Classical Mechanics, Autumn 2017

Assignment #4, Due 7/11/2017 in class

1. Consider a double pendulum constructed with equal masses m and massless rods of equal lengths ℓ. Find the normal modes and their frequencies. Animate the oscillatory modes in Mathematica. What should be the initial conditions such that the oscillations of the double pendulum are periodic with the shortest period ?

2. Consider a water molecule, where the two O-H bonds have an angle of β between them at equilibrium. Model this system as three masses m, m, M joined by two springs of spring constant k. Find the normal modes of oscillations and their corresponding frequencies, by considering small oscillations in all three dimensions. Animate these oscillatory motions in Mathematica. Account for all the degrees of freedom in terms of number of oscillatory modes and number of non-oscillatory degrees of motion.

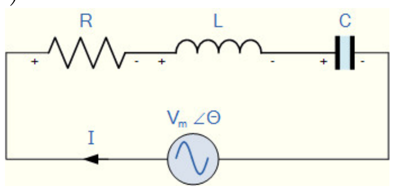
3. Consider a 1D harmonic oscillator with natural frequency ω_0, on which a constant external force F acts for a time τ. Find
   (a) The time dependence of the generalized coordinate x
   (b) The net work done by the external force
   (c) Interpret the above results in the casess $\omega_0\tau \ll 1$, $\omega_0\tau \sim 1$, $\omega_0\tau \gg 1$. In all the three cases, plot $x(t)$ for appropriate values of parameters and initial conditions.
   (d) Plot $Max[x(t)]$ vs. $\omega_0\tau$ and interprete its features in terms of the results above.

4. A particle of mass m moves under the influence of a central force

$$\vec{F} = -\frac{k}{r^2}\left(1 - \frac{\alpha}{r}\right)\hat{r} \ .$$

   For $\alpha = 0$, we know the solution $r(\theta)$ or $u(\theta)$. (The latter would be easier to use in this problem.) Now take α to be small, define an appropriate small perturbation parameter, and determine the first order correction to the period of rotation. Hence estimate the precession of the orbit during one revolution. Comment on the first order corrections to $u(\theta)$.

5. Consider the electrical circuit given in the figure. The voltage source is $V = V_0 \cos(\omega_{\text{ext}} t)$.



(a) Choose appropriate generalized coordinates and describe the system in a Lagrangian framework. Write down the E-L equations of motion.

(b) Solve the E-L equations of motion and analytically find out the values of all resonant frequencies and amplitudes of the resonant oscillations.

(c) Choose appropriate values of parameters, and and plot $V_R(t)$, $V_L(t)$ and $V_C(t)$ (i.e. the voltages across the R, L, and C, respectively) on the same plot. Do this for three frequencies ω_{ext}: close to one of the resonances, smaller than any resonance one for $\omega_{\text{ext}} \approx 0$, and one for $\omega_{\text{ext}} \to \infty$. Comment on your observations.

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