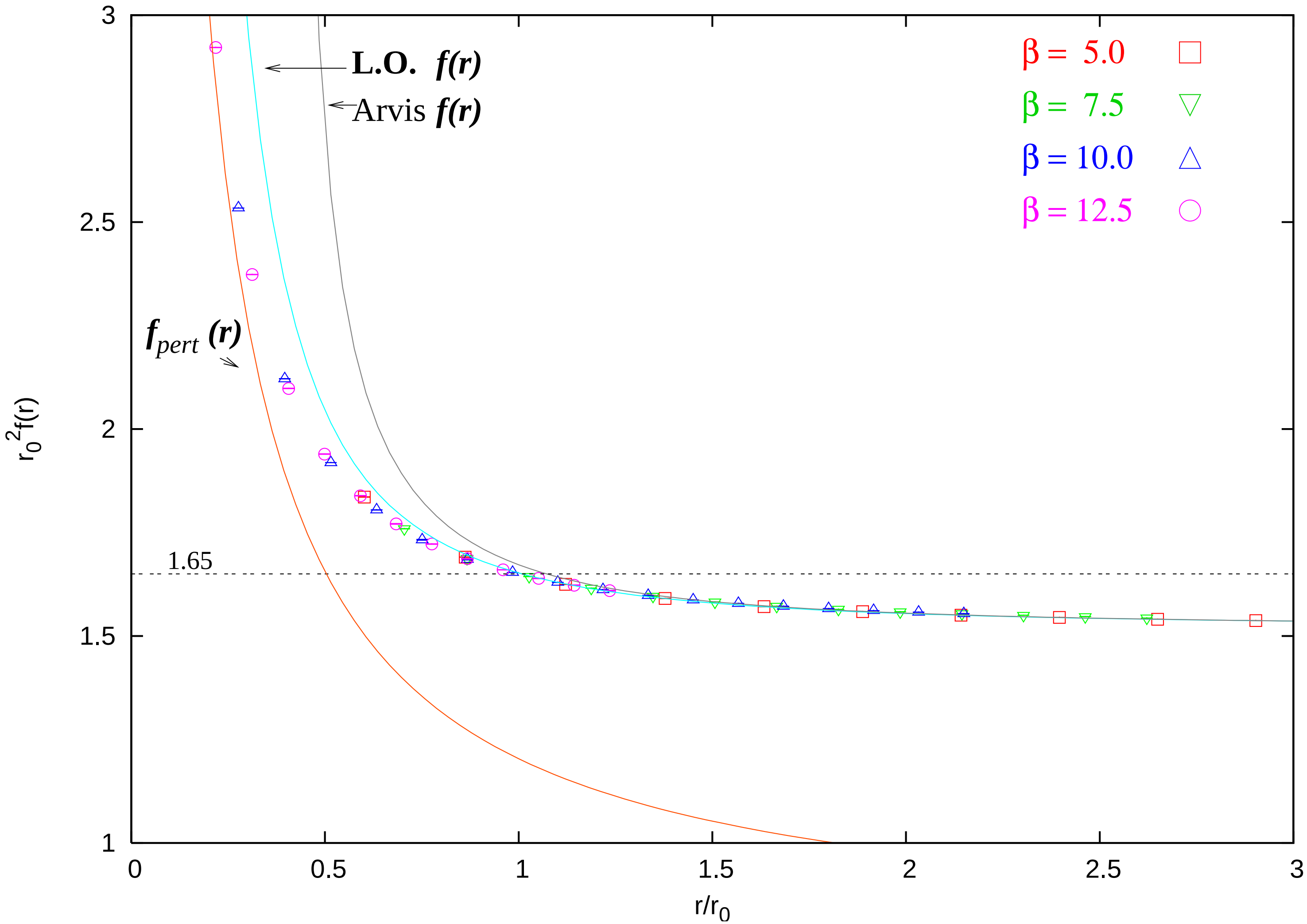
$$\text{N.L.O.}\; f(r) = \sigma + \left(\frac{\pi}{24}\right)\frac{1}{r^2} + \left(\frac{\pi}{24}\right)^2 \frac{3}{2\sigma r^4} \qquad c(r) = -\frac{\pi}{24}\left(1 + \frac{\pi}{8\sigma r^2}\right)$$

$$\text{Arvis}\; f(r) = \sigma\left(1 - \frac{\pi}{12\sigma r^2}\right)^{-1/2} \qquad c(r) = -\frac{\pi}{24}\left(1 - \frac{\pi}{12\sigma r^2}\right)^{-\frac{3}{2}}.$$

