# Heavy Quarks at High Temperature

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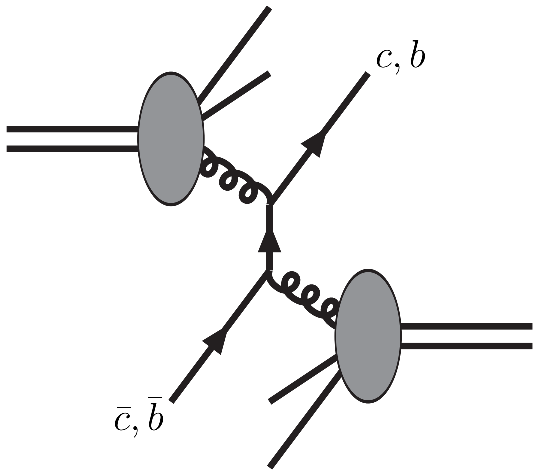
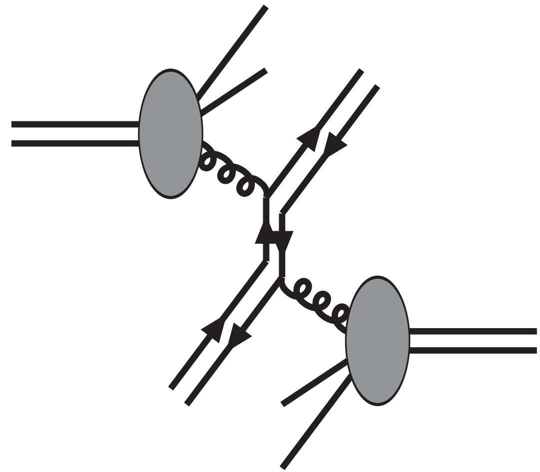
# Initial production



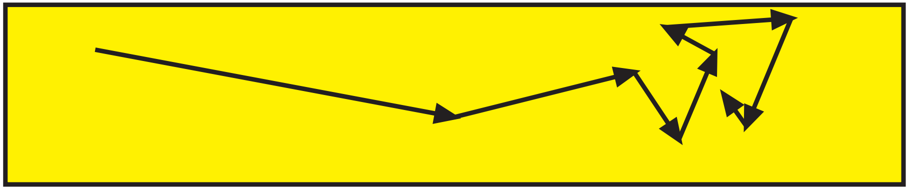
Heavy Quarks

Heavy Quarkonium

Subsequently heavy probes **propagate** through a soft "medium".

In the end they **decay**; heavy quarks often as $c \longrightarrow \ell \nu X$; heavy quarkonium often as $c\bar{c} \longrightarrow \ell^+ \ell^-$; the leptons $\ell$ can be observed.

# Propagation through the medium

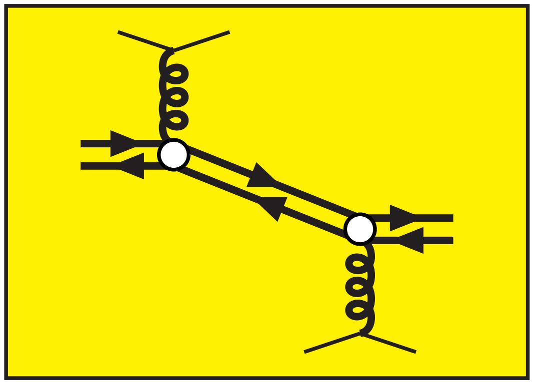


Like heavy particles in Brownian motion, heavy quark jets ("open charm") tend to get stopped ("quenched") by scatterings.

Observables characterizing the stopping are referred to as the momentum diffusion coefficient / the "drag force" / the kinetic thermalization rate.

