

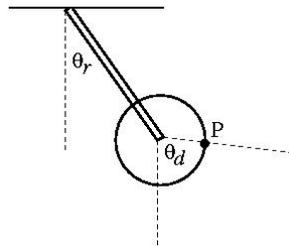
Classical Mechanics (Autumn 2017)

Midterm exam, 14/10/2017

Maximum marks: 100

1. [Total marks: 30]

The “Reaction wheel pendulum” consists of a system where (i) a uniform rod of mass M_r and length L_r is attached at one of its ends with a pivot, and (ii) a uniform disk of mass M_d and radius R_d is attached at the other end of the rod, as shown in the figure.



The disk is free to rotate about its central axis without friction. Take the generalized coordinates θ_r and θ_d as shown in the figure. (Do not assume small oscillations.) P is a fixed reference point on the disk.

- Write down the Lagrangian of the system in terms of the quantities defined above. [5]
- What are the generalized forces Q_r and Q_d along the above coordinates? Write down the Euler-Lagrange equations of motion. Identify any cyclic coordinates and the conservation laws they lead to. [10]
- If the disk is now fixed such that it cannot rotate with respect to the rod any more, write down the resulting constraint on the two generalized velocities. Using the method of Lagrange multipliers, write down the equations of motion for θ_r and θ_d . [5]
- Determine the generalized constraint forces $Q_r^{(c)}$ and $Q_d^{(c)}$ on the rod and the disk, respectively. Give an interpretation for these forces in the Newtonian language. How much work is done by $Q_r^{(c)}$ on the rod while it swings from one extreme to the other (half oscillation) ? [10]

2. [Total marks: 25]

Interactions of charged particles with the electromagnetic scalar potential ϕ and vector potential \vec{A} may be represented by the Lagrangian

$$L = \frac{1}{2}m\vec{v}^2 - e\phi + \frac{e}{c}\vec{A} \cdot \vec{v} .$$

- (a) Determine the generalized momentum \vec{p} and write down the Hamiltonian H . Hence write down Hamilton's equations of motion in terms of \vec{A} and ϕ . [10]
- (b) A charged particle is moving in a circle of radius a in the x - y plane, in the presence of a uniform magnetic field \vec{B} . Choose an appropriate vector potential of the form $\vec{A} = A_x\hat{x}$. Qualitatively sketch the trajectories of the above particle in the $(x-p_x)$ plane and $(y-p_y)$ plane, where \vec{p} is the generalized momentum. [10]
- (c) What is the dimension of the $(\vec{x}-\vec{p})$ phase space of the particle? How many quantities are constants of motion along all the above trajectories? Identify them and write down their values in terms of components of $\vec{x}, \vec{p}, \vec{A}$. [5]

3. [Total marks: 10]

The Rutherford differential cross section, for the scattering of alpha particles over gold nuclei, is calculated to be

$$\frac{d\sigma}{d\cos\Theta}(E) = \left(\frac{k}{4E}\right)^2 \frac{1}{\sin^4(\Theta/2)} .$$

Here Θ is to be measured in the centre-of-mass frame of the two scattering particles. However actually the gold nuclei are initially stationary. The measured angle is therefore α , which is different from Θ .

Using the expression for $\frac{d\sigma}{d\cos\Theta}$ above, determine the differential cross section $\frac{d\sigma}{d\cos\alpha}$. The atomic number of gold is 79 and its mass number is 197. [10]

4. [Total marks: 10]

Let $V(r)$ be a potential that gives rise to an attractive force.

- (a) Determine the conditions for the existence of a stable circular orbit at a radius r_0 , in terms of only r_0 and the derivatives of $V(r)$. [5]
- (b) Hence, if the potential is of the form $V(r) = ar^{(n+1)}$, find the condition on n for the existence of such a stable circular orbit. [5]

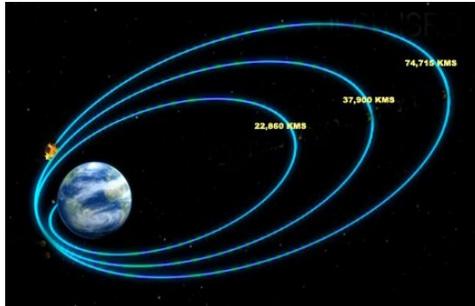
5. [Total marks: 25]

The mass Orbiter Mission (MOM) initially orbited the Earth in an ellipse described by

$$\frac{1}{r} = u = \frac{mk}{\ell^2} (1 + \epsilon \cos \theta) ,$$

where the eccentricity $\epsilon = \sqrt{1 + \frac{2E\ell^2}{mk^2}}$.

- (a) Find the ratio of major and minor axis of the orbit [5]
- (b) Find the ratio of maximum and minimum speeds of the MOM. [5]



- (c) In every “orbit raising” maneuver, the MOM was given a short forward impulse ΔP when it was at its perigee (closest to the Earth), by firing thruster rockets. Determine the new values of E and ℓ . [10]
- (d) Find the ratio of the new apogee (farthest distance from the Earth) to the old apogee, and the ratio of new revolution time to old revolution time of the MOM around the Earth. (All answers in terms of ΔP .) [5]