Perturbation theory

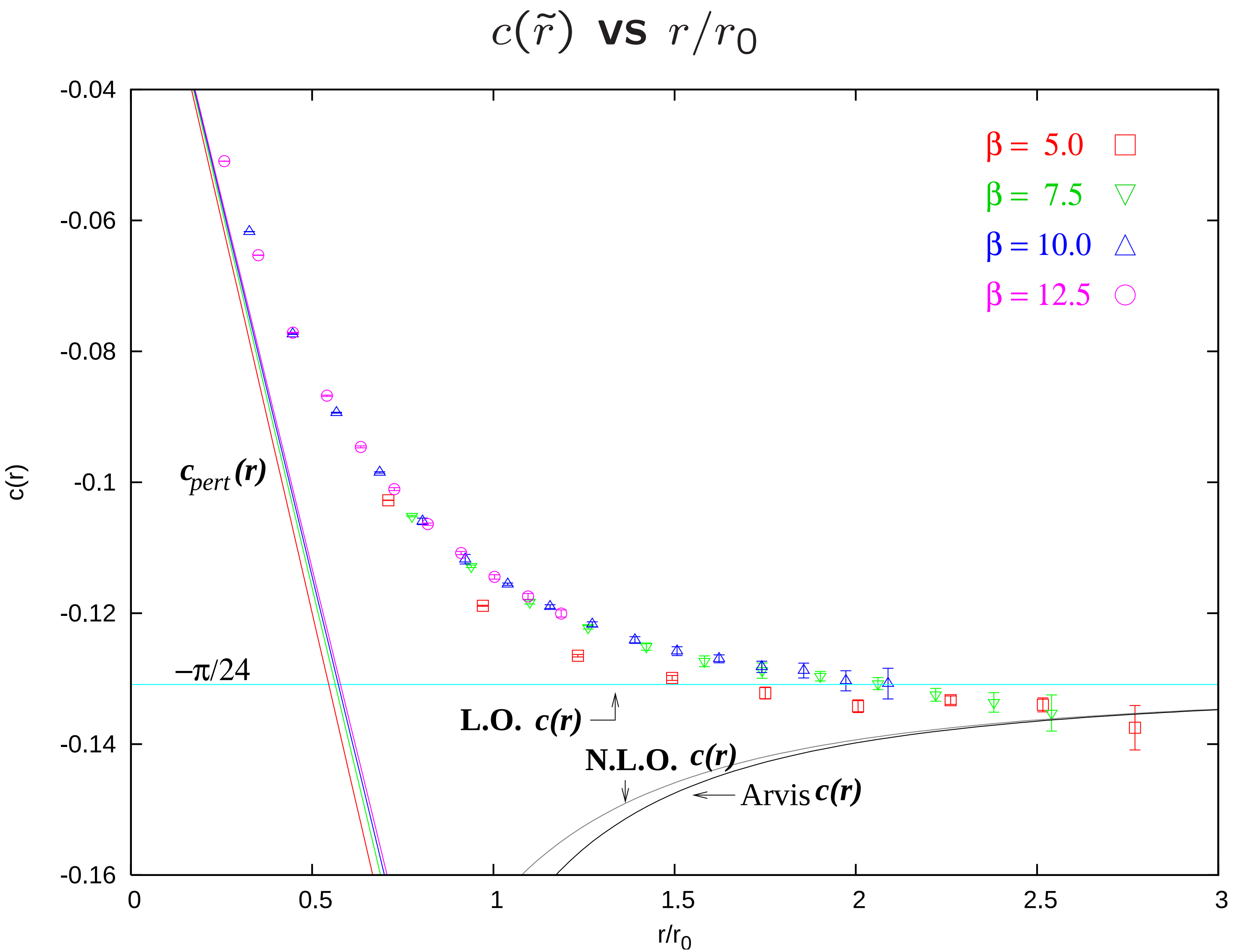
$$V_{\text{pert}}(r) = \sigma_{\text{pert}} r + \frac{g^2 C_F}{2\pi} \ln g^2 r + (\text{higher order terms}) \qquad (1)$$