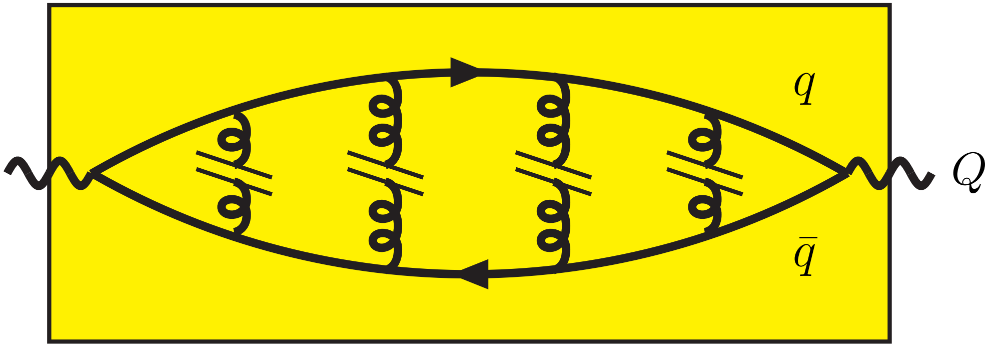
Heavy quarkonium feels less drag than heavy quarks because it has no net colour charge.

However, because of its finite size, it does have a **colour dipole** which leads to kicks and an eventual "decoherence" of its state:



But these effects should be suppressed by $\mathcal{O}(rT)^2$?

On the other hand the Coulomb potential gets Debye-screened within the medium:




So the effective $r$ could be larger than at $T = 0$, and the thermal effects on quarkonium propagation significant.

These effects make themselves visible in the shape of the quarkonium **spectral function**, which determines the thermal component in the dilepton production rate.

**To keep in mind:**

Propagation takes place in Minkowskian time ($t$), with a Minkowskian frequency ($\omega$), but within a thermal system ($\beta = 1/T$).

**Formally:**

$$\left\langle \frac{1}{2}[\hat{\mathcal{J}}^{\mu}(t,\mathbf{x}), \hat{\mathcal{J}}_{\mu}(0,\mathbf{0})] \right\rangle \ ,$$

$$\hat{\mathcal{J}}^{\mu}(t,\mathbf{x}) = e^{i\hat{H}t}\hat{\mathcal{J}}^{\mu}(0,\mathbf{x})e^{-i\hat{H}t} \ ,$$

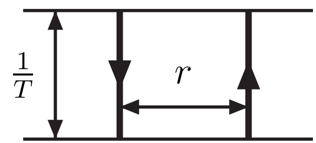
$$\langle \ldots \rangle \equiv \frac{1}{\mathcal{Z}} \mathrm{Tr}\left[ (\ldots) e^{-\beta\hat{H}} \right] \ .$$

These are technically challenging observables!

# I. To which extent is the system perturbative?

A popular lattice observable (having perhaps something to do with heavy quarkonium):

$$\psi_{\mathrm{C}}(r) \equiv \frac{1}{N_{\mathrm{c}}} \langle \mathrm{Tr}[P_{\mathbf{r}} P_{\mathbf{0}}^{\dagger}] \rangle_{\text{Coulomb}} \; .$$




No real time here!

(Gauge invariant alternatives, $\psi_{\mathrm{W}}(r) \equiv \frac{1}{N_{\mathrm{c}}} \langle \mathrm{Tr}[P_{\mathbf{r}} W_0 P_{\mathbf{0}}^{\dagger} W_0^{\dagger}] \rangle$ or $\psi_{\mathrm{T}}(r) \equiv \frac{1}{N_{\mathrm{c}}^2} \langle \mathrm{Tr}[P_{\mathbf{r}}] \, \mathrm{Tr}[P_{\mathbf{0}}^{\dagger}] \rangle$, do **not** reduce to the known zero-temperature static potential at short distances.)

Perturbative expression recently worked out up to $\mathcal{O}(\alpha_s^2)$:

Burnier et al 0911.3480

$$\ln\left(\frac{\psi_\mathrm{C}(r)}{|\psi_\mathrm{P}|^2}\right) \approx \frac{g^2 C_F \exp(-m_\mathrm{E} r)}{4\pi T r}\left\{1 + \frac{g^2}{(4\pi)^2}\left[\frac{11 N_\mathrm{c}}{3}\Big(L_\mathrm{b} + 1\Big) - \frac{2N_\mathrm{f}}{3}\Big(L_\mathrm{f} - 1\Big)\right]\right\}$$

$$+ \frac{g^4 C_F N_\mathrm{c} \exp(-m_\mathrm{E} r)}{(4\pi)^2}\Big[2 - \ln(2 m_\mathrm{E} r) - \gamma_\mathrm{E} + e^{2 m_\mathrm{E} r} E_1(2 m_\mathrm{E} r)\Big] - \frac{g^4 C_F N_\mathrm{c}}{(4\pi)^2}\frac{\exp(-2 m_\mathrm{E} r)}{8 T^2 r^2}$$

