Classical Mechanics, Autumn 2019

Assignment #1, Due 10/09/2019

1. Consider a double pendulum constructed with equal masses m and massless rods of equal lengths ℓ. Take any appropriate values of m and ℓ in arbitrary units. Set up and solve the equations of motions (numerically is OK) with initial conditions of your choice. Plot the trajectories of the two masses in $(\theta_i, \dot{\theta}_i)$ space. (Separate diagrams for the two masses). Comment on the features of these solutions. Can you choose initial conditions such that the trajectories look simple ?

2. Consider a ladder of length L sliding against a wall, without friction. Write down the Lagrangian and the corresponding equations of motion. Solve the equations numerically, and plot the trajectory of the centre of mass in the (x, y) plane, for $\theta_{\text{initial}} = 30^\circ, 60^\circ$ (both in the same figure).

3. Goldstein Chapter 1, Exercise 8

4. Goldstein Chapter 1, Exercise 13. Give a possible interpretation of the result.

5. Consider a rocket that starts moving upwards with an initial speed v_0. It then keeps on discarding mass at a constant rate $dm/dt = R$, till its upward motion stops. Then it stops discarding its mass, and falls down under gravity. Set up the Lagrangian and equations of motion, including effects due to gravity and air drag ($\vec{F} = -\alpha v^2 \hat{v}$) in both its upward and downward motion, Solve the equations and plot (x vs. t) and (v vs t) by choosing appropriate values of quantities.

6. The Lagrangian for a particle with mass m and charge q may be written as
$$L = \frac{1}{2}mv^2 - q\phi + \frac{q}{c}\vec{A} \cdot \vec{v} \ .$$
Derive the equations of motion for the charged particle, in terms of the electric and magnetic fields $\vec{E}$ and $\vec{B}$.