$r_0^2 f(r)$ **vs** r/r_0



$f_{\mathrm{pert}}(r)$: 1-loop perturbation theory.

Dotted line : $r_0^2 f(r) = 1.65$, locates the Sommer scale.



$c_{\text{pert}}(r)$: 1-loop perturbation theory with $\beta = 12.5$ closest to data and $\beta = 5$ farthest.

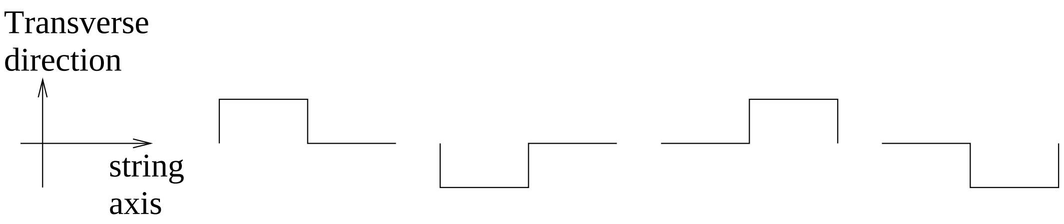
Excited states of the flux tube

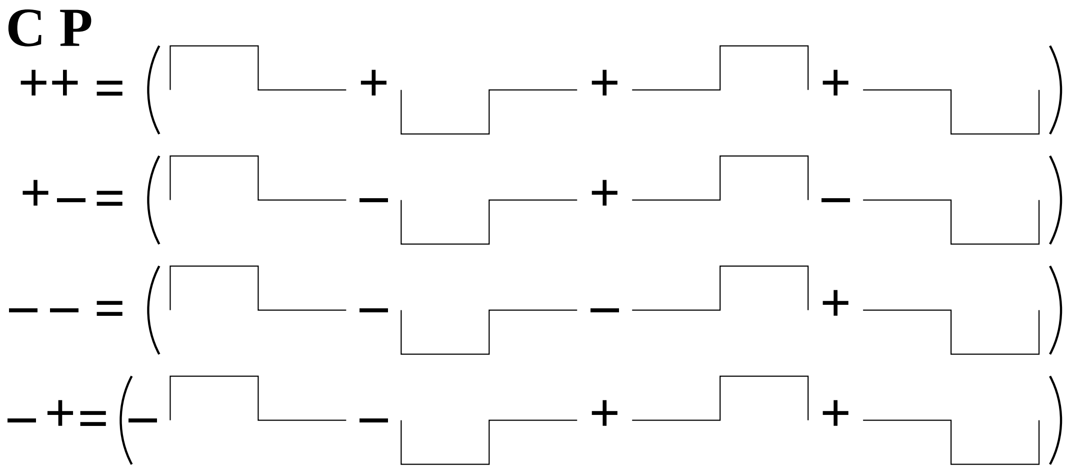
Behaviour under charge conjugation and parity – **CP**