$$+ \frac{g^4 C_F N_\mathrm{c}}{(4\pi)^2}\left[\frac{1}{12 T^2 r^2} + \frac{\mathrm{Li}_2(e^{-4\pi T r})}{(2\pi T r)^2} + \frac{1}{\pi T r}\int_1^\infty \mathrm{d}x \left(\frac{1}{x^2} - \frac{1}{2x^4}\right)\ln\Big(1 - e^{-4\pi T r x}\Big)\right]$$

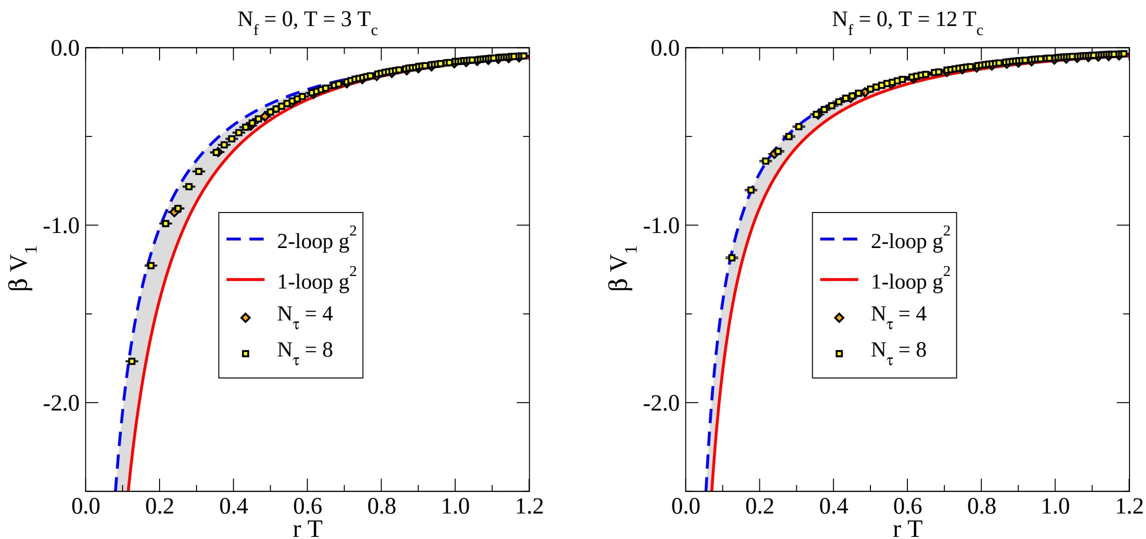
$$+ \frac{g^4 C_F N_\mathrm{f}}{(4\pi)^2}\left[\frac{1}{2\pi T r}\int_1^\infty \mathrm{d}x \left(\frac{1}{x^2} - \frac{1}{x^4}\right)\ln\frac{1 + e^{-2\pi T r x}}{1 - e^{-2\pi T r x}}\right] + \mathcal{O}(g^5)\ ,$$

For $r \ll \frac{1}{\pi T}$, this reproduces the classic $T = 0$ static potential.

Fischler NPB 129 (1977) 157

# Comparison with lattice $[\beta V_1 \equiv -\ln(\psi_{\rm C}/|\psi_{\rm P}|^2)]$:

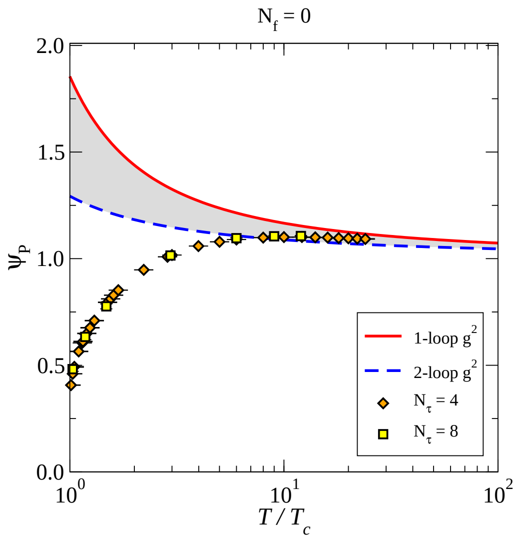
data from Kaczmarek et al hep-lat/0207002




So, somewhat surprisingly, we find reasonable qualitative agreement even at low temperatures ($T_{\rm c} \approx 200$ MeV).