P: Reflect in $q\bar{q}$ axis : $x(\kappa) \longrightarrow -x(\kappa)$

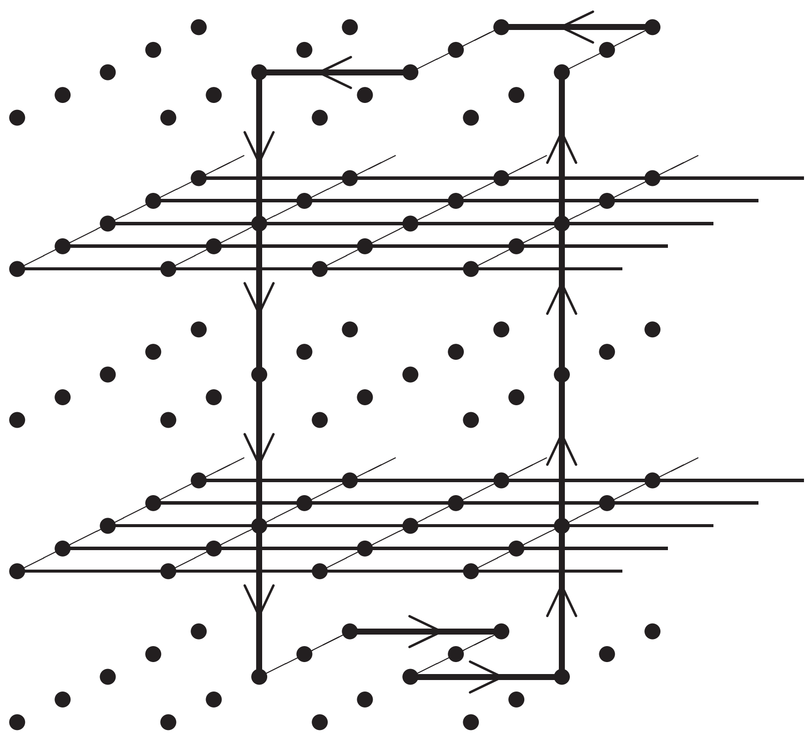
C: Interchange q and $\bar{q}$: $x(\kappa) \longrightarrow x(r-\kappa)$

Combinations $\Leftrightarrow$ symmetry channels.





Algorithm - Excited states



A wilson-loop with different sources at the ends, that lie in the middle of the time-slices. The slices with the solid lines are the time slices with fixed lines during the sublattice updates.

r	W_1 New	W_1 Old	W_2 New	W_2 Old	W_3 New	W_3 Old
4	0.44	0.15	2.7	7.0	9.2	100
5	0.63	0.21	2.7	8.3	8.6	100
6	0.86	0.28	2.7	4.5	8.8	100
7	1.1	0.35	2.9	7.3	8.8	100
8	1.4	0.45	3.1	5.5	9.5	100
9	1.7	0.56	3.6	10	11	100
10	2.1	0.74	4.2	11	14	100
11	2.7	1.0	5.8	27	22	100
12	3.5	1.7	8.6	88	44	100

Percentage errors for Wilson loops for energies E_1, E_2 and E_3.
$\beta = 5$, $T = 8$ with r varying between $4 - 8$. Time $\approx$ 1100 mins.
Old method: 730 mesurements with no source averaging.
New method: 50 mesurements with 12000 updates for source averaging.
2-link averaging was same for both methods.

Energy of the string excited states

$$\textbf{L.O.} \quad E_n = \sigma r + \mu + \frac{\pi}{r}(n - \frac{d-2}{24})$$

$$\textbf{N.L.O} \quad E_n = \sigma r + \mu + \frac{\pi}{r}(n - \frac{d-2}{24}) - \frac{\pi^2}{2\sigma r^3}(n - \frac{d-2}{24})^2$$