(Match is often worse for observables with no perturbative $T = 0$ part, like the expectation value of a single Polyakov loop.)

$$\psi_{\mathrm{P}} = \frac{1}{N_{\mathrm{c}}}\langle \mathrm{Tr}\,[P_{\mathbf{r}}]\rangle = 1 + \frac{g^2 C_F m_{\mathrm{E}}}{8\pi T} + \frac{g^4 C_F}{(4\pi)^2}\left[N_{\mathrm{f}}\left(-\frac{\ln 2}{2}\right) + N_{\mathrm{c}}\left(\ln\frac{m_{\mathrm{E}}}{T} + \frac{1}{4}\right)\right] + \mathcal{O}(g^5)\ .$$




data from Gupta et al 0711.2251

## II. How to really address heavy quarkonium?

In order to get a handle on the many scales appearing, both vacuum and thermal, need to make use of **effective field theories**.

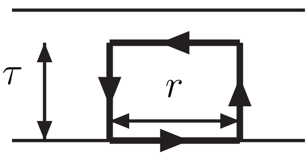
For quarkonium, the relevant framework is that of **NRQCD** (Non-Relativistic QCD) or one of its descendants (pNRQCD etc).

In such frameworks, various heavy quark potentials appear as **matching coefficients**, and can be given a concrete definition (at least within perturbation theory, which now appears reasonable).

in the thermal context: ML et al hep-ph/0611300; Beraudo et al 0712.4394;
Escobedo Soto 0804.0691; Brambilla et al 0804.0993

It is important to keep in mind that the Euclidean $\beta = 1/T$ is "small", while the Minkowskian $t$ is "large" in the "static" limit.

$\Rightarrow$ define a potential from analytic continuation:




$$C_E(\tau, r) \equiv \langle \mathrm{Tr}[W_E(\tau, r)] \rangle \ ,$$
$$i\partial_t C_E(it, r) \equiv V_>(t, r) C_E(it, r) \ .$$

The static limit $V_>(\infty, r)$ through **spectral analysis**.

Hatsuda Sasaki Rothkopf 0910.2321

Position of the spectral peak: average energy, $\mathrm{Re}\, V_>(\infty, r)$.
Its width: decoherence, $\mathrm{Im}\, V_>(\infty, r)$.

Explicitly at $\mathcal{O}(\alpha_s)$:

$$\mathrm{Re}\, V_>(\infty, r) = -\frac{g^2 C_F}{4\pi} \left[ m_{\mathsf{E}} + \frac{\exp(-m_{\mathsf{E}} r)}{r} \right] ,$$

$$\mathrm{Im}\, V_>(\infty, r) = -\frac{g^2 T C_F}{4\pi} \phi(m_{\mathsf{E}} r) ,$$

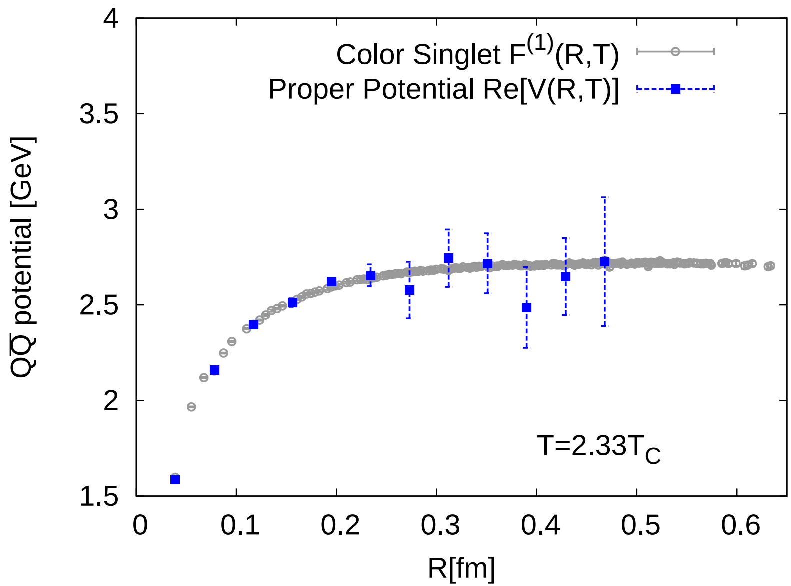
where $m_{\mathsf{E}} \sim gT$ is the Debye mass, $C_F \equiv 4/3$, and

$$\phi(x) = 2 \int_0^\infty \frac{\mathrm{d}z\, z}{(z^2+1)^2} \left[ 1 - \frac{\sin(zx)}{zx} \right] .$$

At $m_{\mathsf{E}} r \ll 1$, $\phi \sim (m_{\mathsf{E}} r)^2$, as is appropriate for a dipole.

# Numerical test for the **real part**:

Hatsuda Sasaki Rothkopf 0910.2321




Low and behold, it does appear to agree with the Coulomb gauge potential! But more precision required for definite conclusions.

## III. How to address single heavy quarks?

In this case the relevant effective framework is that of **HQET** (Heavy Quark Effective Theory).

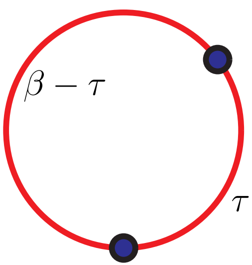
Taking further steps within that theory, the heavy quark kinetic thermalization rate can further be reduced to a **purely gluonic observable** — in analogy with the reduction of heavy quarkonium properties to static potentials within NRQCD!

More precisely, the thermalization rate, $\eta_D$, can be fluctuation-dissipation-related to a force-force transport coefficient, $\kappa$:

$$\eta_D = \frac{\kappa}{2M_{\text{kin}}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{\text{kin}}}\right)\right) \;, \quad \kappa = \lim_{\omega\to 0}\frac{2T\rho_E(\omega)}{\omega} \;.$$

Here $\rho_E$ is the spectral function corresponding to the Euclidean correlator

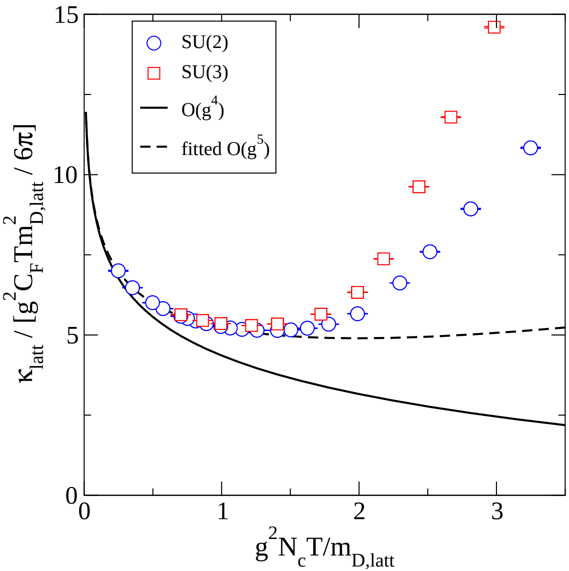
$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re\,Tr}[U_{\beta;\tau}\, gE_i(\tau,\mathbf{0})\, U_{\tau;0}\, gE_i(0,\mathbf{0})]\rangle}{\langle \mathrm{Re\,Tr}[U_{\beta;0}]\rangle} \;.$$




Casalderrey-Solana Teaney hep-ph/0605199;

Caron-Huot ML Moore 0901.1195

So far $\kappa$ has only been measured within "classical lattice gauge theory", but the result is interesting if plotted in terms of a quantity having an analogue in QCD, $m^2_{\text{D,latt}} \sim g^2 T/a$:




$O(g^5)$ in QCD: Caron-Huot Moore, 0708.4232

data: ML Moore Philipsen Tassler, 0902.2856

$\Rightarrow$ the kinetic thermalization rate could be unexpectedly large — as appears to be required by phenomenology.

e.g. Akamatsu et al 0809.1499

## Conclusions

Effective field theory techniques allow to reduce heavy quark related “transport coefficients” to purely gluonic observables.

The convergence of the weak-coupling expansion depends on the observable, but it can in any case yield useful theoretical insights, like those leading to the concept of a “proper” static potential.