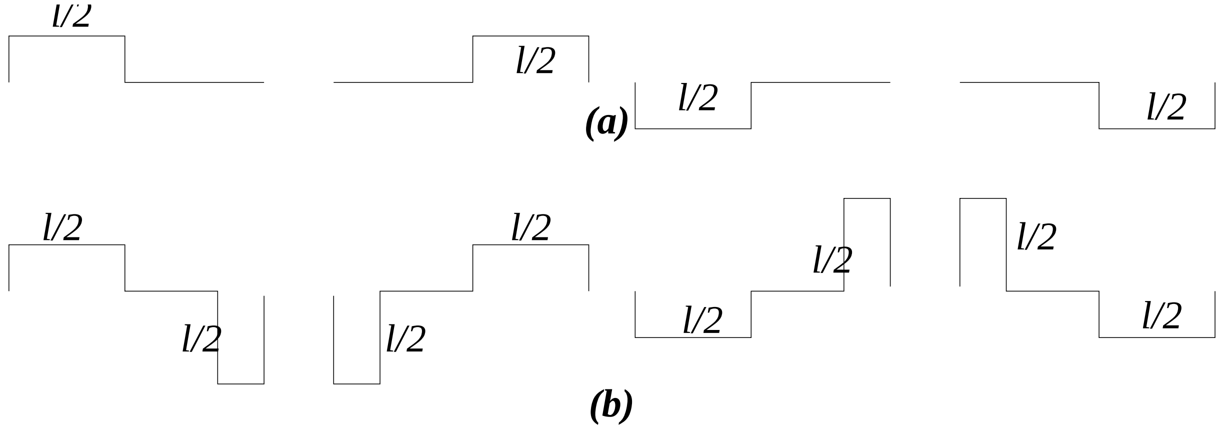
$$\text{Arvis} \quad E_n = \sigma r(1 + \frac{2\pi}{\sigma r^2}(n - \frac{d-2}{24}))^{1/2}$$

We will look mostly at the energy difference $E_n - E_m$.

Correction factors

$$W(T) = \alpha_1 e^{-ET}\left(1 + \frac{\alpha_2}{\alpha_1} e^{-\delta T}\right)$$

$$-\frac{1}{T_2 - T_1} \log \frac{W(T_2)}{W(T_1)} = \bar{E} + \frac{1}{T_2 - T_1}\left[\frac{\alpha_2}{\alpha_1} e^{-\delta T_1}\left(1 - e^{-\delta(T_2 - T_1)}\right)\right].$$



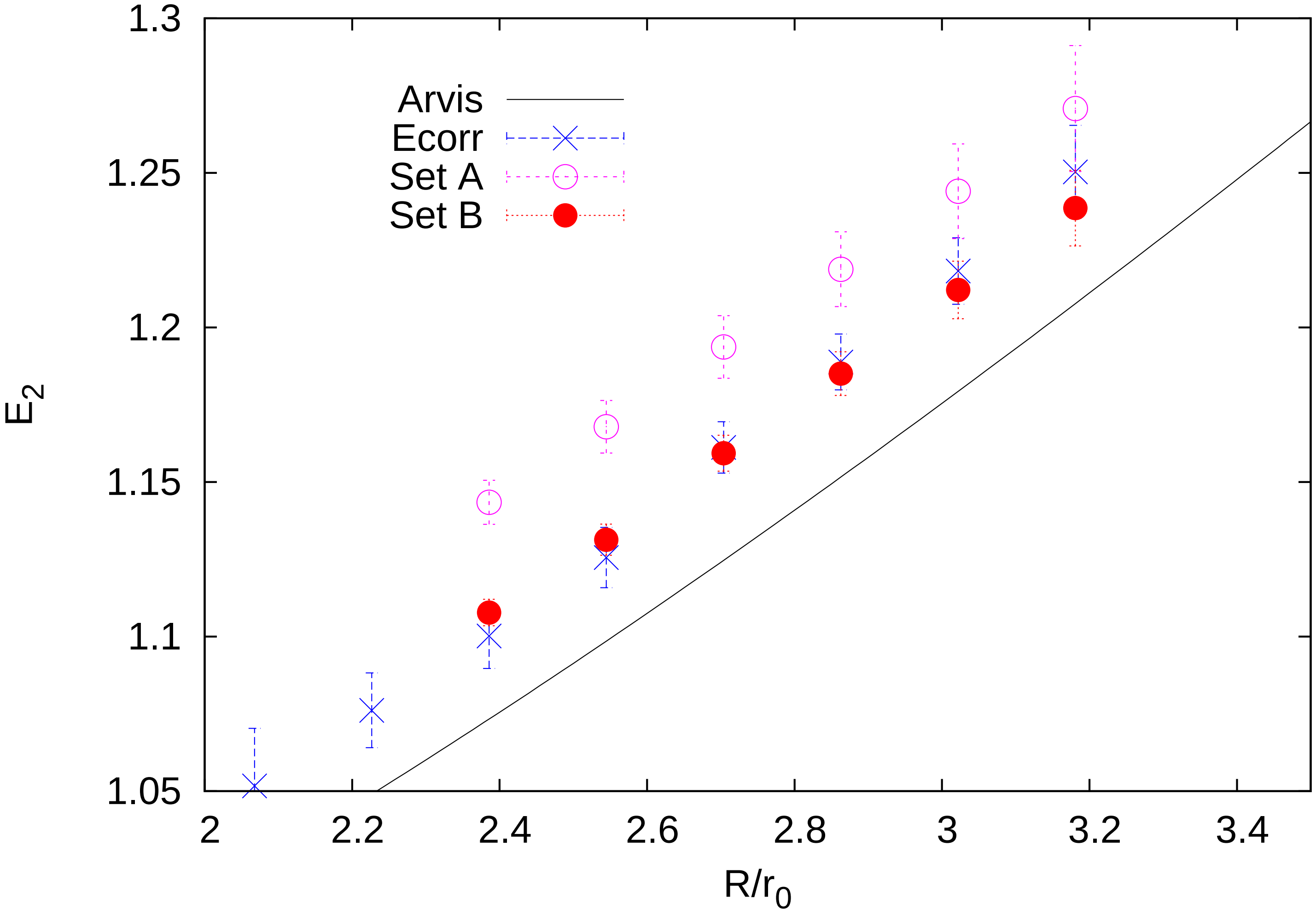
E_2 equires a better "wave function" as we approach the continuum limit. Used source (b) to couple strongly to E_2.

460 measurements with old & new wave functions at $\beta = 7.5$.

T	Lat	t_s	N_s	N_t	R	15	16	17	18	19	20
6	36^3	4	18000	1500	A	0.62	0.73	0.85	0.99	1.23	1.60
10	40^3	4	18000	2500	B	0.39	0.45	0.50	0.60	0.77	0.99

Left: Parameters of the testruns to compare the operators.

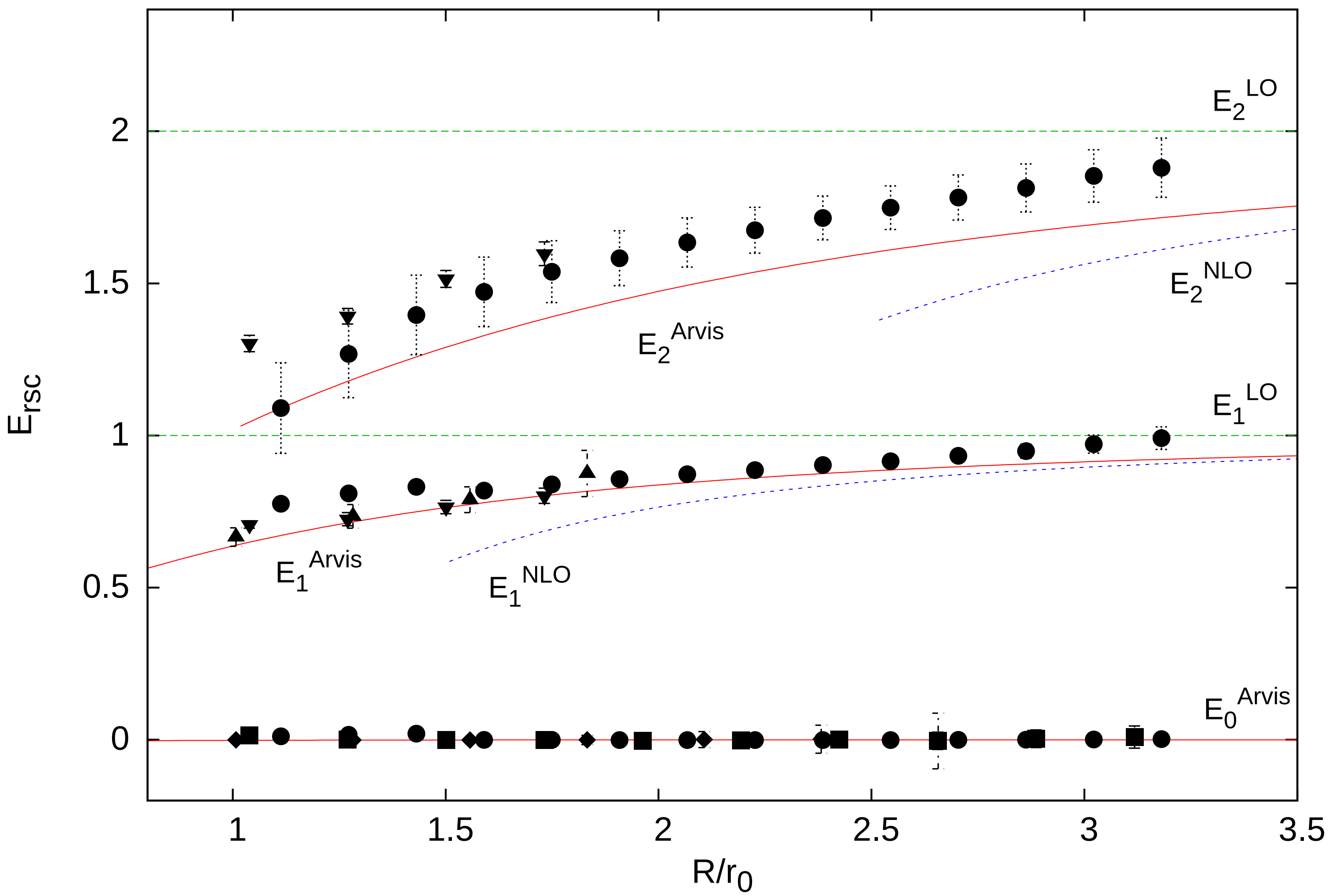
Right: Relative error of $\bar{E}_2$ in % for operator sets A and B.



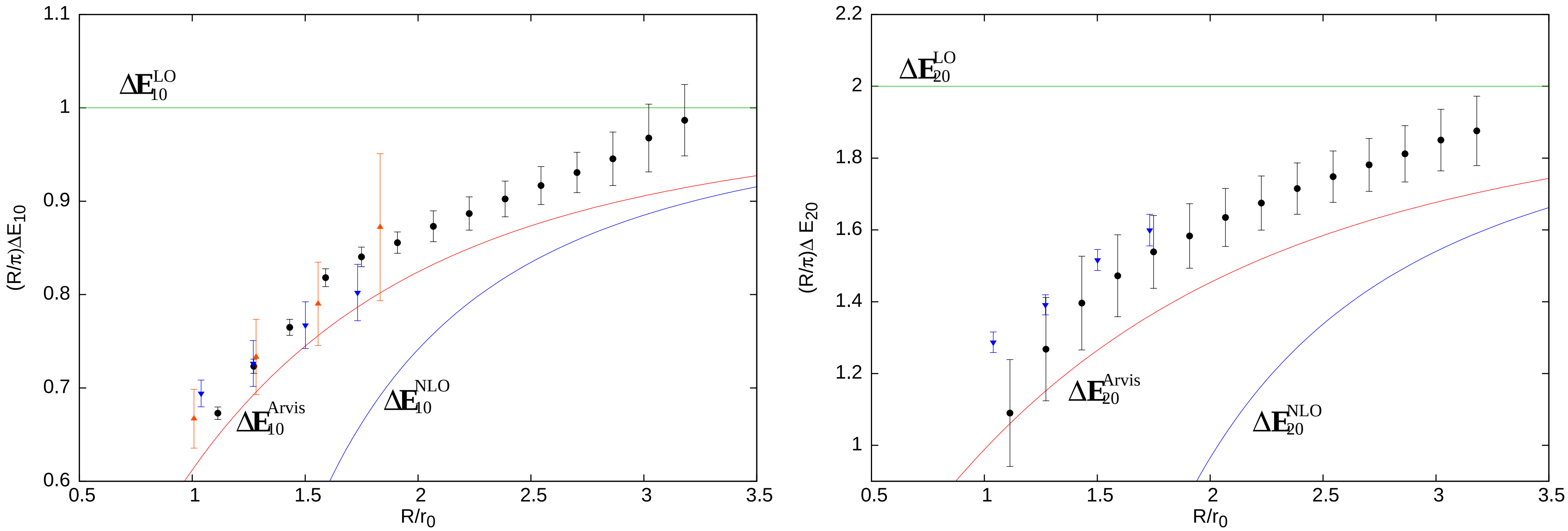
Plot shows E_2 values using source (a) and (b). (b) values coincide with corrected (a) values, but have lower error bars.

β	r_0	$a\,[fm]$	R	$\{S_i\}$	T	$T\,[fm]$	t_s	size	N_s	N_t	#meas
7.5	6.288	0.0795	$7-20$	A	6	0.477	4	38^3	36000	1500	4400
					10	0.795		40^3		3000	6468
					14	1.113		42^3		9000	11176
					18	1.431		54^3		18000	6512
10.0	8.660	0.0581	$9-27$	A	8	0.465	6	40^3	48000	3000	1272, 1272, 1296*
					10	0.581	4	50^3		3000	2352, 2544, 2568
					14	0.814	6	56^3		6000	6384, 6216, 6480
					18	1.046	4	54^3		12000	8664, 8304, 8472
10.0	8.660	0.0581	$9-27$	B	6	0.349	4	48^3	48000	1500	2000, 2000**
					8	0.465	6	48^3		3000	2000, 6000
					10	0.581	4	50^3		6000	2000, 7960
					14	0.814	6	56^3		12000	2000, 2000
12.5	10.92	0.0458	$11-29$	A&B	8	0.366	6	48^3	36000	2000	1000
					10	0.458	4	50^3		3000	4000
					14	0.641	6	56^3		6000	7080
					18	0.824	4	72^3		12000	2080

The number of measurements marked with * corresponds to the R values $9-15, 17-21, 23-27$ and the ones marked with ** to $9-15, 17-27$ respectively.



Results for the total energies E_n. The $\bullet$'s are the values for $\beta = 7.5$, $\blacksquare$'s for $\beta = 10.0$ and $\blacklozenge$'s for $\beta = 12.5$. $\blacktriangledown$ are corrected values for $\beta = 10.0$ and $\blacktriangle$ for $\beta = 12.5$. The results have been rescaled, such that $E_n^{\mathrm{LO}} = n$.



Left: Results for the energy difference ΔE_{10}.
Right: Results for the difference ΔE_{20}. The $\bullet$'s are the values for $\beta = 7.5$, while $\blacktriangledown$ are corrected values for $\beta = 10.0$ and $\blacktriangle$ for $\beta = 12.5$.

Deviations from Nambu-Goto predictions at higher orders have been reported by Giudice, Gliozzi and Lottini in JHEP 0903 (2009) 104, arXiv 0901.0748 for gauge duals of random percolation problems.

Conclusions

- For the Lüscher term, the asymptotic value is approached in a non-monotonic way with r.

- Almost impossible to distinguish the type of the string from the force data. Differences are at the level of 0.1% at $2r_0$.

- $c(r)$ suggests that a Nambu-Goto like behaviour is good beyond $2.5r_0$.

- We have found a way to use the multilevel philosophy for the excited states as well.

- We need to use both the multilevel technique as well as improved wave functions to go ahead. We have taken a first step to show how it can be done.